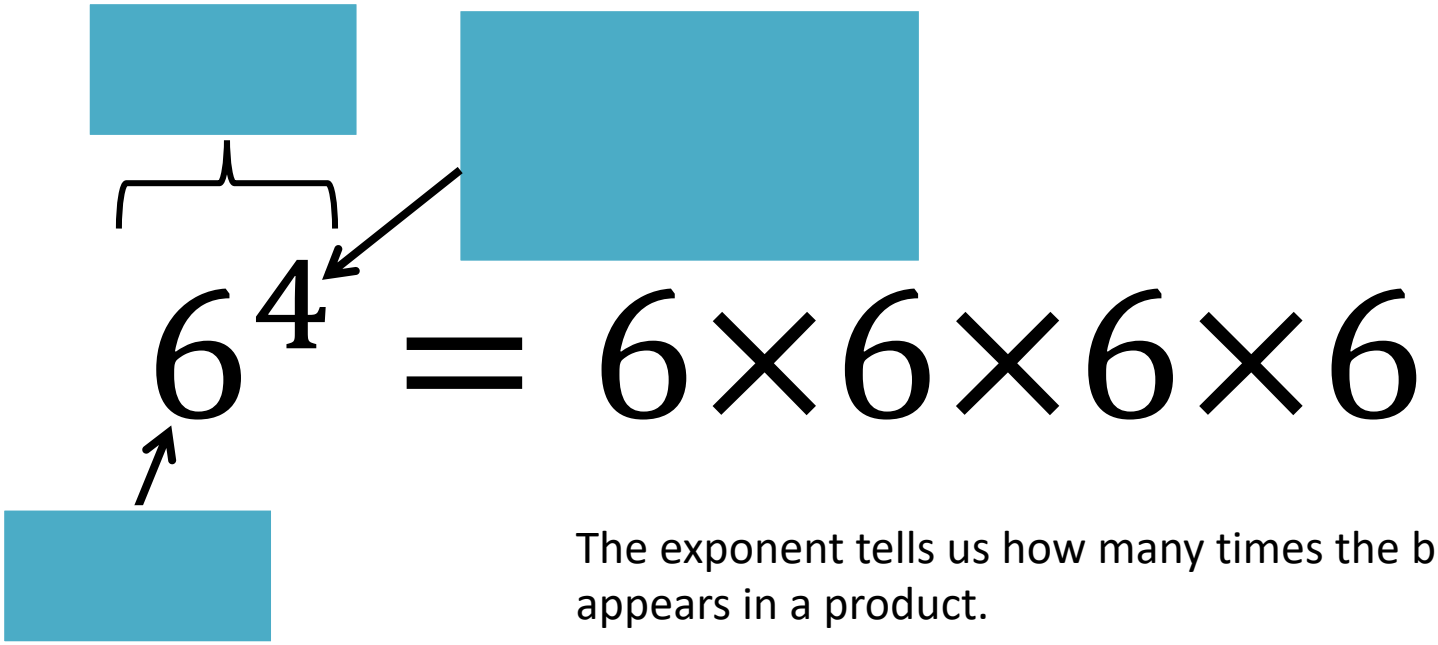
Laws of Indices

Terminology



The exponent tells us how many times the base appears in a product.

Overviews:

Multiplying powers

How would we simplify this?

$$x^3 \times x^2$$

1st law of indices:

$$x^a \times x^b = x^{a+b}$$

i.e. when we multiply two powers, we add the exponents.

$$x^5 \times x^4 =$$

$$x^2 \times x^3 \times x^4 =$$

$$x \times x^2 \times x^9 =$$

$$y^k \times y^2 =$$

$$q \times q^a =$$

How would we simplify this?

$$\frac{x^5}{x^3}$$

2nd law of indices:

$$x^a \div x^b = x^{a-b}$$

i.e. when we divide two powers, we subtract the exponents.

$$2^{100} \div 2^{2} =$$

$$\frac{x^{10}}{x^{3}} =$$

$$\frac{y^{20}}{y^{-1}} =$$

$$x^{15} \div x =$$

$$\frac{x^{3} \times x^{3}}{x^{2}} =$$

Raising a Power to a Power

How would we simplify this?

$$(x^3)^4 = \\ = $$

3rd law of indices:

$$(x^a)^b = x^{ab}$$

i.e. when we raise a power to a power, we multiply the exponents.

$(y^3)^5 =$

$(p^7)^8 =$

$(m^{10})^4 =$

$(a^{-b})^{-c} =$

1 What is half of 2^7?

2 What is a quarter of 4^x?

3 $4^x + 4^x + 4^x + 4^x = 4^{16}$ What is x?

Exercise

1 Simplify the following.

a $\left(\frac{x^{2y}}{x^{y}}\right)^{3} =$

b $x \times (x \times (x^{3})^{2})^{2}$

=

2 If $\frac{2^{x} \times 2^{y}}{2^{3}} = 2^{7}$, what is x in terms of y?

3 If $x = 2^p$, $y = 2^q$, express the following in terms of x and/or y:

(i) $2^{p+q} =$

(ii) $2^{2q} =$

(iii) $2^{p-1} =$

4 Simplify the following.

a) $3^y + 3^y + 3^y =$

b) $2^{2y} + 2^{2y} =$

5 Solve $\frac{(2^x)^5}{2^3} = \frac{2}{(2^4)^x}$

3^0 3^{-1}

Is there a pattern we can see that will help us out?

$3^3 = 27$ ÷ 3

$3^2 = 9$ ÷ 3

$3^1 = 3$ ÷ 3

$3^0 =$

$3^{-1} =$

$3^{-2} =$

$2^{-3} =$

$3^{0} =$

$4^{-1} =$

$5^{-3} =$

$\left(\frac{1}{2}\right)^{-1} =$

$\left(\frac{3}{5}\right)^{-2} =$

$\left(\frac{2}{3}\right)^{-2} =$

Exercise

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Let $x = 2^p$, $y = 2^q$,

If xy = 32 and $2xy^2 = 32$, find the value of p and the value of q .

A reminder of the Laws of Indices

$a^b \times a^c =$

$\frac{a^b}{a^c} =$

$(a^b)^c =$

$a^{-b} =$

$a^0 =$

$a^1 =$

$$x^{\frac{1}{2}} = \sqrt{x}$$

And how could we prove this?

$$x^{\frac{1}{3}} =$$

$$x^{\frac{1}{n}} =$$

Examples

$64^{\frac{1}{2}} =$

$64^{\frac{1}{3}} =$

$81^{\frac{1}{4}} =$

$16^{0.25} =$

$(\sqrt[3]{x})^2 =$

$(-1000)^{\frac{1}{3}} =$

What if the numerator is not 1?

$$9^{\frac{3}{2}} =$$

$$32^{\frac{2}{5}} =$$

$$16^{-\frac{3}{4}} =$$

$$27^{\frac{2}{3}} =$$

$$9^{-\frac{3}{2}} =$$

$$\left(\frac{125}{64}\right)^{-\frac{1}{3}} =$$

$$\left(\frac{4}{9}\right)^{-\frac{3}{2}} =$$

$$(ab)^2 =$$

$$(a+b)^2 =$$

$$=$$

$$(2x)^2 =$$

$$(3x^2y)^3 =$$

$$(9x^6)^{\frac{1}{2}} =$$

$$(8x^6y)^{\frac{1}{3}} =$$

Simplify $(3x^2y^3)^2$

Simplify $(9x^4y)^{\frac{1}{2}}$

Law of Indices Backwards

Solve $x^{\frac{3}{4}} = 27$

The ‘thinking backwards’ method

The ‘cancelling the power’ method.

Further Examples

Solve $x^{-\frac{2}{3}} = 2\frac{7}{9}$

Solve $y^{-3} = 3\frac{3}{8}$

Solve $x^{\frac{2}{3}} = 9$

Solve $x^{-\frac{3}{2}} = \frac{8}{27}$

1

$x^{\frac{3}{2}} = 8$ and $y^{-2} = \frac{25}{4}$.

Work out the value of $\frac{x}{y}$

2

$p^{-2} = q^6 \times r^4$

Write p in terms of q and r.

Give your answer in its simplest form.

What do you notice about all of the numbers:

$$2, 8, 4$$

Changing bases

1 Solve $4^x = 2^{10}$

2 Solve $2^x = \frac{8^3}{\sqrt{2}}$

1 If $9\sqrt{3} = 3^k$, find k.

2 Solve $3^x = 9^{2x-1}$

1

$$16^{\frac{1}{5}} \times 2^{x} = 8^{\frac{3}{4}}$$

Work out the exact value of x.

2

Solve

$$\sqrt[3]{9} \times \sqrt[4]{27} = \sqrt[x]{3}$$