

Factorising Quadratics



1: Factorising out a common bracket TOP Maths IGCSE

Sometimes the **common factor might be an entire bracket!**

The principle remains the same...

$$(x + 3)^2 + 2(x + 3)$$

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In this particular case, it may be easier to expand everything out first before 'refactorising':

$$x^2 + 6x + 9 + 2x + 6$$

Factorise $(p - q)^2 + (p + 3)(p - q)$

Factorise the following:

$$(x + y)^2 + (x + y)(2x + 5y)$$

Factorise $5(x + 2)^2 + 3(x + 2)$

1 Factorise fully
 $(x + 3)^2 + 4(x + 3)$

3 Factorise fully
 $(x + 4)^3 + 2(x + 4)^2$

2 Factorise $(2y + 1)^2 + (x + y)(2y + 1)$

4 Factorise fully
 $2(x + y)^{10} + (x - y)(x + y)^9$

Seven different types of factorisation

1. Factoring out a single term

$$2x^2 + 4x = 2x(x + 2)$$

2. $x^2 + bx + c$

$$x^2 + 4x - 5 = (x + 5)(x - 1)$$

3. Difference of two squares

$$4x^2 - 1 = (2x + 1)(2x - 1)$$

4. $ax^2 + bx + c$

$$2x^2 + x - 3 = (2x + 3)(x - 1)$$

5. Combining techniques

$$\begin{aligned}x^3 - x &= x(x^2 - 1) \\ &= x(x + 1)(x - 1)\end{aligned}$$

6. Intelligent Guesswork

$$\begin{aligned}x^2 + y^2 + 2xy + x + y \\ &= (x + y + 1)(x + y)\end{aligned}$$


7. "Pairwise"

$$\begin{aligned}x^3 + 2x^2 - x - 2 \\ &= x^2(x + 2) - 1(x + 2) \\ &= (x^2 - 1)(x + 2) = \dots\end{aligned}$$

Patterns when expanding brackets

Expand the following brackets. Do you notice anything about the relationship between the coefficient of x and constant term, and the numbers in the original brackets?

i.e. How could we get these numbers instantly from the 2 and the 3 in the brackets?

$$(x + 2)(x + 3) = x^2 + 5x + 6$$


$$(x + 4)(x + 2) =$$

$$(x + 7)(x + 5) =$$

$$(x + 8)(x - 3) =$$

The term in the middle term (“the coefficient of x ”) is the **sum** of the numbers in the two brackets.

The third term (the constant term) is the **product** of the numbers in the two brackets.

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

a and b add
to give 3.

a and b
times to
give 2.

How does this suggest we can factorise say $x^2 + 3x + 2$?

$$x^2 + 3x + 2 =$$

$$x^2 + 11x + 30 =$$

/ \
30 1
15 2
10 3
6 5

$$x^2 - 2x - 24 = (\quad)(\quad)$$

/	\
1	24
2	12
3	8
4	6

$$x^2 - 5x + 6 = (\quad)(\quad)$$

1 $x^2 + 6x + 5 =$

2 $x^2 + 5x - 14 =$

3 $x^2 - 6x + 9 =$

1 $x^2 + 4x + 3 =$

2 $y^2 - y - 56 =$

3 $x^2 - 14x + 49 =$

Challenge Problems

a $x^4 - 6abx^2 + 9a^2b^2$

b $x^{11} + 2x^9 + x^7$

Firstly, what is the square root of:

$$\sqrt{4x^2} =$$

$$\sqrt{25y^2} =$$

$$\sqrt{16x^2y^2} =$$

$$\sqrt{x^4y^4} =$$

$$\sqrt{9(z-6)^2} =$$

3: Difference of two squares

We have '**the difference of two squares**' when, unsurprisingly, we have two 'square' terms, and the **difference** between them!

$$4x^2 - 9$$

Always start with two brackets, one with a +, one with a -

$$= (\quad + \quad) (\quad - \quad)$$

$$1 - x^2 =$$

$$y^2 - 16 =$$

$$x^2y^2 - 9a^2 =$$

$$25 - p^2 =$$

$$4x^2 - 9y^2 =$$

Factorise:

$$x^2 - 36 =$$

$$y^2 - 49 =$$

$$4 - 9x^2 =$$

$$1 - 4p^2q^2 =$$

$$(x + 1)^2 - (x - 1)^2 =$$

$$49 - (1 - x)^2 =$$

$$51^2 - 49^2 =$$

$$(2t + 1)^2 - 9(t - 6)^2 =$$

$$1 \quad 4p^2 - 1 =$$

$$2 \quad 1 - 9p^2q^2 =$$

$$3 \quad p^6 - 1 =$$

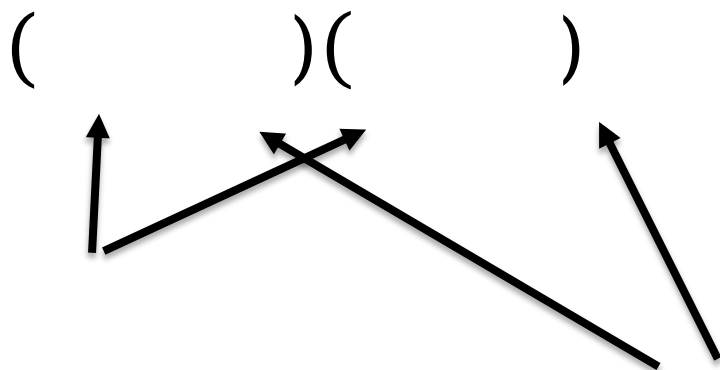
$$4 \quad 50(2x + 1)^2 - 18(1 - x)^2 =$$

$$5 \quad 32x^8 - 162 =$$

$$2x^2 + x - 3$$

Factorise using:

a. 'Going commando'



b. Splitting the middle term

$$2x^2 + x - 3 \quad \begin{array}{l} \oplus 1 \\ \otimes -6 \end{array}$$

The diagram shows the quadratic expression $2x^2 + x - 3$ with arrows pointing from the middle term x to two blank lines below. To the right, a small table shows the factors of -6 that sum to 1: $\oplus 1$ and $\otimes -6$.

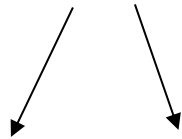
$$12x^2 + 17x - 5$$



$$\begin{array}{r} \oplus 17 \\ \otimes -60 \\ \quad / \quad \backslash \\ 60 \quad 1 \\ 30 \quad 2 \\ 20 \quad 3 \end{array}$$

= 

$$4x^2 - 9x - 9$$



$$\begin{array}{r} \oplus -9 \\ \otimes -36 \\ \quad / \quad \backslash \\ 36 \quad 1 \\ 18 \quad 2 \\ 12 \quad 3 \end{array}$$

=

$$2x^2 + 11x + 12 =$$

$$6x^2 - 7x - 3 =$$

For this one splitting the middle term is difficult! Use 'intelligent guessing' of the two brackets.

$$2x^2 - 5xy + 3y^2 =$$

1 $2x^2 + 3x + 1 =$

2 $2x^2 + x - 15 =$

3 $15y^2 - 13y - 20 =$

4 $25y^2 - 20y + 4 =$

5: Combining Factorisation Techniques

$$x^3 - x =$$

$$2x^2 - 50 =$$

$$x^3 + 3x^2 + 2x =$$

1 $8x^2 - 2 =$

2 $x^4 - x^3 - 6x^2 =$

3 $x^4 - 1 =$

Factorise the following expressions.

$$1 \quad 3y^2 - 27 =$$

$$2 \quad 2x^3 - 8x =$$

$$3 \quad x^{10} - 4x^9 + 4x^8 =$$

$$4 \quad x^8 - 1 =$$

$$x^2 + ax + bx + ab$$

$$=$$

$$ab - a + b - 1$$

$$=$$

$$1 \quad xy + 3x - 2y - 6$$

$$2 \quad a^2b - a + ab - 1$$

$$3 \quad x^2 + y^2 + 2xy + xz + yz$$

We saw earlier with splitting the middle term that we can factorise different parts of the expression separately and hope that a common term emerges.

$$\underline{x^2 + ax + bx + ab}$$

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$$\underline{x^3 - 2x^2 - x + 2}$$

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$$\underline{x^2 - y^2 + 4x + 4y}$$

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=

$$\begin{aligned} 1 \quad & x^2 - xy + 2x - 2y \\ & = \\ & = \end{aligned}$$

$$\begin{aligned} 2 \quad & x^3 - 3x^2 - 4x + 12 \\ & = \\ & = \\ & = \end{aligned}$$

$$\begin{aligned} 3 \quad & a^2 + b^2 + 2ab + ac + bc \\ & = \\ & = \end{aligned}$$

$$x^2 + y^2 + 2xy - 1 =$$

$$x^3 + 2x^2 - 9x - 18 =$$

$$xy^2 + 3y^2 + x + 3 =$$

$$xy - x - y + 1 =$$

Factorise the following using either 'pairwise factorisation' or 'intelligent guessing'.

1 $ab - 2a + 4b - 8 =$

2 $x^3 + x^2 + x + 1 =$

3 $x^2 + 2xy + y^2 =$

4 $a^2 - b^2 - 3a + 3b =$