

Solving Quadratic Equations



There are 4 ways in which we can solve quadratic equations.

$$2x^2 - 5x - 3 = 0$$

1 By Factorising

$$\begin{aligned} 2x^2 - 5x - 3 &= 0 \\ (2x + 1)(x - 3) &= 0 \\ x &= -\frac{1}{2} \text{ or } x = 3 \end{aligned}$$

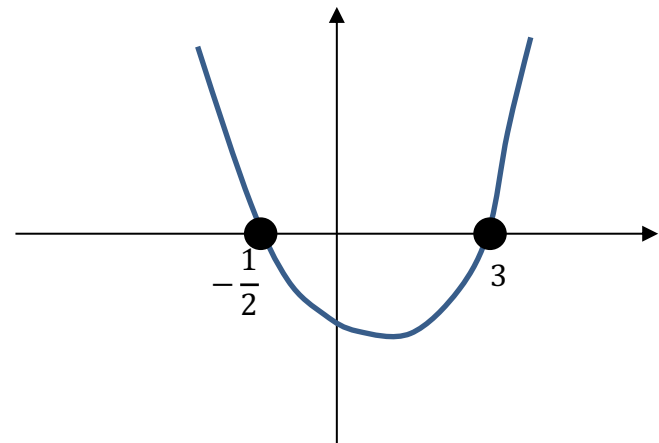
2 By using the Quadratic Formula

$$\begin{aligned} x &= \frac{5 \pm \sqrt{25 - (4 \times 2 \times -3)}}{4} \\ x &= -\frac{1}{2} \text{ or } x = 3 \end{aligned}$$

3 By 'Completing the Square'

$$\begin{aligned} x^2 - \frac{5}{2}x - \frac{3}{2} &= 0 \\ \left(x - \frac{5}{4}\right)^2 - \frac{25}{16} - \frac{3}{2} &= 0 \\ \left(x - \frac{5}{4}\right)^2 &= \frac{49}{16} \\ x - \frac{5}{4} &= \pm \frac{7}{4} \\ x &= -\frac{1}{2} \text{ or } x = 3 \end{aligned}$$

4 Approximating by using a Graph




$$\square \times \square = 0$$

If two things multiply to give 0, what do you know?
At least one of those things must be 0.

Therefore, how could we make this equation true?

$$(x + 3)(x - 2) = 0$$



$x = \square$ **or** $x = \square$

Solving the following.

$$(x - 1)(x + 2) = 0$$



$$x(x - 6) = 0$$



$$(6 - x)(5 + x) = 0$$



$$(2x + 1)(x - 3) = 0$$



$$(3x - 2)(5x + 1) = 0$$



$$(1 - 4x)(3x + 2) = 0$$



We've seen that solving equations is not too difficult when we have it in the form:

$$[\textit{factorised expression}] = 0$$

$$\text{Solve } x^2 + 2x = 15$$

$$x^3 = x$$

Solve the following.

$$x^2 = 4$$

Method 1

Method 2

Solve the following equations.

a $x^6 = 9x^3 - 8$

b $x^3 = x^2 + x - 1$

$$2x(x - 1) = (x+1)^2 - 5$$

$$\text{Solve } \frac{5(2x+1)^2}{4x+5} = 5x-1$$

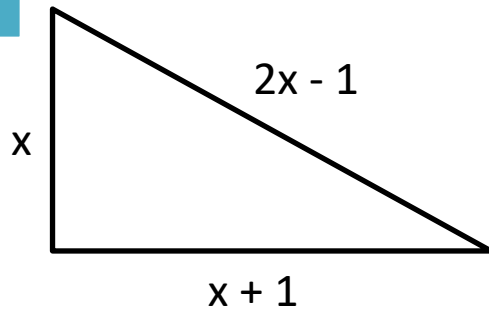
Solve the following equations.

X $10x = 1 + \frac{3}{x}$

Y $4x + \frac{7}{x} = 29$

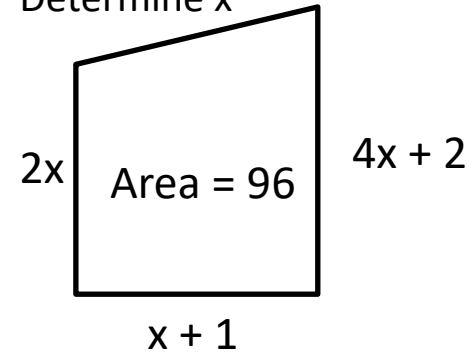
1


Determine x



2

Determine x




 If $ax^2 + bx + c = 0$

Then:

$x =$

Solve $x^2 - 3x - 5 = 0$ giving your answers to 3 significant figures.

 If $ax^2 + bx + c = 0$

Then:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Equation	a	b	c	Solutions (to 3sf)
$x^2 + 5x + 1 = 0$				
$x^2 - x - 1 = 0$				
$4x^2 + 4x + 1 = 0$				
$3 - 5x - x^2 = 0$				
$x^2 + x + 1 = 0$				

A Solve the following. Use exact values.

$$\frac{(2x - 3)^2}{x + 1} = 3$$

B The sides of a rectangle are $3x + 1$ and $4x + 1$. Its area is 40. Determine x .

Express the following in the form $(x + p)^2 + q$

a $x^2 + 5x - 1$

b $x^2 - 9x + 20$

So far the coefficient of the x^2 term has been 1. What if it isn't?

Express $3x^2 + 12x - 6$ in the form $a(x + b)^2 + c$

Express $2 - 4x - 2x^2$ in the form $a - b(x + c)^2$

1. Sketching Parabolas

You can find this in the ‘Sketching Quadratic Equations’ .

Completing the square allows us to find the minimum or maximum point of a curve, and is especially useful for sketching when the quadratic has no ‘roots’.

2. Solving Quadratics

$$x^2 + 2x - 5 = 0$$

$$(x + 1)^2 - 6 = 0$$

$$(x + 1)^2 = 6$$

$$x + 1 = \pm\sqrt{6}$$

$$x = -1 \pm \sqrt{6}$$

Solve the following **by completing the square**, giving your answers **to 3sf**.

$$(x + 3)^2 + x = 10$$

Solve the equation $x^2 + 7x - 18 = 0$ by:

1 Factorising

$$(x + 9)(x - 2) = 0$$

$$x = -9 \text{ or } x = 2$$

2 Using the Quadratic Formula

$$a = 1, b = 7, c = -18$$

$$x = \frac{-7 \pm \sqrt{49 + 72}}{2} = \frac{-7 \pm 11}{2}$$

$$= -9 \text{ or } 2$$

3 Completing the Square

$$\left(x + \frac{7}{2}\right)^2 - \frac{49}{4} - 18 = 0$$

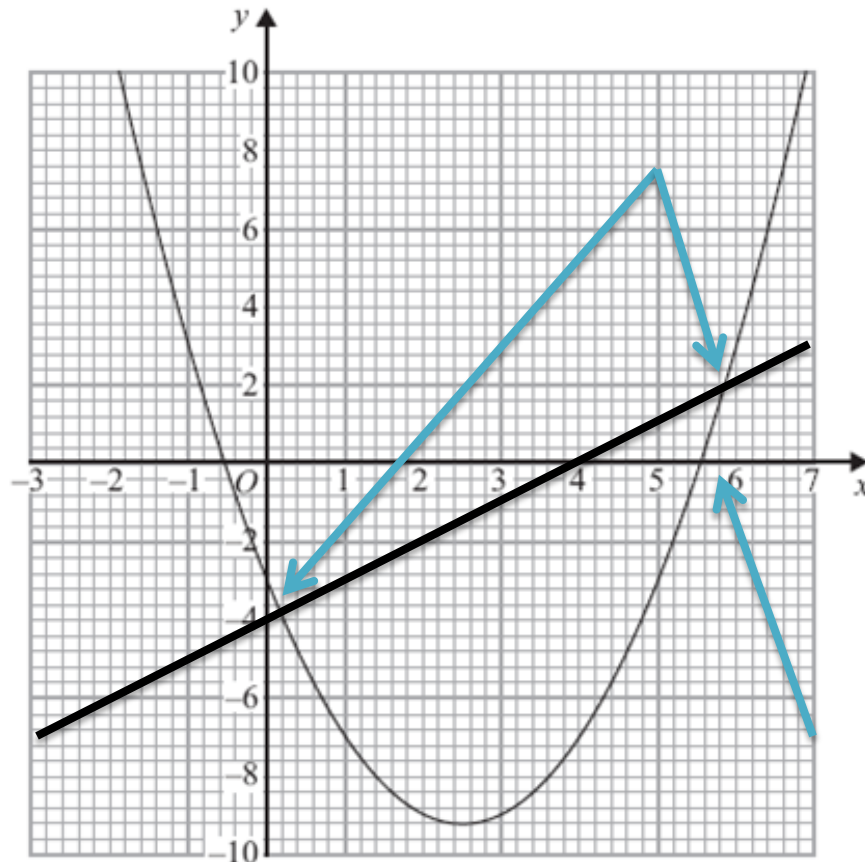
$$\left(x + \frac{7}{2}\right)^2 = \frac{121}{4}$$

$$x + \frac{7}{2} = \pm \frac{11}{2}$$

$$x = \frac{-7 \pm 11}{2}$$

Solving Quadratics by using a Graph - Preview

The diagram shows the graph of $y = x^2 - 5x - 3$



a) Use the graph to find estimates for the solutions of

i. $x^2 - 5x - 3 = 0$

**Accept -0.6 to -0.5 ,
 5.5 to 5.6 .**

ii. $x^2 - 5x - 3 = 6$

$x = -1.4, 6.4$

b) Use the graph to find estimates for the solutions of the simultaneous equations:

$$y = x^2 - 5x - 3$$

$$y = x - 4$$

$x = 0.2, y = -3.8$

$x = 5.8, y = 1.6$