# Algebraic Fractions

Are these algebraic steps correct?

$$\frac{40 - x}{3} = x + 4 \quad \rightarrow \quad \frac{40}{3} = 2x + 4$$

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$$2(4 - 2x) = 3x - 2 \quad \rightarrow \quad 2(4) = 5x - 2$$

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$$\sqrt{2 - x} = 2x + 3 \quad \rightarrow \quad \sqrt{2} = 3x + 3$$

Are these algebraic steps correct?

$$\frac{a^2b}{a+b} \rightarrow \frac{ab}{b}$$

To cancel or not to cancel, that is the question?

$$\frac{y^2 + \cancel{x}}{2 + \cancel{x}}$$

$$\frac{\cancel{s}(4 + z)}{\cancel{s}}$$

$$\sqrt{x^2 + \cancel{2}} = y + \cancel{2}$$

$$\frac{(2x + 1)\cancel{(x - 2)}}{\cancel{x - 2}}$$

$$\frac{\cancel{pq}(r + 2) + 1}{\cancel{pq}}$$

$$\frac{\cancel{1} + r}{2} - \cancel{1}$$

# What did we learn?



You can't add or subtract a term which is 'trapped' inside a bracket, fraction or root.

$$2\sqrt{a-x} = 2x+1$$
$$\rightarrow \quad 2\sqrt{a} = 3x+1 \quad ☹$$

In a fraction, we can only <u>divide top and bottom by something</u>, not add/subtract.

$$\frac{a+b}{c+b} \quad \rightarrow \quad \frac{a}{c} \quad ☹$$

# Simplifying Algebraic Fractions



Just factorise top and bottom, then cancel!

$$\frac{x^2 + x}{x + 1} =$$

$$\frac{2x^2 + 4x}{x^2 - 4} =$$

$$\frac{3x + 3}{x^2 + 3x + 2} =$$

$$\frac{2x^2 - 5x - 3}{6x^3 - 2x^4} =$$

$$\frac{x(x+1)}{x^2-1} \rightarrow$$

$$\frac{2x^2+5x-3}{x^2-9} \rightarrow$$

$$\frac{x^2+2xy+y^2}{2x+2y} \rightarrow$$

$$-(4-y)$$

$$-(2x-9)$$

$$\frac{1-x}{x-1}$$

$$\frac{(3-2x)(2-x)}{(2x-3)(x+1)}$$

# Simplifying Algebraic Fractions

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**a** $\dfrac{(x+10)^5}{(x+10)^4}$

**b** $\dfrac{x^2+2x+1}{x^2+3x+2}$

**c** $\dfrac{6x^2-x-1}{4x^2-1}$

**d** $\dfrac{2y^2+4y}{3y^2+7y+2} \times \dfrac{9y^2-1}{3y^2-y}$

What's our usual approach for adding fractions?

$$\frac{2}{3}+\frac{1}{2}=$$

Questions:

$\frac{2}{5}+\frac{5}{6}=$

$\frac{6}{7}-\frac{2}{3}=$

$\frac{5}{8}+\frac{3}{5}=$

Sometimes we don't need to multiply the denominators. We can find the [redacted] of the denominators.

$$\frac{1}{3} + \frac{1}{9} = [redacted]$$

# Adding/Subtracting **Algebraic** Fractions

The same principle can be applied to algebraic fractions.

$$\frac{x}{3}+\frac{2x+1}{2}=$$

$$\frac{x+1}{2}-\frac{x-1}{3}=$$

$$\frac{1}{x}+\frac{2}{x^2}=$$

# Adding/Subtracting Fractions

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$$\frac{3}{x-1} - \frac{4}{x} =$$

$$=$$

$$=$$

$$\frac{2x+1}{x^2-1}+\frac{1}{x-1}$$

$$\frac{1}{x^2-x}+\frac{1}{xy-y}$$

# Multiplying and Dividing

$$\frac{1}{x} \div \frac{x^2}{2} =$$

$$\frac{x^2 - x}{2xy} \times \frac{4x^2}{x - 1} =$$

Simplify fully $\dfrac{3x^2 - x - 14}{9x^2 - 4} \div \dfrac{x + 2}{3x^2 + 2x}$

1 $\dfrac{x^2+4x}{x^2-1} \div \dfrac{x+4}{x+1}$

2 $\dfrac{2x^2-x-6}{16x^2-25} \div \dfrac{x-2}{4x^2-5x}$

# Exercise

1 Fully simplify the following:

$$\frac{3}{2x+4}+\frac{1}{x+2}=$$ ☐

2 Simplify fully:

$$\frac{3x}{(x-3)(x+6)}-\frac{2}{x+6}=$$ ☐

3 Simplify:

$$\frac{x}{x+1}\times\frac{3}{x^2}=$$ ☐

4 Simplify $\frac{x^2+4x-12}{x^2-25}\div\frac{x+6}{x^2-5x}=$ ☐

# Solving Equations with Algebraic Fractions

When asked to solve an equation with fractions:

- **Combine fractions into single fraction** then **multiply through by denominator**.
- But if multiplying everything by $x^2$ turns equation into a quadratic, this is simpler.

x

$$\frac{8}{x^2} + \frac{4}{x} = 4$$

y

$$\frac{x}{2x-3} + \frac{4}{x+1} = 1$$

# Solving Equations with Algebraic Fractions

a

$$\frac{x}{3}+\frac{10}{x-1}=4$$

b

Solve, giving your answer to 3sf.

$$\frac{1}{x}+5=\frac{2}{x^2}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

# Exercise

Find exact solutions to:

**c** $x + \frac{3}{x} = 7$

**d** $\frac{x-2}{5} - \frac{6}{x} = 1$