Linear Inequalities

คิดถึงครูอ้น(C9)
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Why we need inequalities in maths

Inequalities are needed in mathematics when we need to represent a **range of values**.

Equation:	Number of Solutions:
$x + 5 = 7$	
$x^2 = 9$	
$x + 3 = x$	
$x > 4$	

This means "x is more than 4".

Reading inequalities

Notice the symbol is taller on the side which is larger.

What it means

$x > 7$

$x \geq 7$

$x < 10$

$x \leq 8$

Further Examples

Are the following inequalities true or false?

$3 < 4$ **True** **False**

$-5 > 1$ **True** **False**

$5 \leq 5$ **True** **False**

Write the following as inequalities:

1 "x is greater than 3"

2 "b is less than or equal to 7"

3 "a is greater than or equal to 5"

4 "y is smaller than 8"

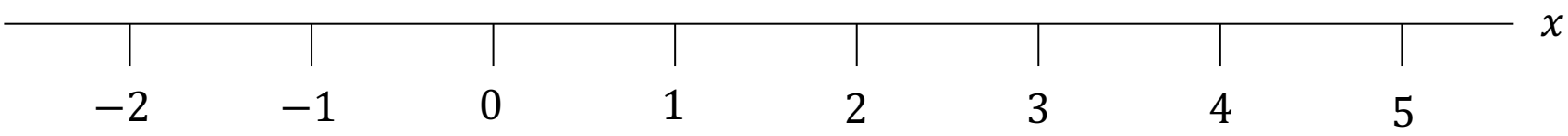
5 "p is at most 6"

6 "k is at least -2"

Inequalities on Number Lines

$x = 4$

We can use a filled circle on a number line to indicate we want to include the value.

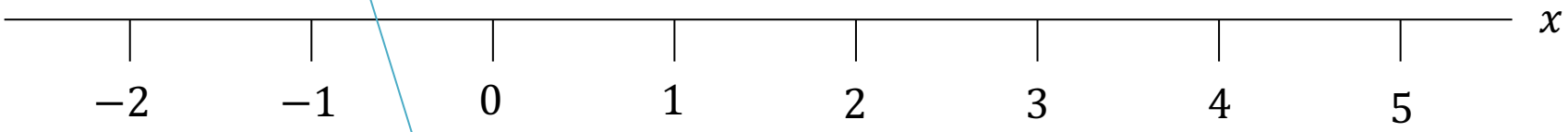


But what about:

$x \leq 4$

We again use a filled circle to indicate that we want to include 4.

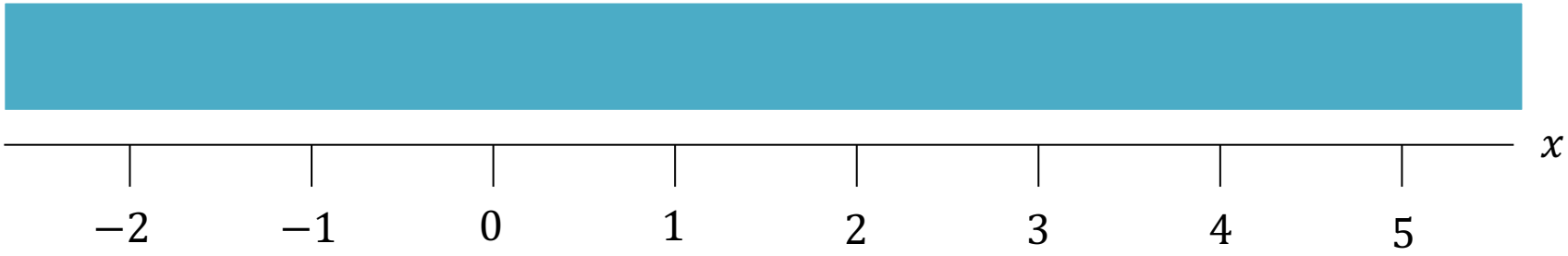
?



But we also have an arrow pointing left to say we also want any value less than 4.

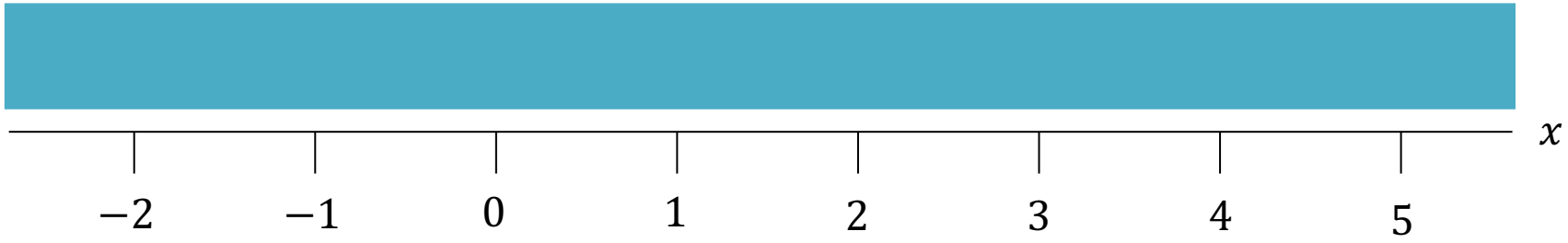
$x \geq 1$

We again want to include 1, but our arrow is right this time to indicate values greater than 1.



$x < 2$

We again have an arrow left to indicate "less than 2", but this time we **DON'T** want to include 2 itself. We use an **unfilled circle** to indicate that 2 is excluded.

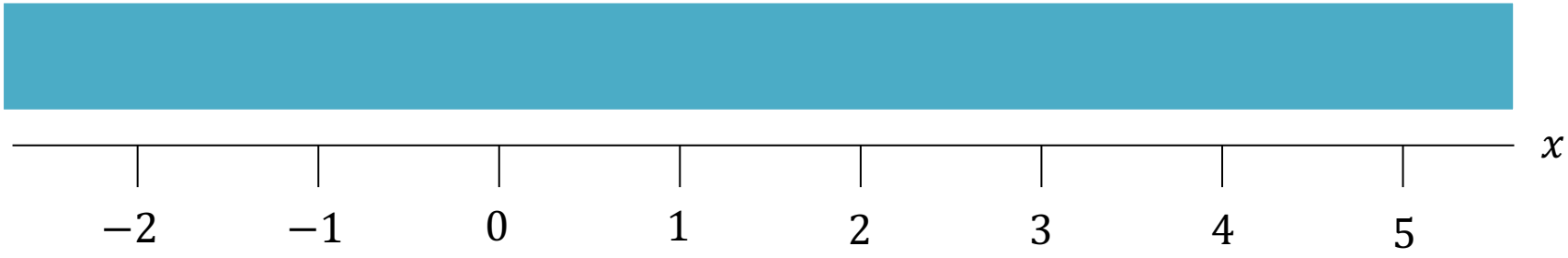


Two-ended inequalities

$$-1 \leq x < 3$$

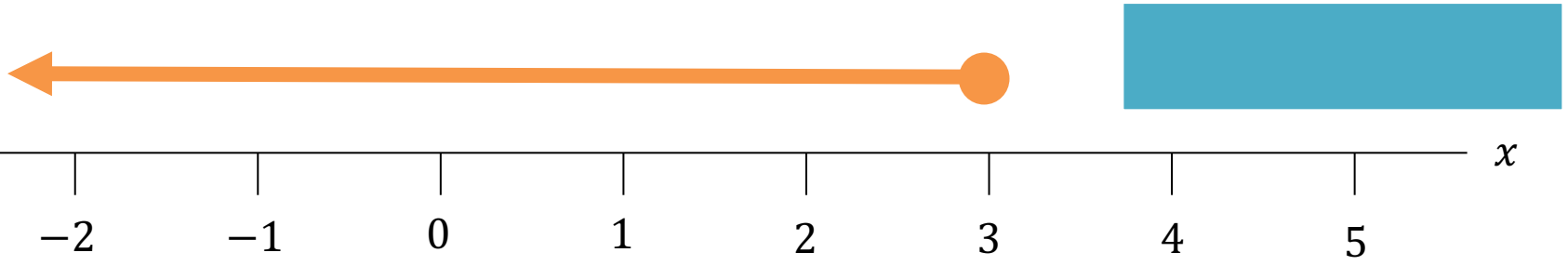
What does this mean in words?

On a number line…



x is an integer (whole number) and $-1 \leq x < 4$.
List all the numbers that x could be.

1



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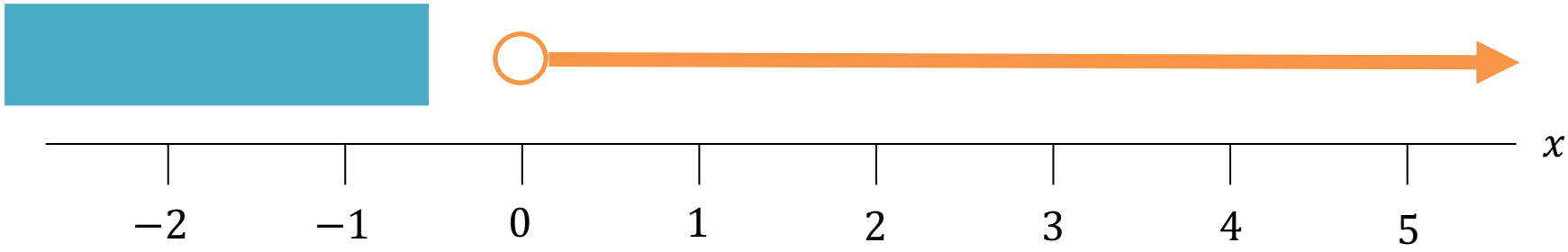

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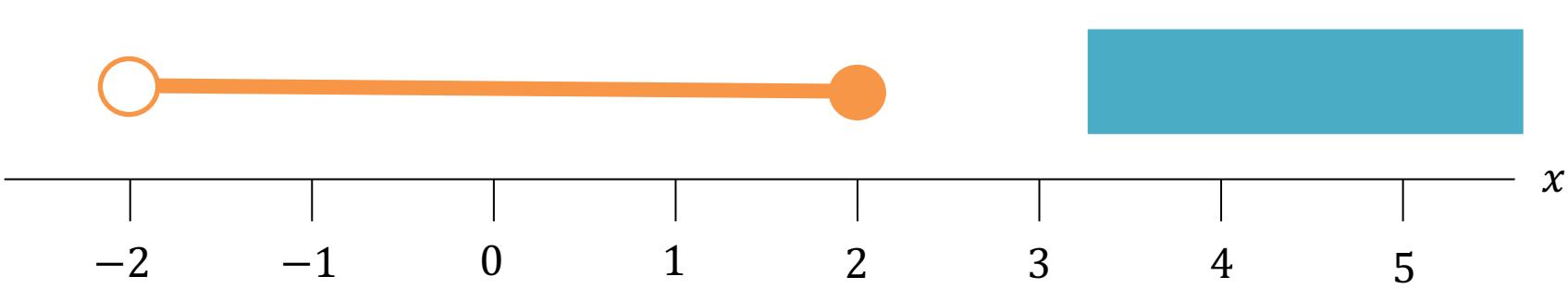

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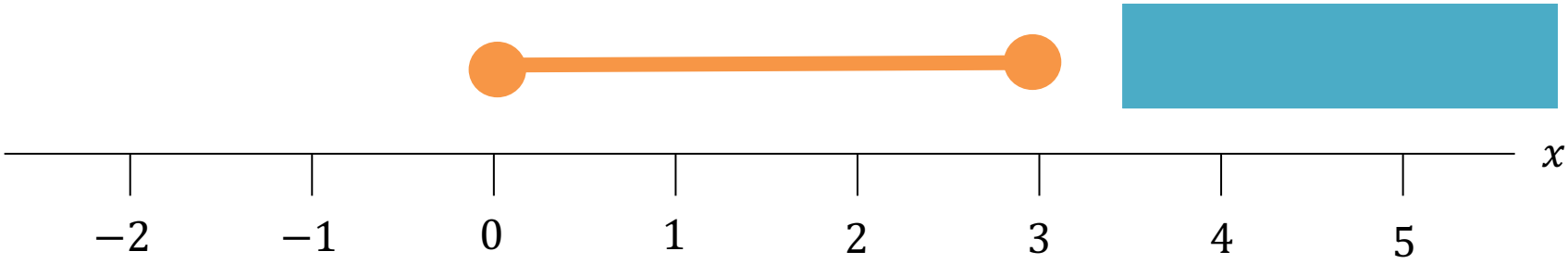


Exercise

1 Write each of the following as inequalities:

a "z is at least 6"

b "a is more than 0"

c "b is greater than or equal to -1"

d "x between 2 and 3 (inclusive)"

e "y is between -1 and 2 (exclusive)"

2 If x is an integer (whole number), then list the numbers in each range:

a $2 \leq x \leq 5$

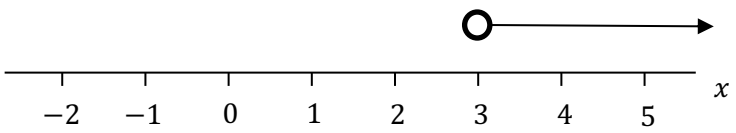
b $2 \leq x < 5$

c $2 < x < 5$

d $-1.5 < x \leq 2$

3 Represent the following inequalities on a number line.

a



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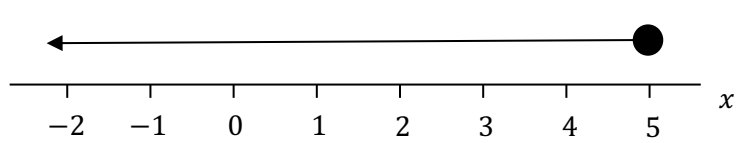
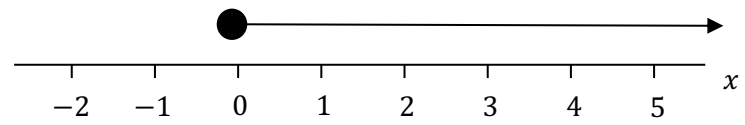


c



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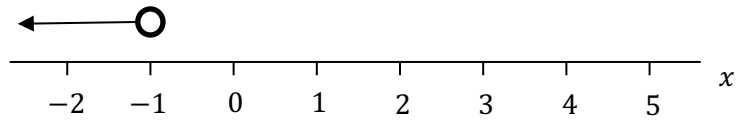

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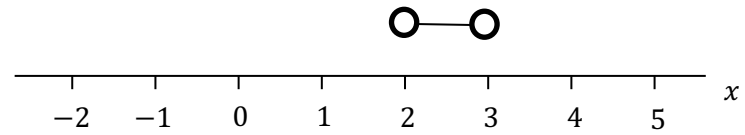

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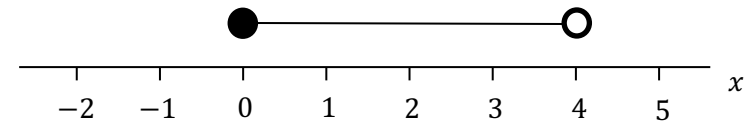

Inequalities behave in a similar way to equations: whatever we do to one side of the equation, we have to do the same to the other.

$$x - 1 \geq 3$$

$+1$ ⇓ ⇓ $+1$

$$x \geq 4$$

Example values…

$$2x \geq 6$$

$\div 2$ ⇓ ⇓ $\div 2$

$$x \geq 3$$

Example values…

Examples

Solve $3x - 2 < 16$

Solve $3 \geq \frac{x+1}{5}$

Variable on both sides

If the variable appears on both sides of the equation, again we can solve in a similar way to how solve equations.

Often the best strategy is to first get all the variable terms (e.g. x) on the side of the equation where there is more of them.

Solve $4x - 2 > x + 10$

Negative number on front of the x

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We might think that we can divide both sides of the equation by a negative number to solve:

$$-2x < 6$$

$\div(-2)$? $\div(-2)$

$$x < -3$$

Think of a number that works with this inequality.

Does it work with the simplified inequality?

No! It is not true that $1 < -3$ However, if we reversed the direction of the inequality, it is true that $1 > -3$

When you divide or multiply both sides of an inequality by a negative number, **reverse the direction of the inequality**.

$$-2x < 6$$

$\div(-2)$ $\div(-2)$

$$x > -3$$

But it's probably easiest to avoid needing to divide by a negative number in the first place…

Example

$$10 - 3x \geq 22$$

Method 1: Dividing by a negative number

Method 2: Put the variable term on the side where it'll be positive.

Solving double-ended inequalities

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We saw earlier that an inequality like $3 \leq x \leq 5$ means "x is **between** 3 and 5".
Sometimes we need to solve these type inequalities.

$$6 \leq 2x < 10$$

$\div 2 \qquad \div 2 \qquad \div 2$

$$\mathbf{3 \leq x < 5}$$

$$-2 < \frac{x-3}{4} < 5$$

$\times 4 \qquad \times 4 \qquad \times 4$

$$-8 < x - 3 < 20$$

$+3 \qquad +3 \qquad +3$

$$\mathbf{-5 < x < 23}$$

Exercise

1 Solve the following inequalities.

a $-2 < 3x - 5 < 16$

b $1 < \frac{x-5}{3} \leq 5$

2 Solve the following inequalities.

$3 - \frac{4x}{5} > \frac{3}{5}(2 - 6x)$

Combining Inequalities

We've already seen examples where we've combined inequalities together:

"x is greater than 2 and less than 5."

$$x > 2$$
$$x < 5$$
$$\Rightarrow 2 < x < 5$$

Alice says: "**Charles is at least 12 years old".**
Bob says "**Charles is at least 15 years old**".

What do we know about Charles' age?

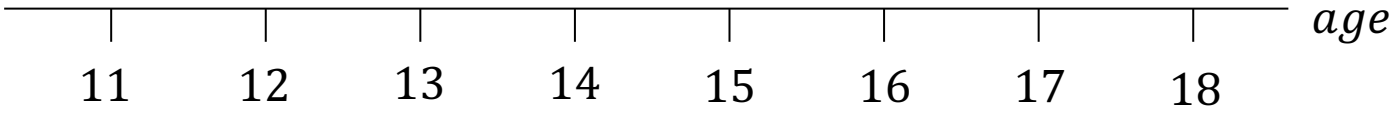
Shota says: "**Tasmin is between 10 and 14".**
Fleur says "**Tasmin is at least 13**".

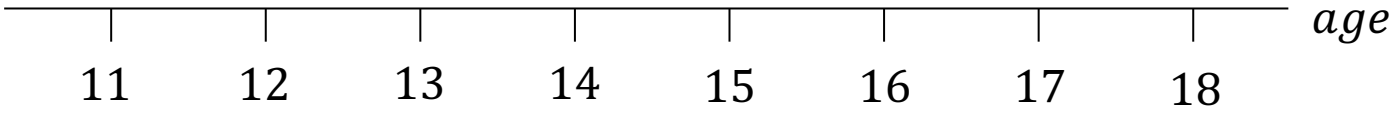
What do we know about Tasmin's age?

Combining Inequalities using Number Lines

"Charles is at least 12 years old."

"Charles is at least 15 years old."

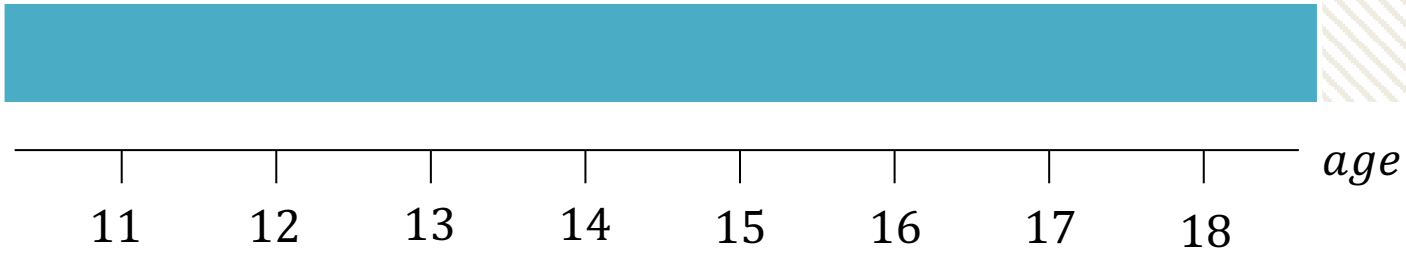




$12 \leq a \leq 17$

$a > 15$

Combined

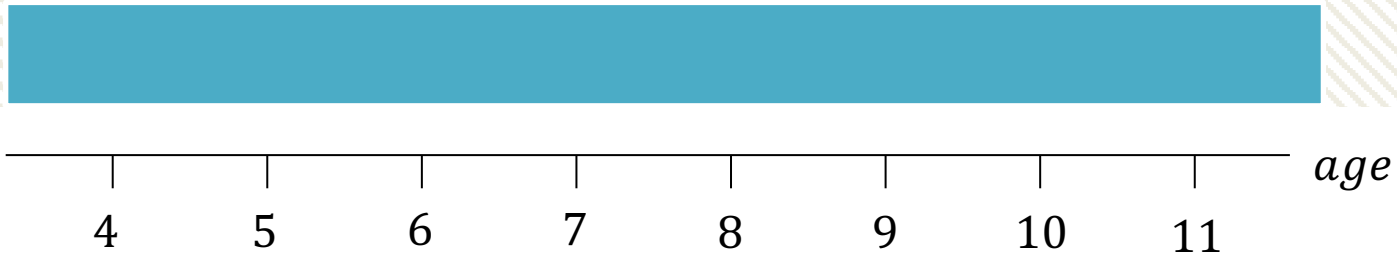


Harder Examples

$x \leq 6$ or $x \geq 8$

$5 < x \leq 9$

Combined



Combined inequality:

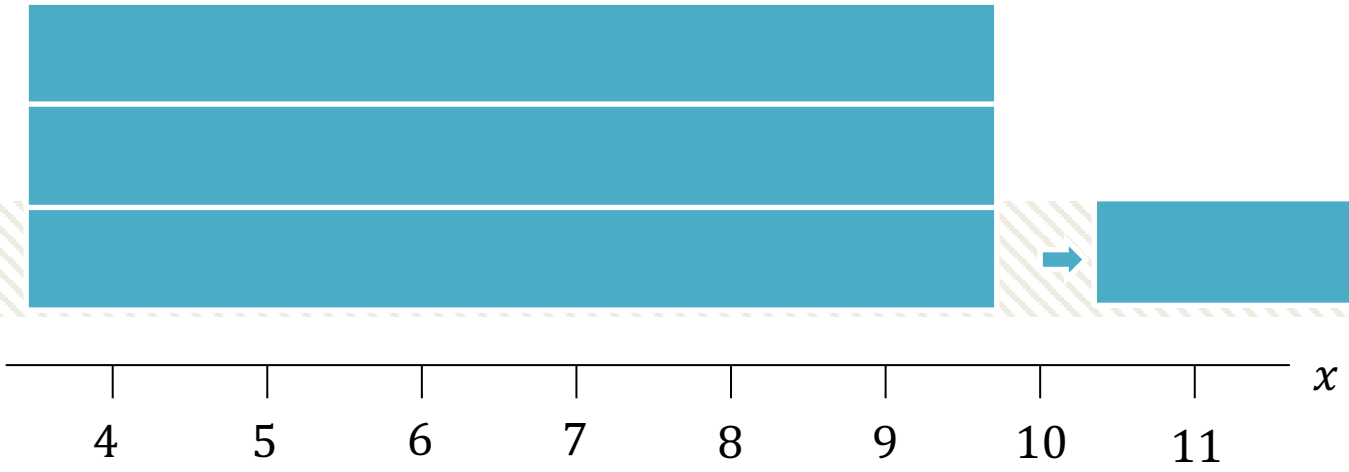
Harder Examples

1

$x \leq 8$

$6 \leq x < 9$

Combined

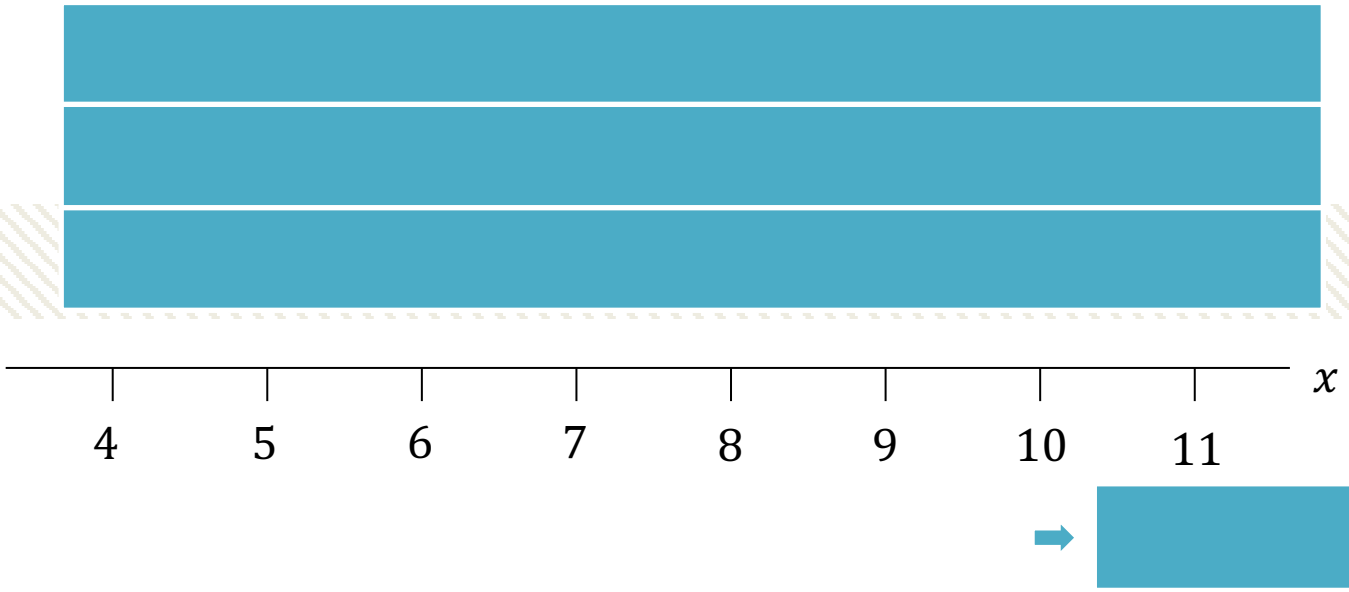


2

$x \leq 6$ *or* $9 < x$

$7 \leq x \leq 10$

Combined



Solving Harder Double-Ended Inequalities

Solve

$$3 - x \leq 2 < 10 - 2x$$

$3 - x \leq 2$

$3 \leq x + 2$

$x \geq 1$

$2 < 10 - 2x$

$2x < 8$

$x < 4$

$$\mathbf{1 \leq x < 4}$$

Split into two separate inequalities.

Solve each separately.

Combine together.

Solve

$$1 + x < 5 \leq 7 + 5x$$

Exercise

1 By drawing suitable number lines or otherwise, combine the following inequalities.

$x < 2 \; or \; x > 3, \quad x < 1 \; or \; x > 4$

2 Solve the following inequalities.

$2 - x < x < 8 - x, \quad x \leq 3$