Simultaneous Equations

METHOD 1: Graphical method

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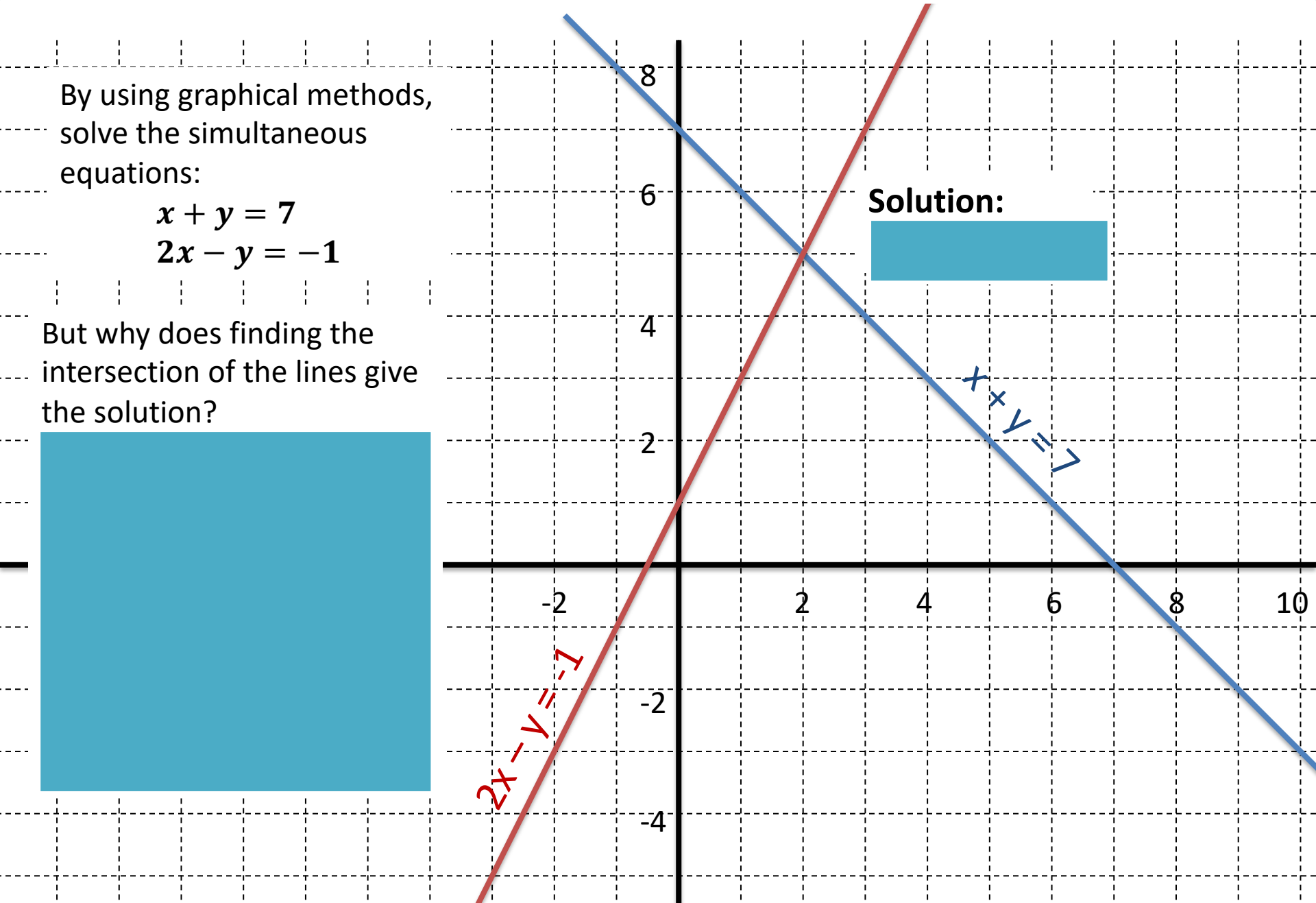
By using graphical methods, solve the simultaneous equations:

$$x + y = 7$$
$$2x - y = -1$$

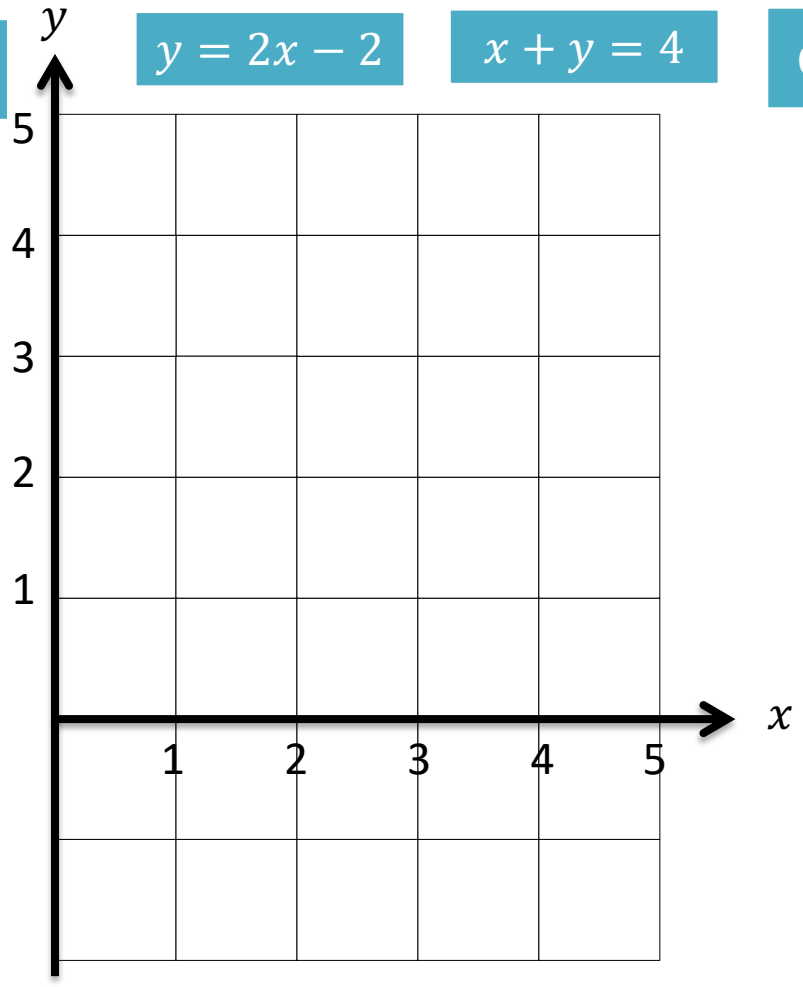
But why does finding the intersection of the lines give the solution?



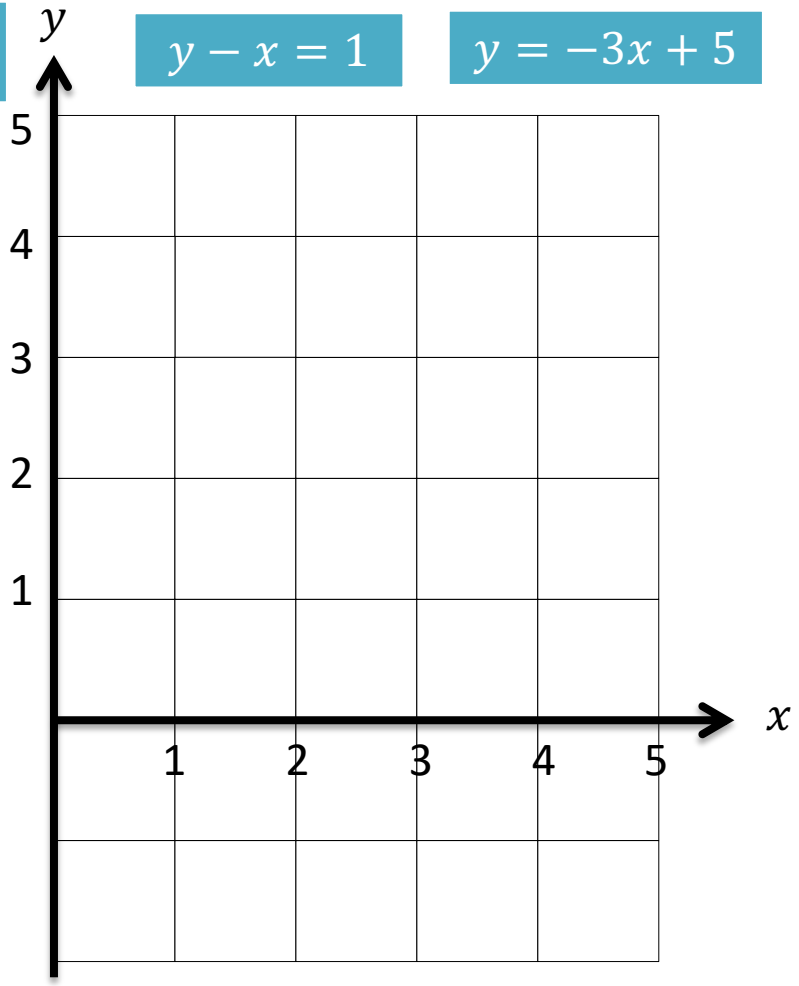
Solution:

Solve the simultaneous equations.

Q1 $y = 2x - 2$ $x + y = 4$



Q2 $y - x = 1$ $y = -3x + 5$



* Remember that the easiest way is to pick two points and join up, e.g. when $x = 0$ and when $y = 0$.

Thinking graphically…

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For two simultaneous equations, when would we have…

0 solutions for x and y?

Unique solution for x and y?

Infinitely many solutions for x and y?

METHOD 2: Solving by Elimination

By either adding or subtracting the equations, we can 'eliminate' one of the variables.

$$2x + y = 6 \quad (1)$$
$$3x - y = 9 \quad (2)$$

$$4x + y = 6 \quad (1)$$
$$6x + y = 4 \quad (2)$$

Solving by Elimination

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$$5x + 2y = 13$$
$$2x + 2y = 4$$

$$2x + 3y = 5$$
$$5x - 2y = -16$$

Challenge Problem

Solve the system of equations for x, y, z:

$$x^2 - z - y^2 = 8$$
$$x^2 + z + y^2 = 10$$
$$-x^2 + z - y^2 = -12$$

The positive integers x,y and z satisfy

$$xy = 14$$
$$yz = 10$$
$$xz = 35$$

What is the value of $x+y+z$?

Exercise

A The sum of two numbers is 2. The difference between them is 4. What is their product?

B Two cats and a dog cost £91. Three cats and two dogs cost £159. How much does a cat cost?

C Solve:

$$a^2 + b^2 = 30$$
$$a^2 - b^2 = 20$$

Challenge Problem

Mars, his wife Venus and grandson Pluto have a combined age of 192. The ages of Mars and Pluto together total 30 years more than Venus' age. The ages of Venus and Pluto together total 4 years more than Mars's age. Find their three ages.

METHOD 3: Solving by Substitution

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We currently have two equations both involving two variables.

$$3x - 2y = 0$$
$$2x + y = 7$$

Perhaps we could **put one equation in terms of x or y**, then **substitute this expression into the other**.

Why do you think we chose this equation to rearrange?

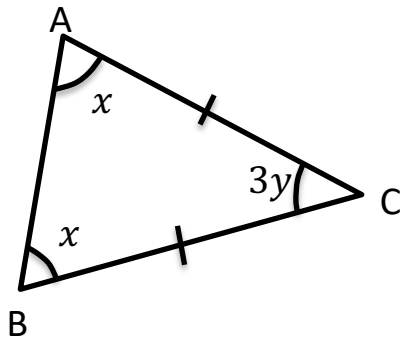
$$2x + y = 7 \Rightarrow y = \square$$

Solve for x and y, using substitution.

$$2x + y = 5$$
$$x + 3y = 5$$

$$3x - 2y = 16$$
$$x + y = 2$$

Use substitution only to solve the following simultaneous equations.



The angle at A is 12° greater than the angle at C. Find x and y.

Challenge Problem

Use substitution only to solve the following simultaneous equations.

* Solve the simultaneous equations:

$$x + y = 3$$
$$x^3 + y^3 = 9$$