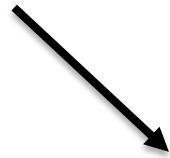


# Sequences



4, 9, 14, 19, 24, 29, ...

$n^{\text{th}}$  term of sequence



$u_n$

The coefficient of  $n$  is the difference between the numbers.



=



$n$



$n$  means the position in the sequence, so for the first term,  $n = 1$ .

# The idea of 'adjusting' an initial formula

$n$

1

2

3

4

5

$u_n$

2

5

8

11

14

...

Adjustment

Therefore formula:

$u_n =$




Find formulae for the  $n^{\text{th}}$  term of each of these sequences:


2, 5, 8, 11, 14, ...

$$u_n = \boxed{\phantom{000000}}$$

12, 9, 6, 3, 0, -3, ...

$$u_n = \boxed{\phantom{000000}}$$

 A **linear** sequence (or 'arithmetic sequence') is one where the difference between terms is constant.  $u_n = an + b$

  $u_n$  means the  $n^{\text{th}}$  term in the sequence.  
(often  $a_n$  or  $x_n$ )

If we're currently considering the  $n^{\text{th}}$  term, how would we refer to the term before that?



$n$	1	2	3	4	5	6	
$u_n$	2	5	8	11	14	17	...

This is the 'position'.



This is the 'term'.

i.e. The term at the 4<sup>th</sup> position is 11.



 A geometric sequence is one in which we multiply by a constant each time.

(In contrast to an *arithmetic* sequence where we add each time)

$n$	1	2	3	4	5
$u_n$	4	8	16	32	64 ...

Adjustment

Therefore formula:

$$u_n =$$

For geometric sequences where we're multiplying by  $k$  each time, start the formula as  $k^n$ .

$n$	1	2	3	4	5	
$u_n$	2	6	18	54	162	...



Adjustment



Therefore formula:

$$u_n = \boxed{\phantom{a \cdot k^n}} = \boxed{\phantom{a \cdot k^n}}$$

For geometric sequences where we're multiplying by  $k$  each time, start the formula as  $k^n$ .

A

30, 300, 3000, 30000, ...

B

4, 20, 100, 500, ...

For each of the following determine the position-to-term formula

1 0, 1, 2, 3, 4 ...

$$u_n = \text{[ ]}$$

2 6, 11, 16, 21, ...

$$u_n = \text{[ ]}$$

3 4, 16, 64, 256 ...

$$u_n = \text{[ ]}$$

4 3, 6, 12, 24, ...

$$u_n = \text{[ ]}$$

For each of the following determine the position-to-term formula

**A** 8, 24, 72, 216, 648 ...

$u_n =$

**B** -1, +1, -1, +1, -1 ...

$u_n =$

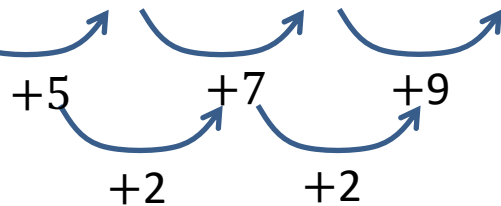
**C** 0.5, 0.25, 0.125, 0.0625 ...

$u_n =$

What do you notice about the difference?

3, 8, 15, 24, 35, ...

$n$	1	2	3	4	5
$u_n$	3	8	15	24	35
$1n^2$	1	4	9	16	25
Adjust	+2	+4	+6	+8	+10



**STEP 1:** Write out  $n$  and  $u_n$

**STEP 2:** Work out second difference.

**STEP 3:** Halve this to find coefficient of  $n^2$  term.

**STEP 4:** Work out what we need to add to get from this to correct term. Work out its formula.

$$u_n = \text{[Redacted]}$$

3, 9, 19, 33, 51, ...

$u_n =$

4, 15, 32, 55, 84, ...

$u_n =$

$$u_n = ?$$

**A** 4, 5, 4, 1, -4, -11, ...

**B** 3.5, 7, 9.5, 11, 11.5, 11, ...

# Finding a formula using simultaneous equations

You're given the first three terms of a quadratic (second difference) sequence:

$$u_1 = 3, \quad u_2 = 7, \quad u_3 = 15$$

We know that we can use:

$$u_n = an^2 + bn + c$$

What equations can we form?



Solve by elimination: