Sequences

4, 9, 14, 19, 24, 29, …

n^{th} term of sequence

The coefficient of n is the difference between the numbers.

$$u_n = \square\, n \;\square$$

n means the position in the sequence, so for the first term, $n = 1$.

The idea of 'adjusting' an initial formula

n	1	2	3	4	5
u_n	2	5	8	11	14 …
Adjustment					

Therefore formula:

$u_n =$

Find formulae for the n^{th} term of each of these sequences:

$2, 5, 8, 11, 14, \ldots$ $\qquad u_n = \square$

$12, 9, 6, 3, 0, -3, \ldots$ $\qquad u_n = \square$

Terminology

A **linear** sequence (or 'arithmetic sequence') is one where the difference between terms is constant. $u_n = an + b$

u_n means the n^{th} term in the sequence.
(often a_n or x_n)

If we're currently considering the n^{th} term, how would we refer to the term before that?

This is the '**position**'.

n	1	2	3	4	5	6	
u_n	**2**	**5**	**8**	**11**	**14**	**17**	...

This is the '**term**'.
i.e. The term at the 4th position is 11.

Geometric Sequences

TOP Maths IGCSE

✎ A geometric sequence is one in which we multiply by a constant each time.

(In contrast to an *arithmetic* sequence where we add each time)

n	1	2	3	4	5
u_n	4	8	16	32	64 …
Adjustment					

Therefore formula:

$u_n =$

For geometric sequences where we're multiplying by k each time, start the formula as k^n.

n	1	2	3	4	5	
u_n	2	6	18	54	162	...
Adjustment						

Therefore formula:

$u_n =$ $=$

For geometric sequences where we're multiplying by k each time, start the formula as k^n.

A

$$30, 300, 3000, 30000, \ldots$$

B

$$4, 20, 100, 500, \ldots$$

Exercise

For each of the following determine the position-to-term formula

1 $0, 1, 2, 3, 4 \ldots$

$u_n =$

2 $6, 11, 16, 21, \ldots$

$u_n =$

3 $4, 16, 64, 256 \ldots$

$u_n =$

4 $3, 6, 12, 24, \ldots$

$u_n =$

Exercise

For each of the following determine the position-to-term formula

A

$8,\ 24,\ 72, 216, 648 \ldots$

$u_n =$

B

$-1, +1, -1, +1, -1 \ldots$

$u_n =$

C

$0.5, 0.25, 0.125, 0.0625 \ldots$

$u_n =$

Second Difference

What do you notice about the difference?

$$3, 8, 15, 24, 35, \ldots$$

Working out u_n

n	1	2	3	4	5
u_n	3	8	15	24	35
		+5	+7	+9	
			+2	+2	
$1n^2$	1	4	9	16	25
Adjust	+2	+4	+6	+8	+10

$u_n =$

STEP 1: Write out n and u_n

STEP 2: Work out second difference.

STEP 3: Halve this to find coefficient of n^2 term.

STEP 4: Work out what we need to add to get from this to correct term. Work out its formula.

More Examples

$3, 9, 19, 33, 51, \ldots$ $\quad u_n =$

$4, 15, 32, 55, 84, \ldots$ $\quad u_n =$

A $4, 5, 4, 1, -4, -11, \ldots$

B $3.5, 7, 9.5, 11, 11.5, 11, \ldots$

Finding a formula using simultaneous equations

You're given the first three terms of a quadratic (second difference) sequence:

$$u_1 = 3, \qquad u_2 = 7, \qquad u_3 = 15$$

We know that we can use:

$$u_n = an^2 + bn + c$$

What equations can we form?

Solve by elimination: