Counterexamples & Proofs

คิดถึงครูอ้น(C9)
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Bad examples of proof

"Show that the sum of any three consecutive integers is a multiple of 3."

Kyle's proof:

"5 + 6 + 7 = 18, which is divisible by 3".

Key Terms: An integer just means a whole number.

Counterexamples

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A counterexample is an example used to disprove a statement.

Discussing in pairs, find counterexamples for the following statements:

	Statement	Possible counterexample
1	Prime numbers are always odd.	
2	The square root of a number is always smaller than the number itself.	
3	If p is prime then $p + 2$ is prime.	
4	$2n^2 + 11$ is prime for all integer values of n	

Example

"Show that the sum of any three consecutive integers is a multiple of 3."

Example

"Show that the sum of any four consecutive integers is even."

Representing other numbers

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How could I algebraically represent:

Consecutive integers	
An odd number	
An even number	
Two consecutive odd numbers	
Two odd numbers	
One less than a multiple of 5.	
A number that when divided by 4, gives you a remainder of 2.	

"Prove that the sum of three consecutive even numbers is a multiple of 6."

Harder Example

"Prove that the product of two consecutive odd numbers is always one less than a multiple of 4."

Harder Example

"Prove that the product of two odd numbers is odd."

Consecutive odd numbers

"Prove that the sum of three consecutive odd numbers is 3 more than a multiple of 6."

Proofs involving Squares

"Prove that the sum of the squares of two consecutive numbers is odd."

X Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

Y Prove that the sum of the squares of any three consecutive odd numbers is always 11 more than a multiple of 12.

Z Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.