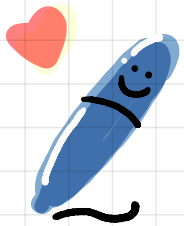
# 3.3) Harder Graph



# Graphs:Harder Graphs.



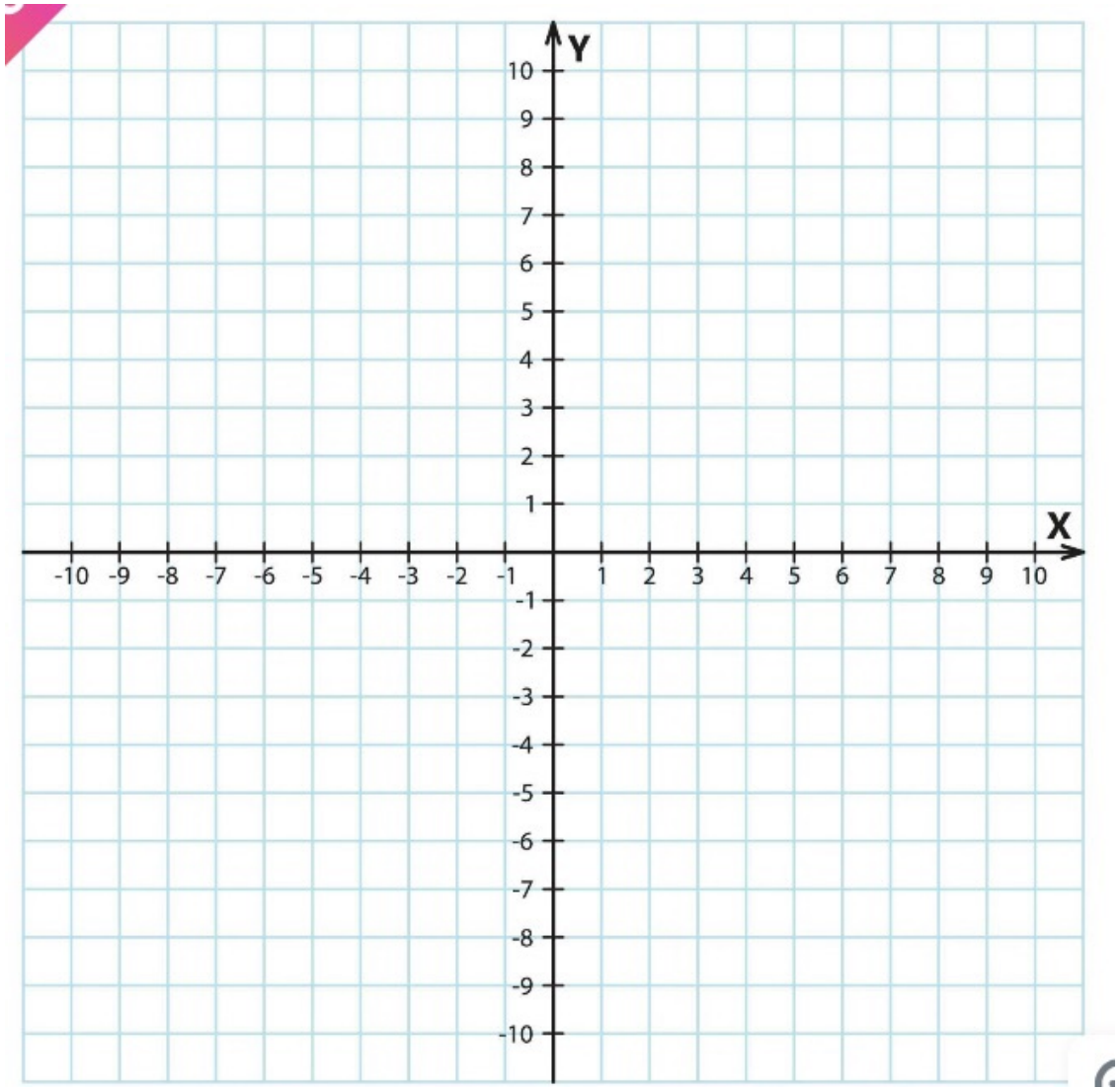
## Cubic Graphs

$y = x^3$

| x | | | | | |
|---|---|---|---|---|---|
| y | | | | | |

$y = -\frac{1}{2}x^3$

| x | | | | | |
|---|---|---|---|---|---|
| y | | | | | |



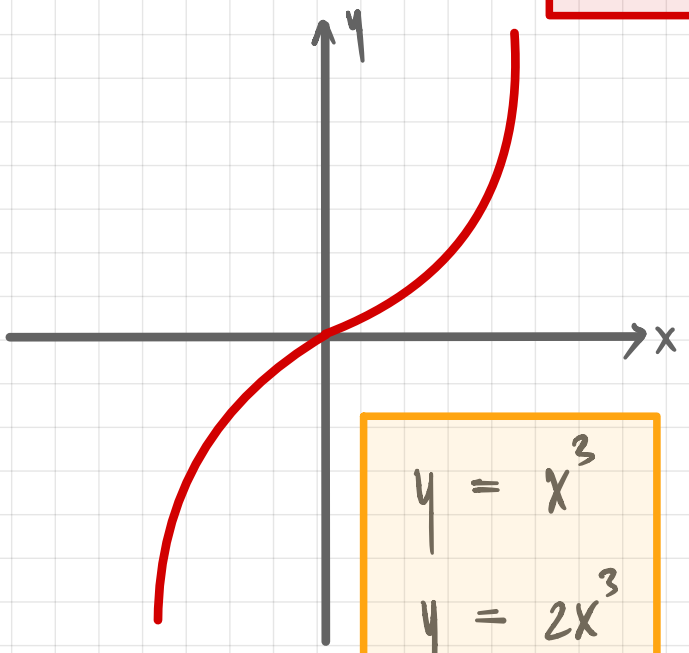
# Graphs:Harder Graphs.



## Cubic Graphs

$y = ax^3$

$a > 0$

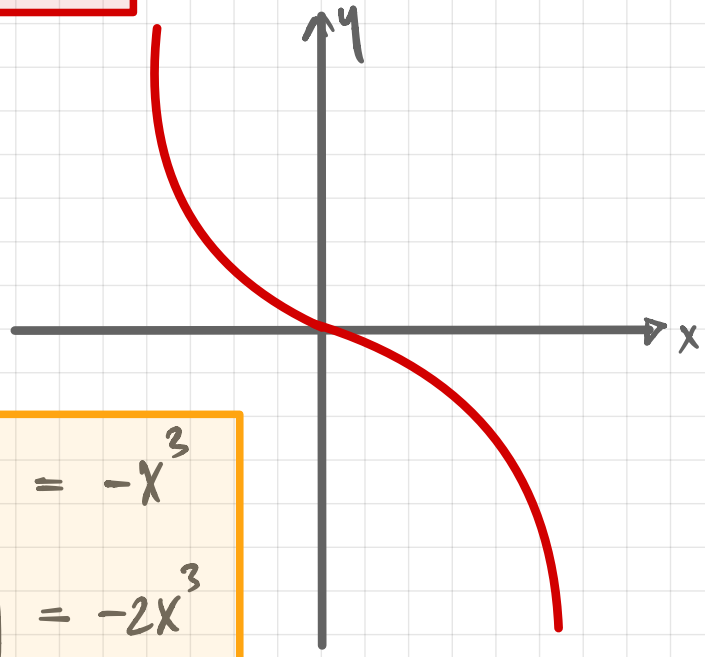



$y = x^3$

$y = 2x^3$

$y = \frac{1}{2}x^3$

$a < 0$




$y = -x^3$

$y = -2x^3$

$y = -\frac{1}{2}x^3$

# Graphs:Harder Graphs.



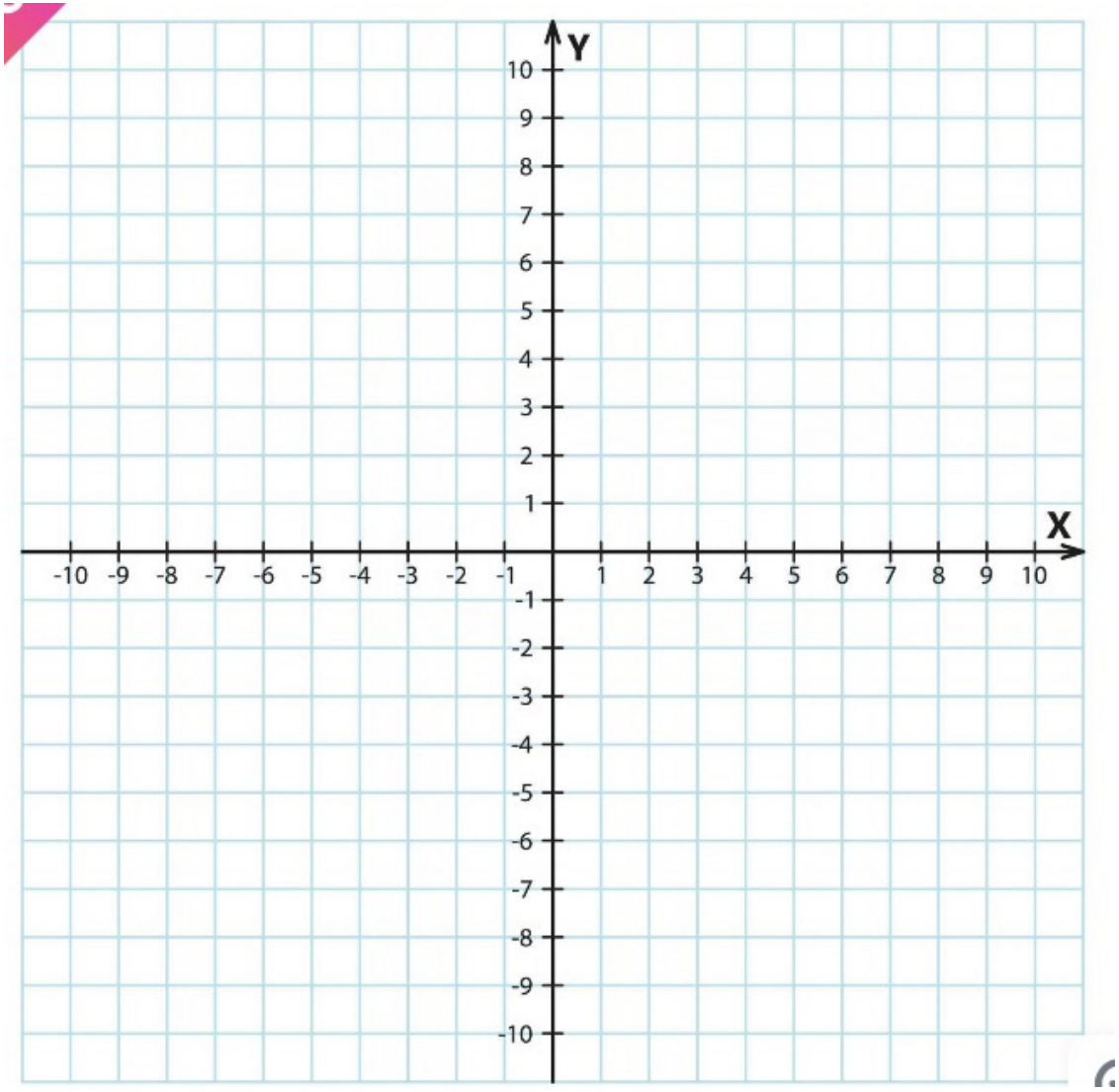
## Cubic Graphs

$y = (x-1)(x+1)(2-x)$

| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |

$y = x^3 + 2x^2 - 3x$

| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |



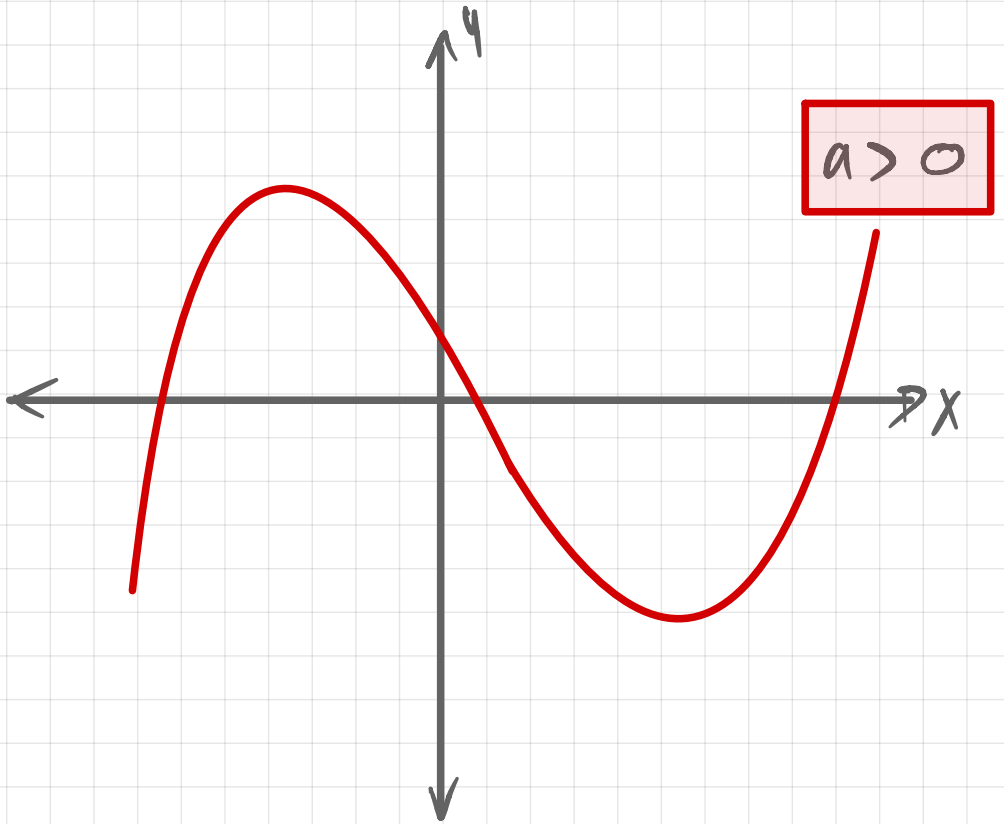
# Graphs:Harder Graphs.

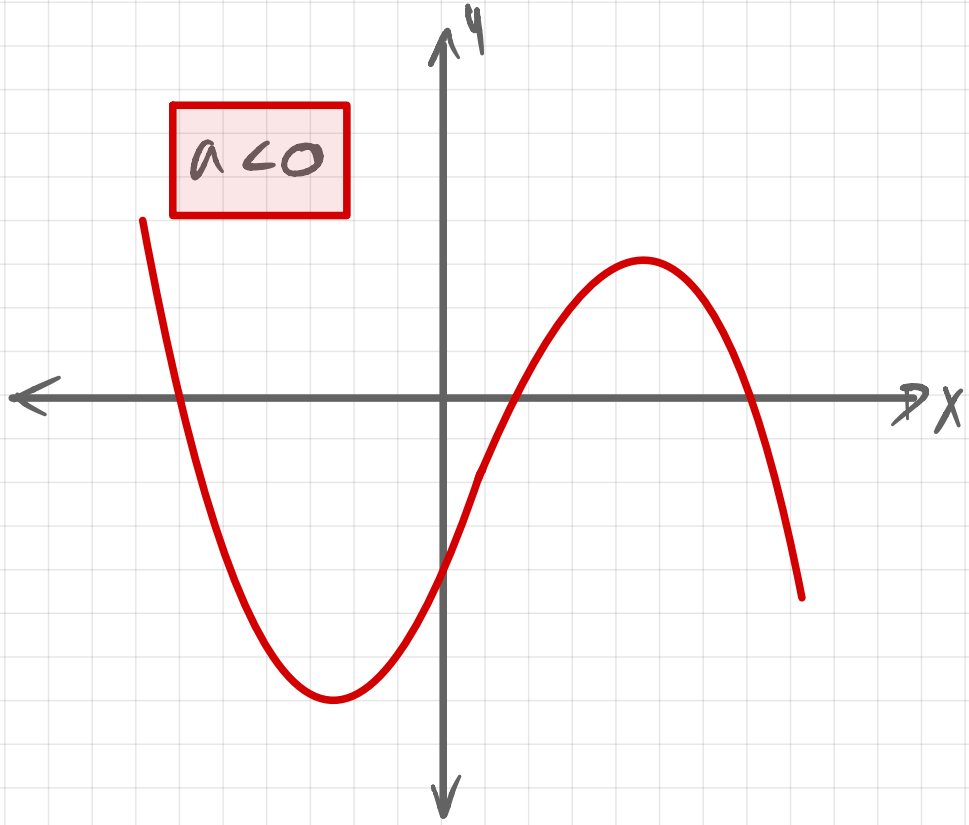


## Cubic Graphs

$$y = ax^3 + bx^2 + cx + d$$

$$y = (\quad)(\quad)(\quad)$$



# Graphs:Harder Graphs.



Example: Sketch $y = (x-2)(x+1)(x-1)$

# Graphs:Harder Graphs.



Example: Sketch $y = (x+2)(3-x)(x-1)$

# Graphs:Harder Graphs.



Example: Sketch $y = x^3 - 2x^2 - 3x$

# Graphs:Harder Graphs.



**Example:** Sketch $y = (x+1)(x-2)^2$

# Graphs:Harder Graphs.



**Review:**

Reciprocal of $x \rightarrow$

Reciprocal of $5 \rightarrow$

Reciprocal of $\frac{1}{3} \rightarrow$

Reciprocal of $\frac{4}{5} \rightarrow$

# Graphs:Harder Graphs.



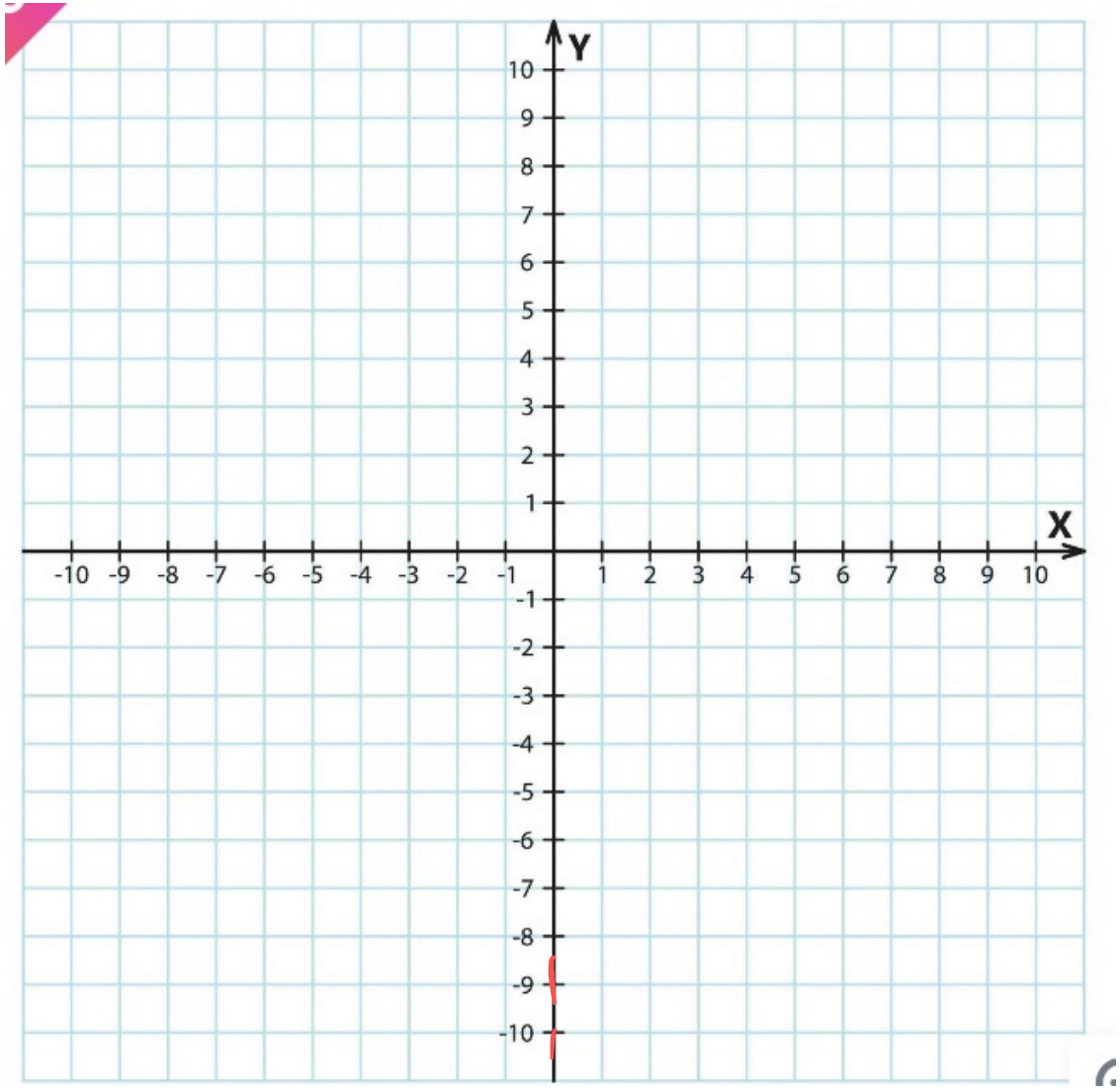
## Reciprocal Graphs

$y = \frac{1}{x}$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |

$y = \frac{-2}{x}$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |



# Graphs:Harder Graphs.

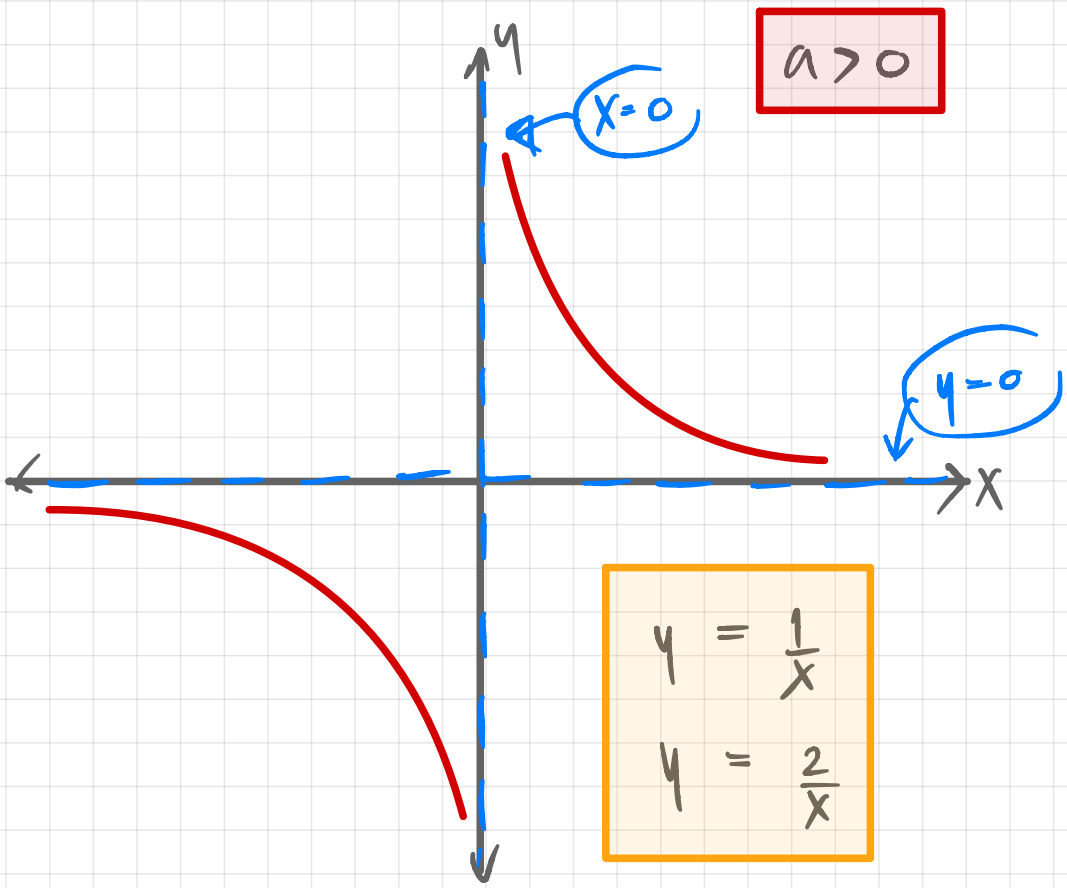


## Reciprocal Graphs

$y = \frac{a}{x}$ → Asymptote → $x = 0$, $y = 0$ → $xy = a$

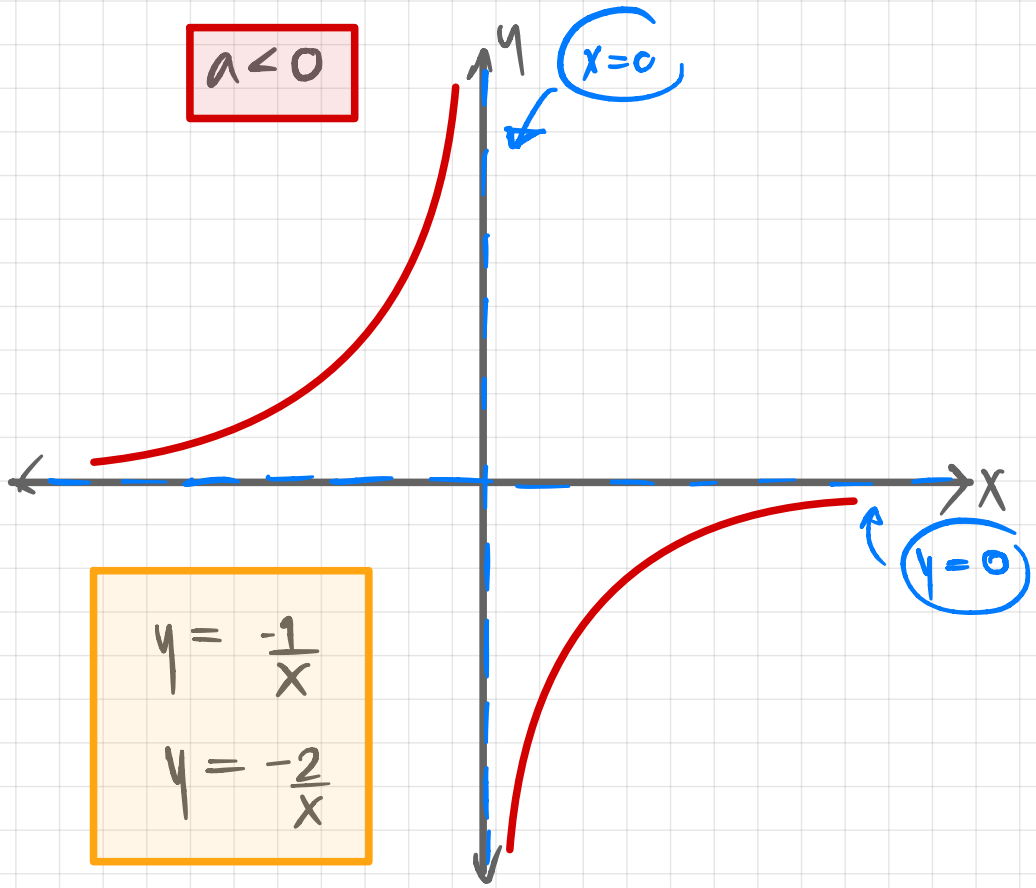
$xy = 5$

$xy = -3$




$y = \frac{1}{x}$

$y = \frac{2}{x}$




$y = \frac{-1}{x}$

$y = -\frac{2}{x}$

# Graphs:Harder Graphs.

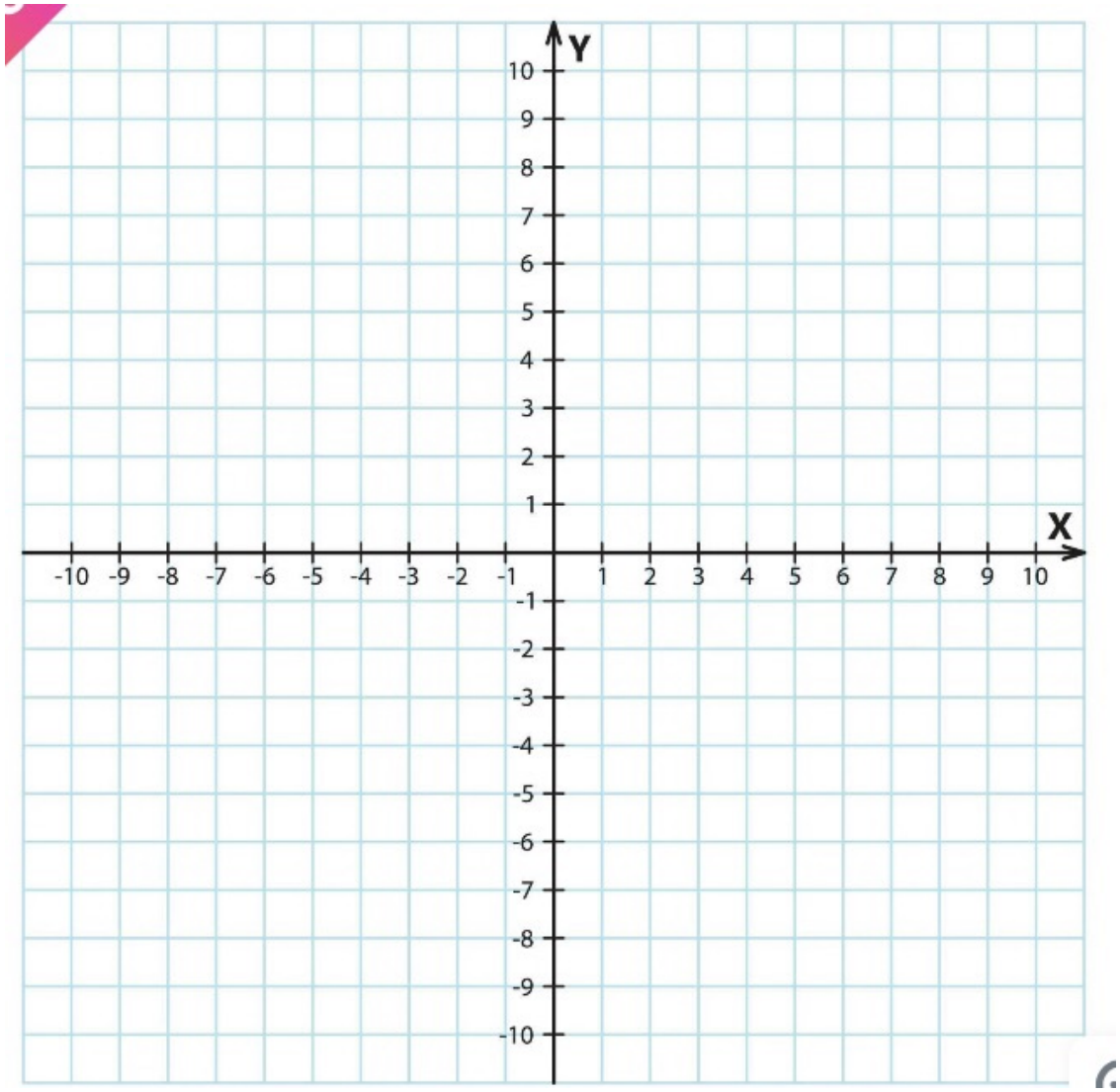


$y = \frac{1}{x^2}$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |

$y = \frac{-2}{x^2}$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |



# Graphs:Harder Graphs.

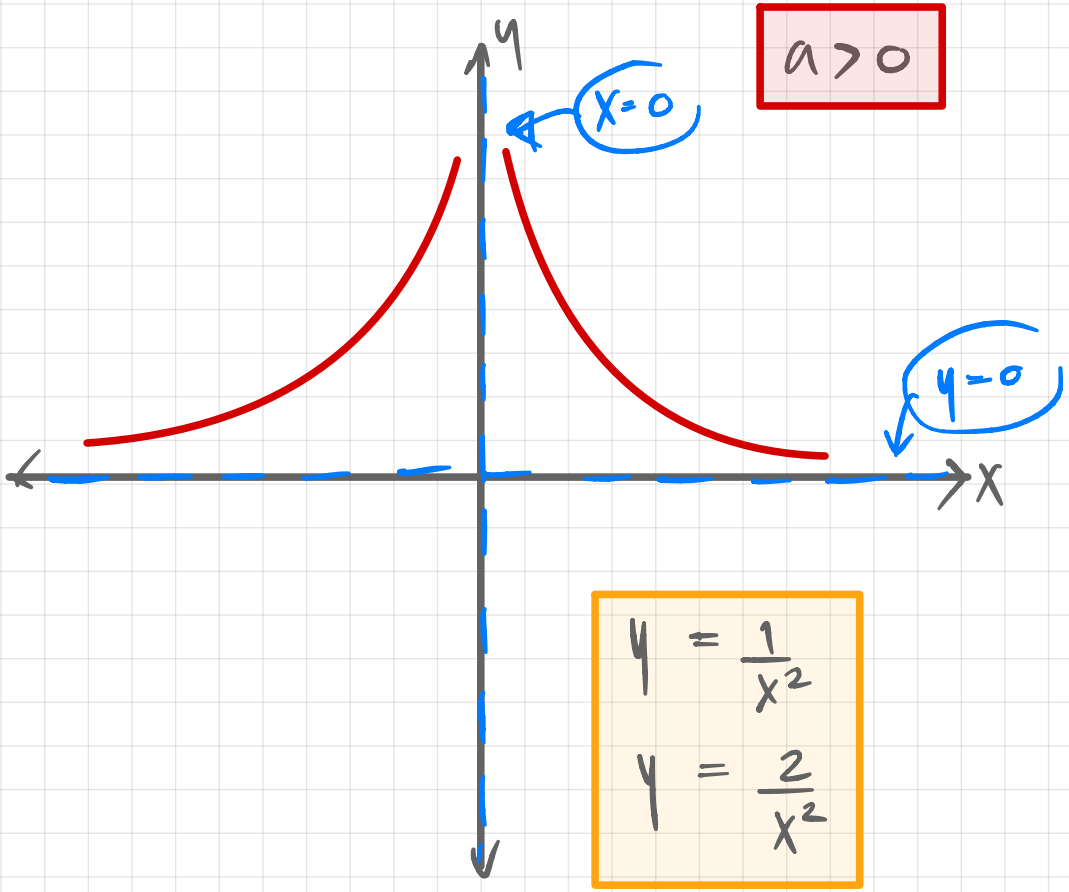


$y = \frac{a}{x^2}$ → Asymptote → $x = 0$, $y = 0$ → $yx^2 = a$

$yx^2 = 3$

$yx^2 = -5$

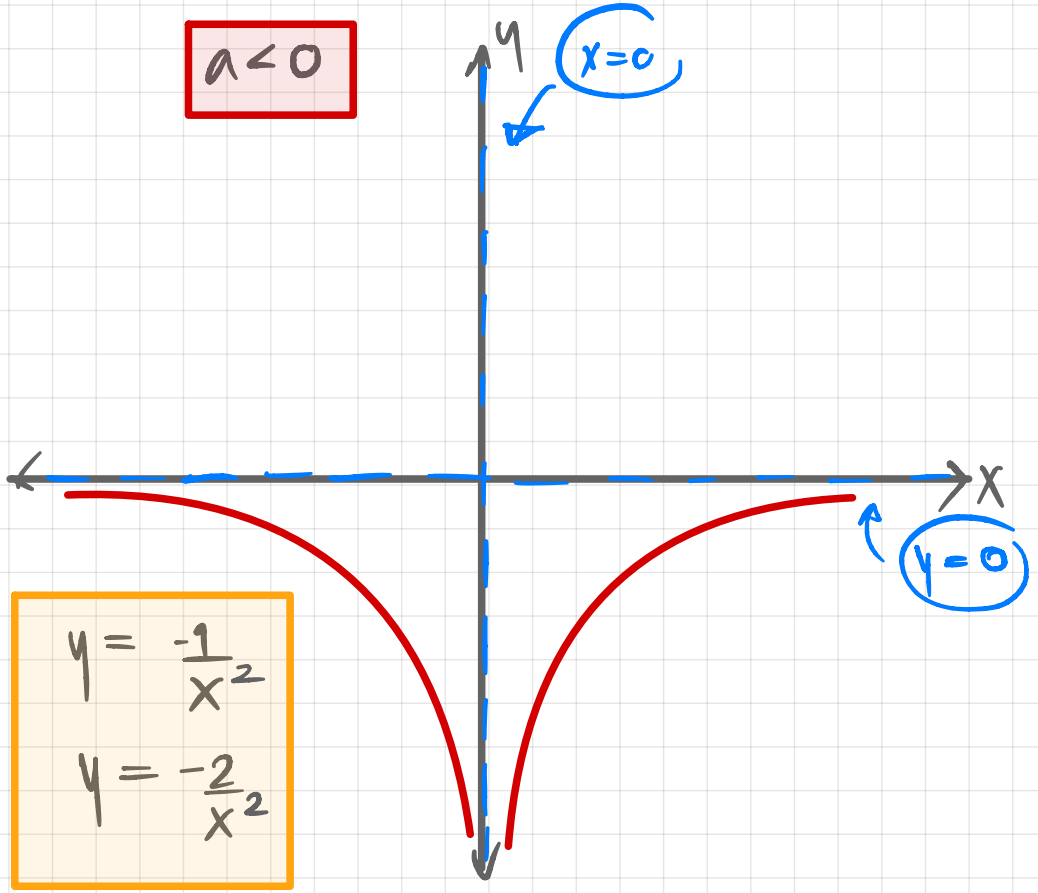
$a > 0$




$y = \frac{1}{x^2}$

$y = \frac{2}{x^2}$

$a < 0$




$y = \frac{-1}{x^2}$

$y = \frac{-2}{x^2}$

# Graphs:Harder Graphs.



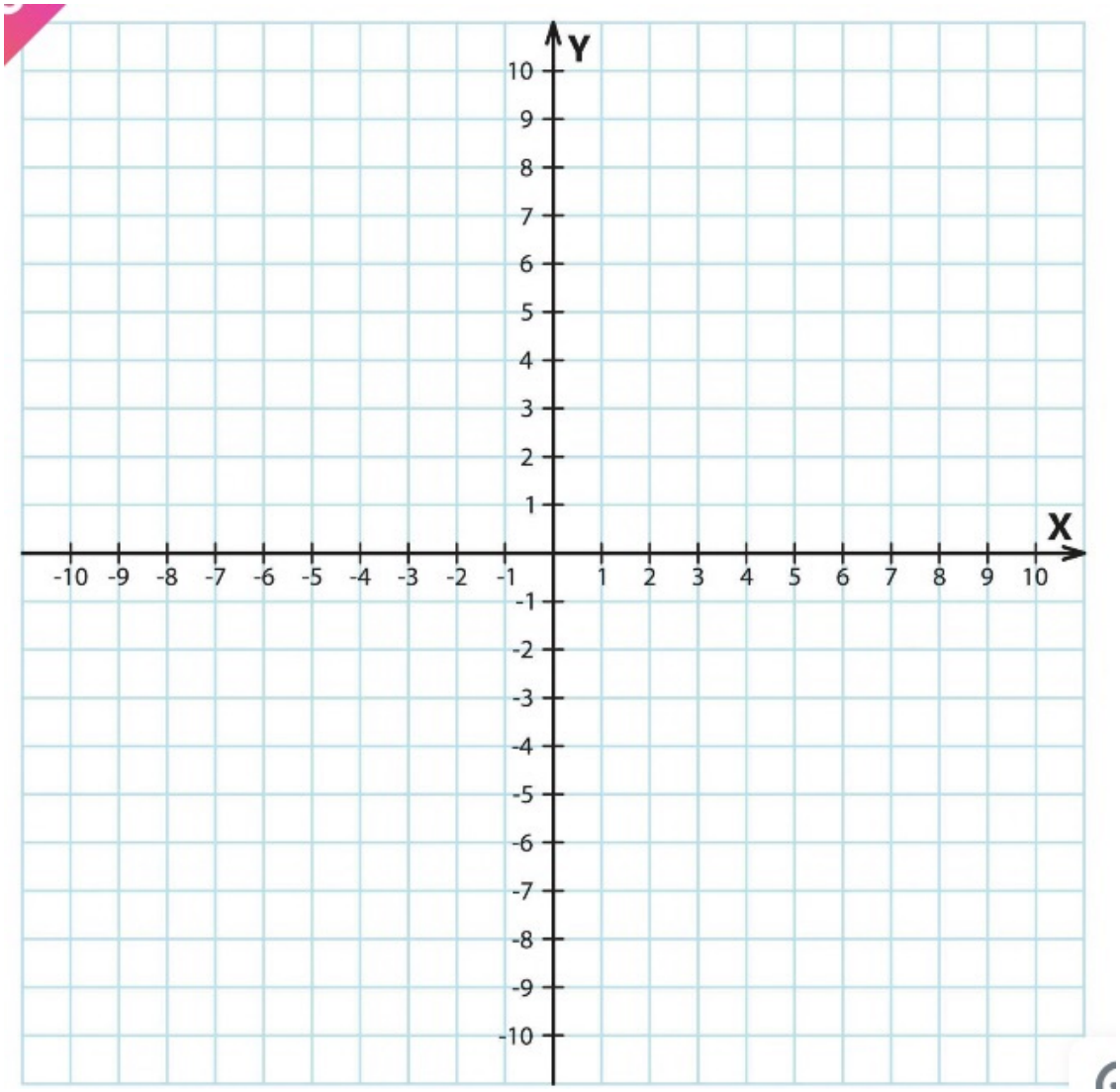
## Exponential Graphs

$y = 2^x$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |

$y = \left(\frac{1}{2}\right)^x$

| x | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y | | | | | |



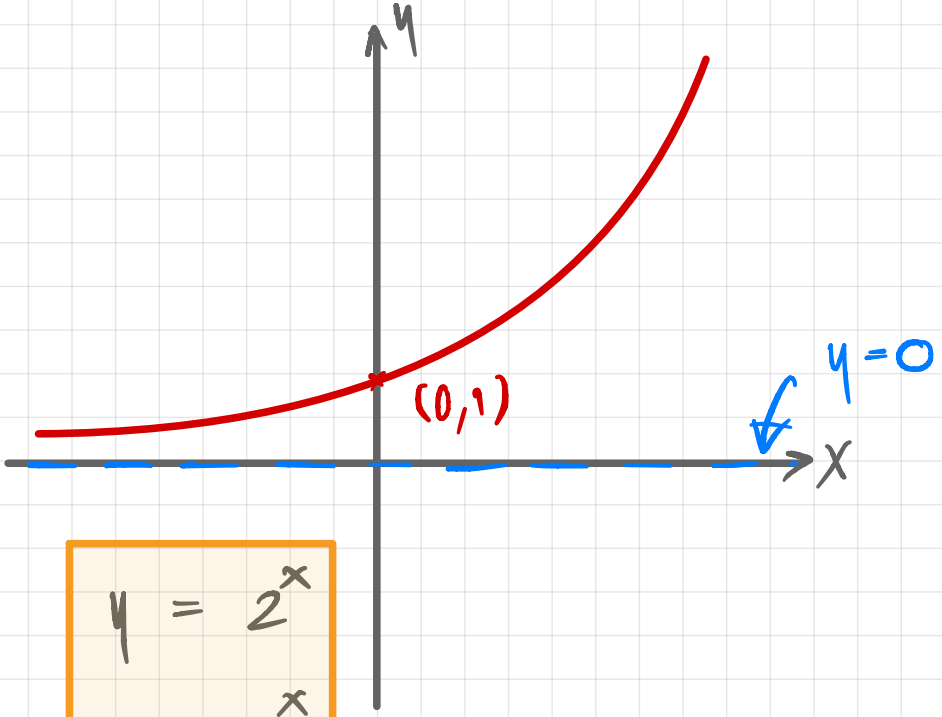
# Graphs:Harder Graphs.



## Exponential Graphs

$y = a^x$ ⟶ Asymptote ⟶ $y = 0$

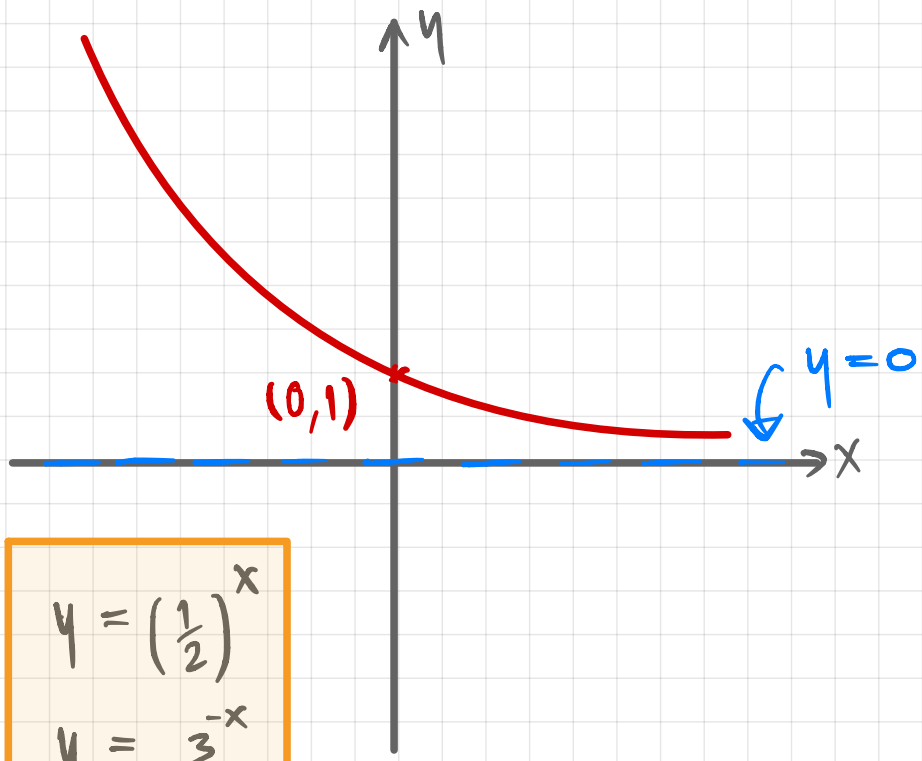
$a > 1$




$y = 2^x$

$y = e^x$

$0 < a < 1$




$y = \left(\frac{1}{2}\right)^x$

$y = 3^{-x}$

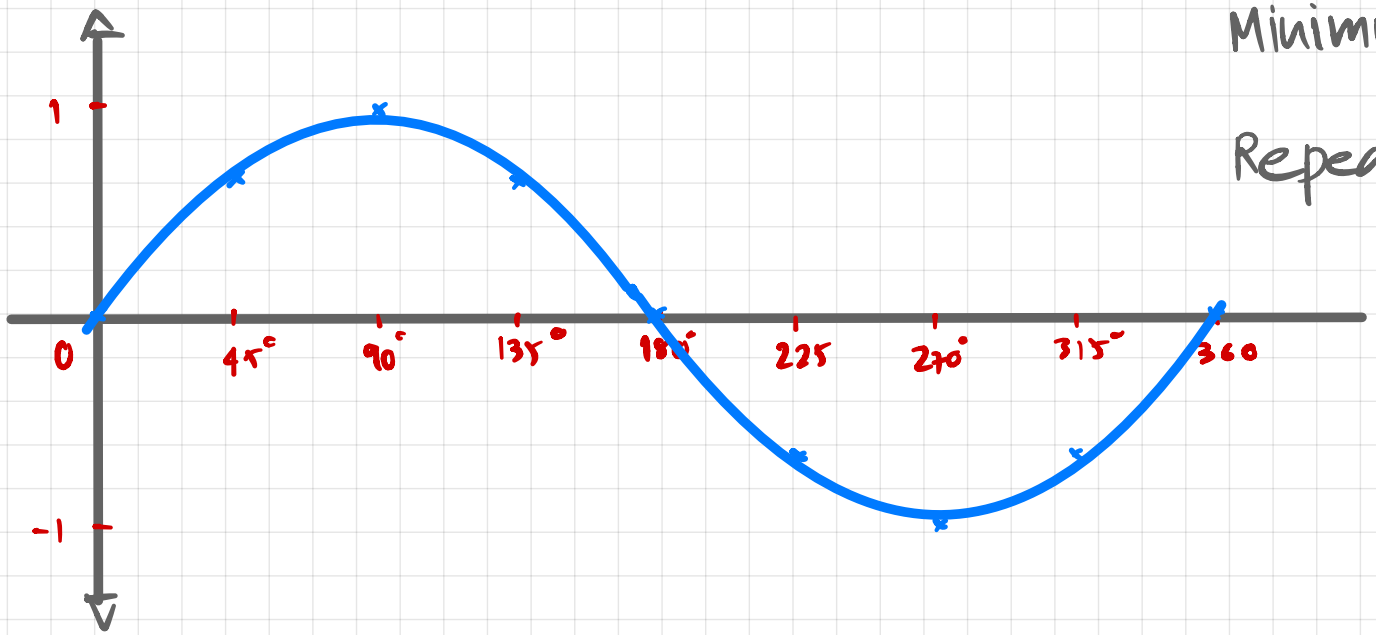
# Graphs:Harder Graphs.



## Trigonometry Graphs

$y = \sin(x)$ for $0 \leq x \leq 360$

| x | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| y | 0 | 0.7 | 1 | 0.7 | 0 | -0.7 | -1 | -0.7 | 0 |




Maximum = 1
Minimum = -1
} Amplitude = 1

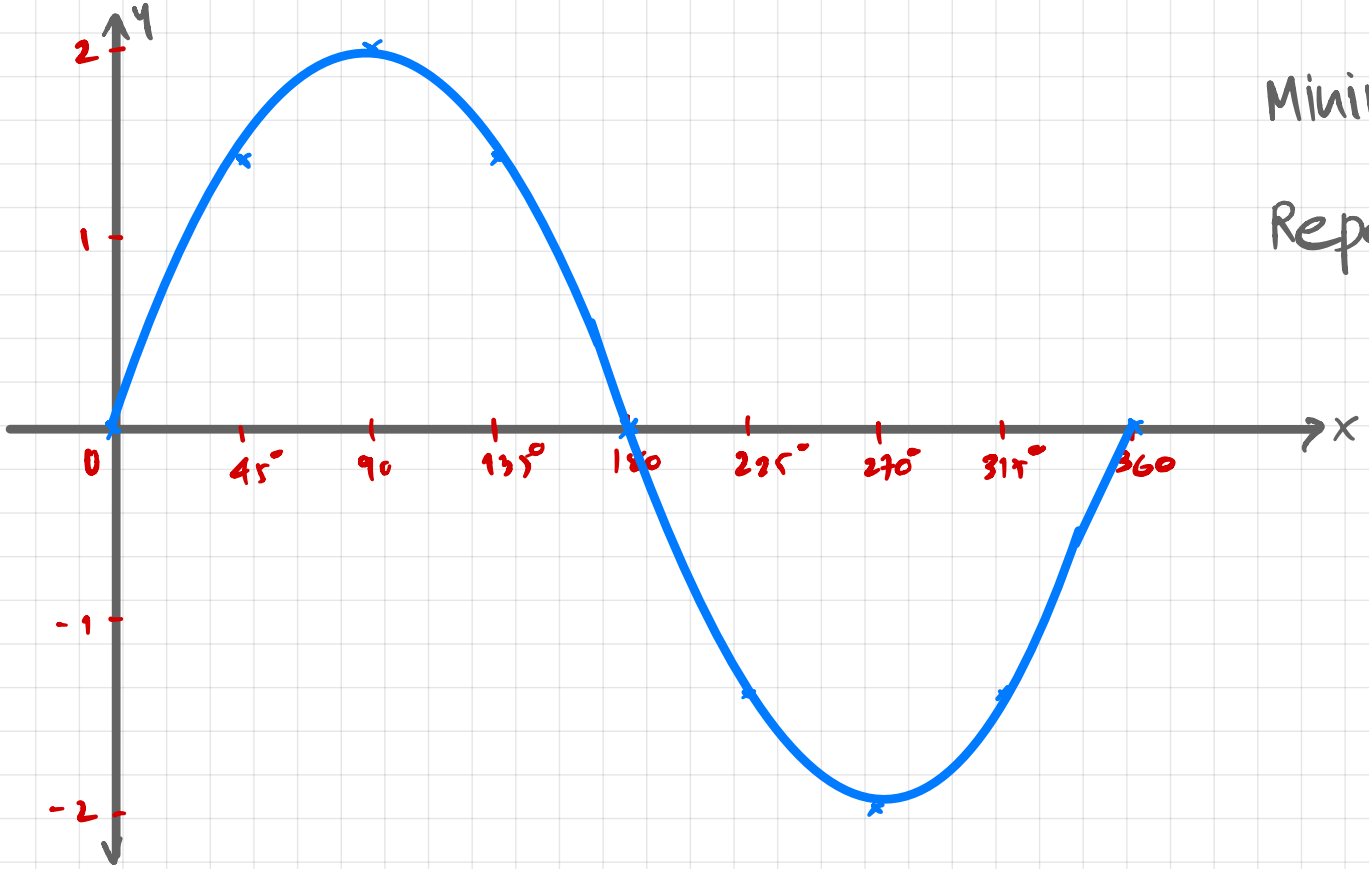
Repeats every 360°
↳ Period = 360°

# Graphs:Harder Graphs.



$y = 2\sin(x)$ for $0 \leq x \leq 360$

| x | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| y | 0 | 1.4 | 2 | 1.4 | 0 | -1.4 | -2 | -1.4 | 0 |




Maximum = 2
Minimum = -2
} Amplitude = 2

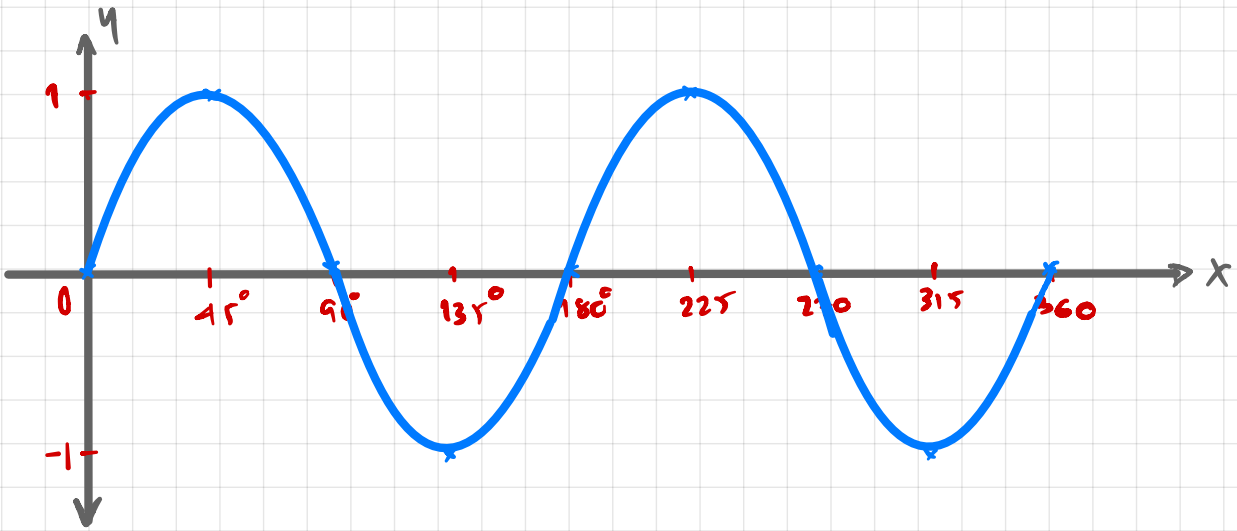
Repeats every 360°
↳ Period = 360°

# Graphs:Harder Graphs.



$y = \sin(2x)$ for $0 \leq x \leq 360$

| x | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| y | 0 | 1 | 0 | -1 | 0 | 1 | 0 | -1 | 0 |




Maximum = 1
Minimum = -1
} Amplitude = 1

Repeats every = 180°

↳ Period = 180°

$180 = \frac{360}{2}$

# Graphs:Harder Graphs.



$A > 0$

$y = A\sin(BX)$ → Maximum $y = A$, Minimum $y = -A$ } Amplitude $= A$

Repeats every (Period) $= \frac{360}{B}$

$y = 3\sin(2X)$

$y = \frac{1}{2}\sin(3X)$

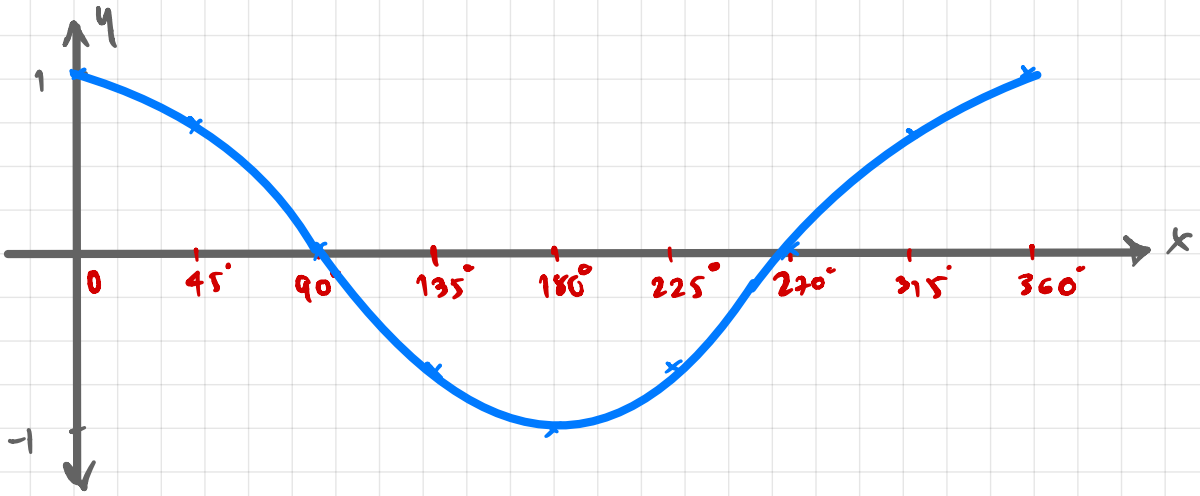
$y = 5\sin(\frac{1}{2}X)$

# Graphs:Harder Graphs.



$y = \cos(x)$ for $0 \leq x \leq 360$

| x | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| y | 1 | 0.7 | 0 | -0.7 | -1 | -0.7 | 0 | 0.7 | 1 |




Maximum = 1
Minimum = -1
} Amplitude = 1

Repeats every 360°
↳ Period = 360°

# Graphs:Harder Graphs.



$A > 0$

$y = A\cos(Bx)$ $\longrightarrow$ Maximum $y = A$, Minimum $y = -A$ } Amplitude $= A$

Repeats every (Period) $= \frac{360}{B}$

$y = 2\cos\left(\frac{1}{3}x\right)$

$y = \frac{1}{3}\cos(2x)$

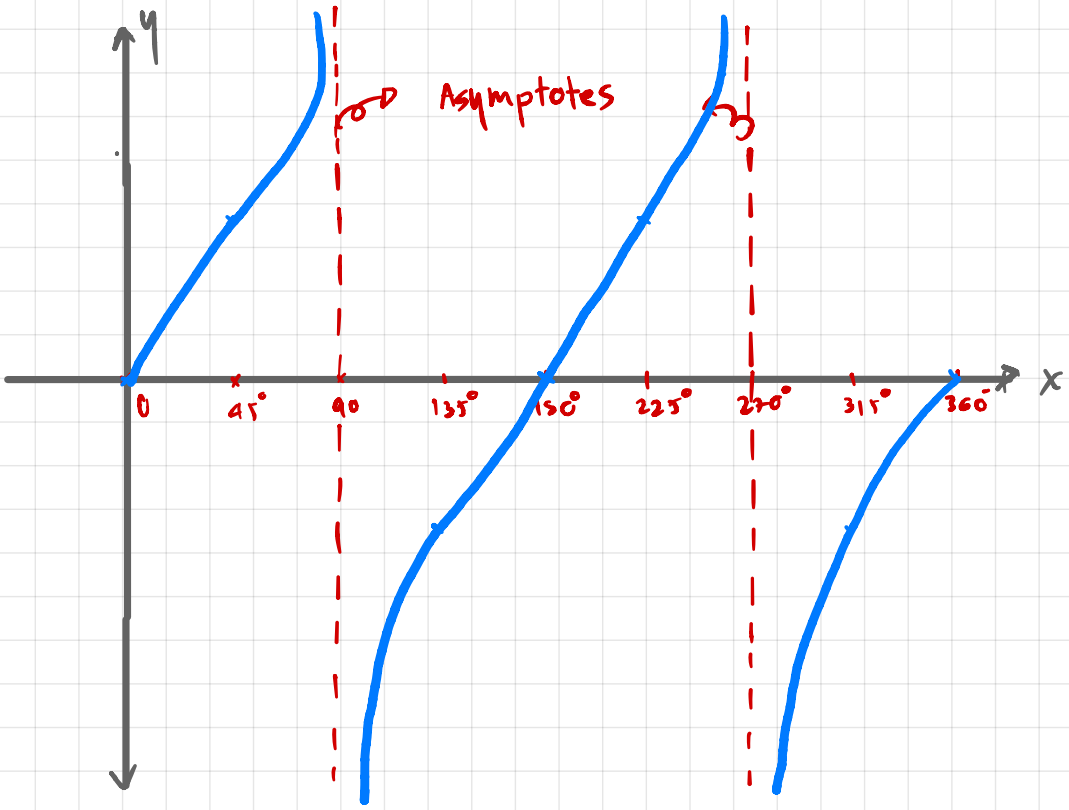
$y = 5\cos(x)$

# Graphs:Harder Graphs.



$y = \tan(x)$ for $0 \leq x \leq 360$

| x | 0° | 45° | 90° | 135° | 180° | 225° | 270° | 315° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| y | 0 | 1 | – | -1 | 0 | 1 | – | -1 | 0 |




Maximum –

Minimum –

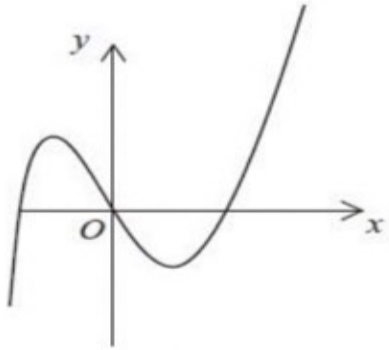
Repeats every = 180°

Asymptotes $x = 90$ , $x = 270$

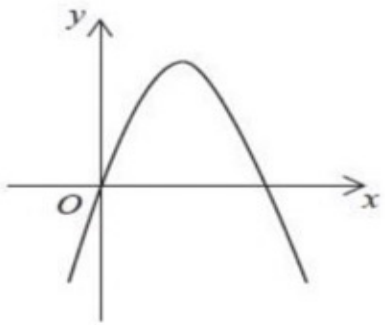
# Graphs:Harder Graphs.



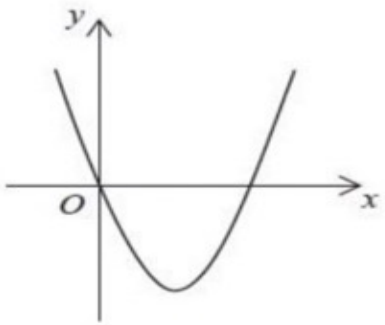
**Example:**



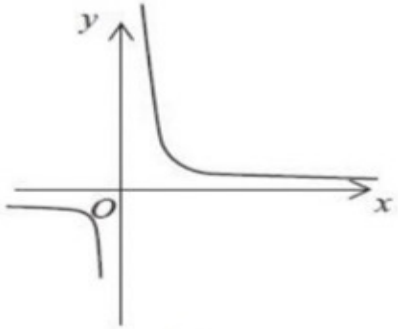

A



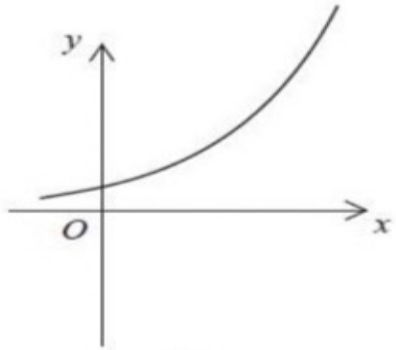

B




C




D




E

The table shows the equations of some graphs.

| Equation | Graph |
|---|---|
| $y = 4^x$ | |
| $y = -x(x - 4)$ | |
| $y = x^3 - x^2 - 2x$ | |
| $xy = 8$ | |
| $y = x^2 - 4x$ | |

Match the letter of the graph with its equation.