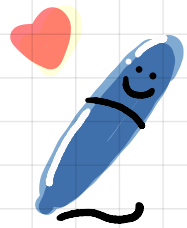
3.6) Graph Transformation



Graph Transformations.

Transformations

Translations

Translations by $\begin{pmatrix} x \\ 0 \end{pmatrix}$ x direction

$\begin{pmatrix} 0 \\ y \end{pmatrix}$ y direction

Reflections

Reflections in the x-axis $(y=0)$

y-axis $(x=0)$

Stretches

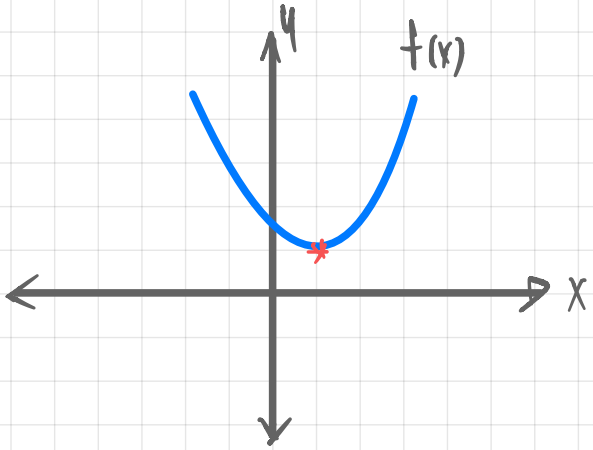
Stretches in the x direction → scale factor = ...

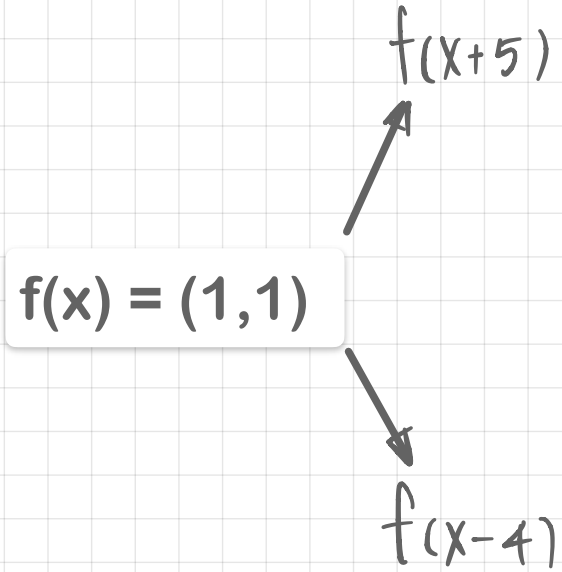
y direction

Graph Transformations.

f(x+a)

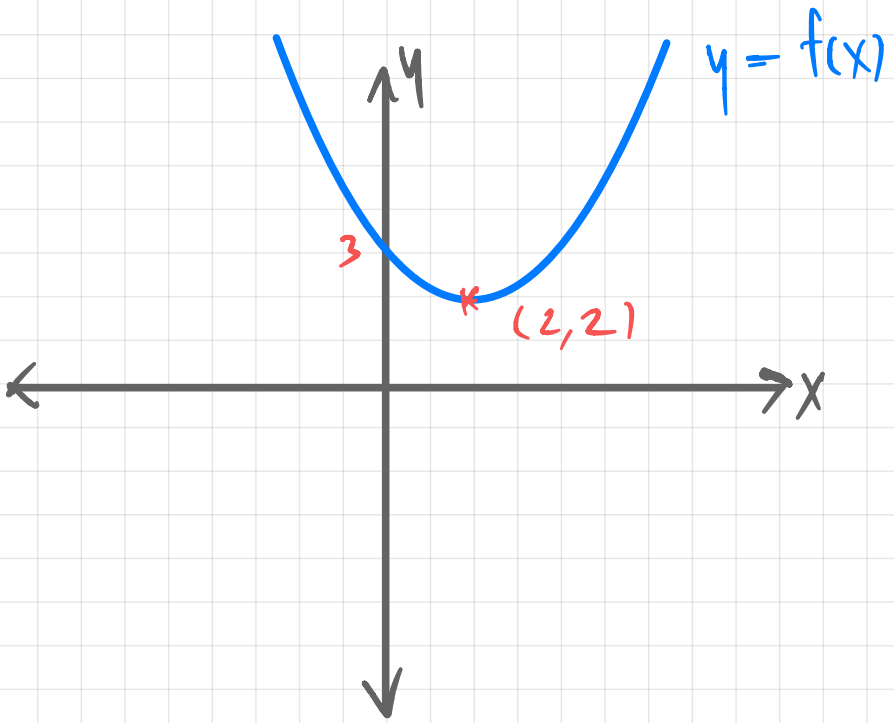
(x,y) >>>>> (x-a,y) >>>>> Translation in the x-direction

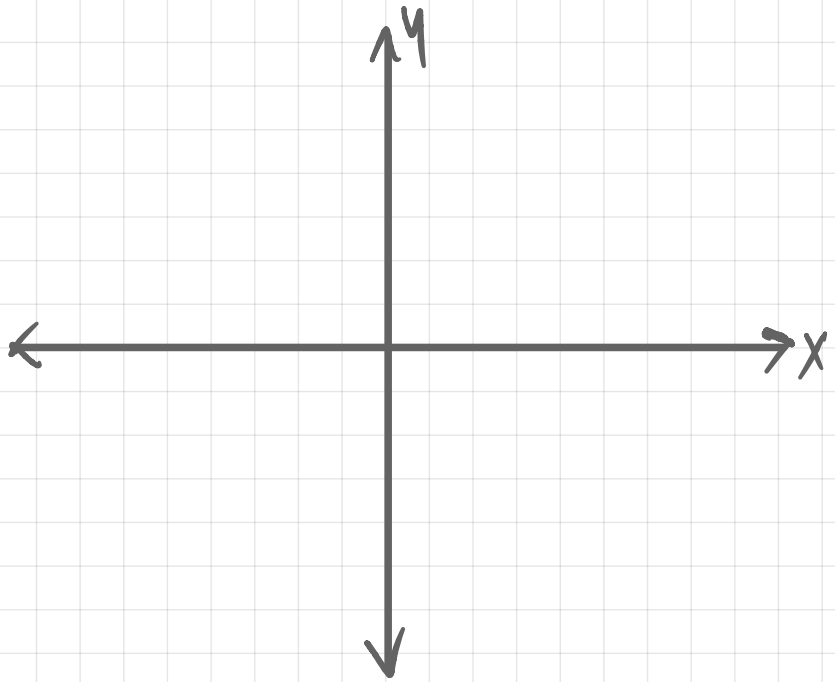




Graph Transformations.

Example: Sketch $y = f(x+2)$

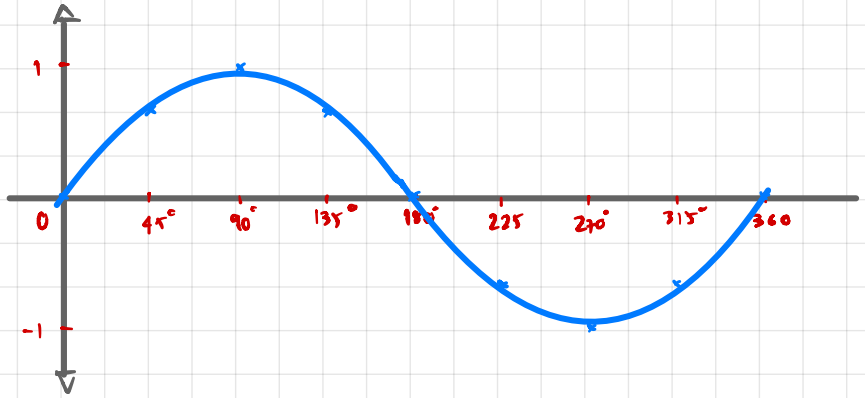




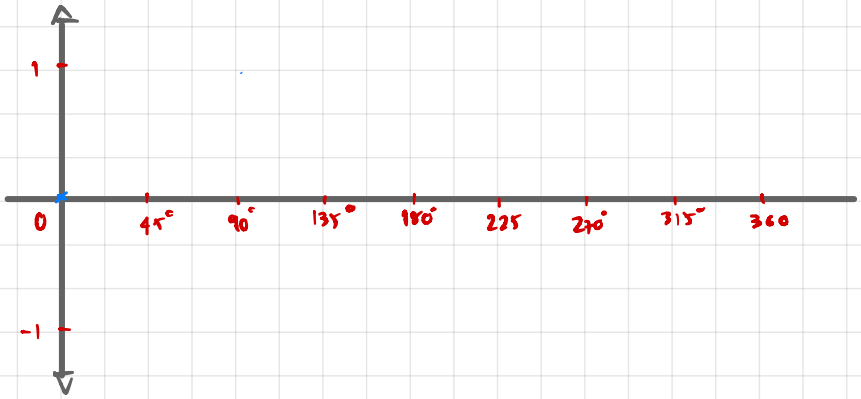
Graph Transformations.

Example:

$y = \sin(x)$



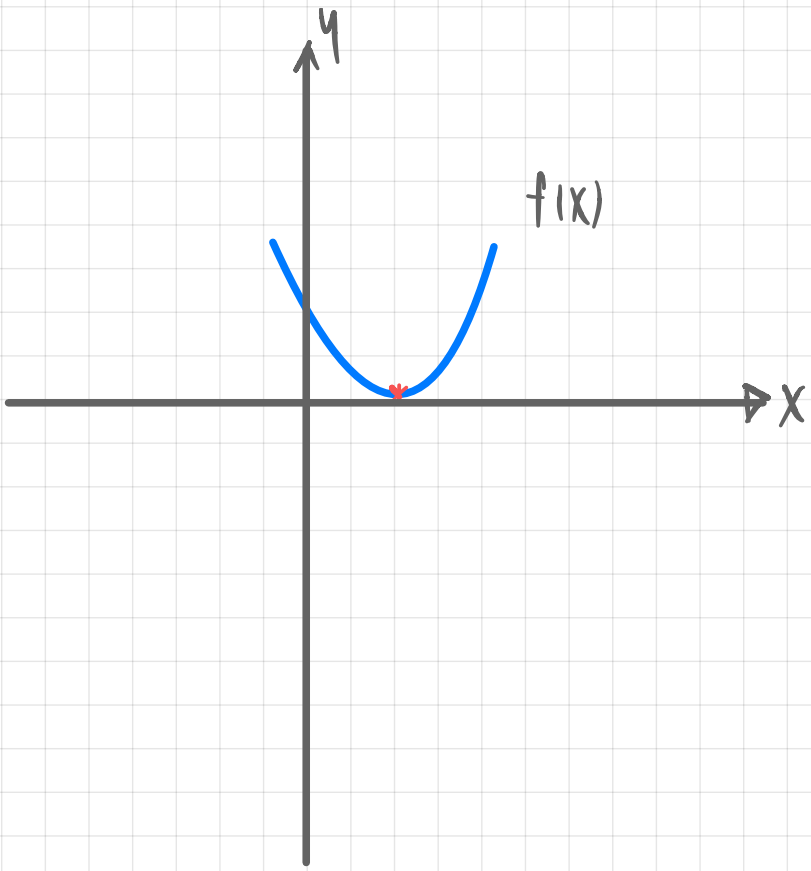
$y = \sin(x - 45°)$



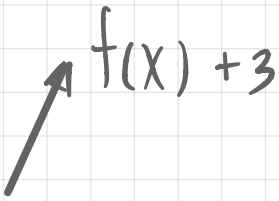
Graph Transformations.

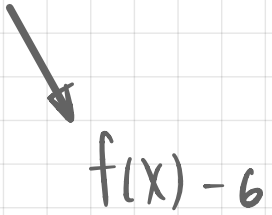
f(x)+a

(x,y) >>>>> (x,y+a) >>>>> Translation in the y-direction



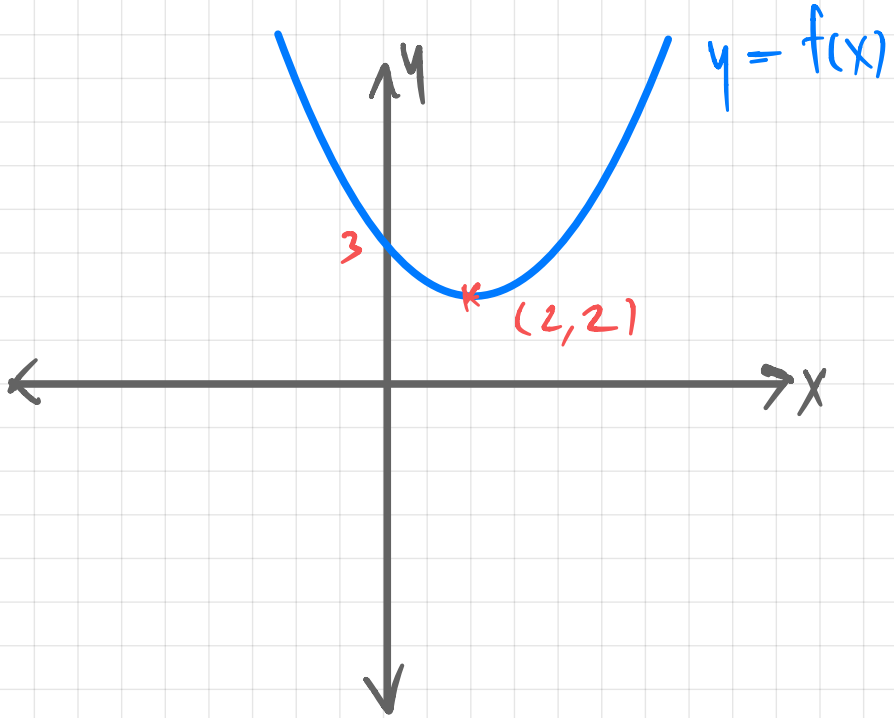
f(x) = (2,0)

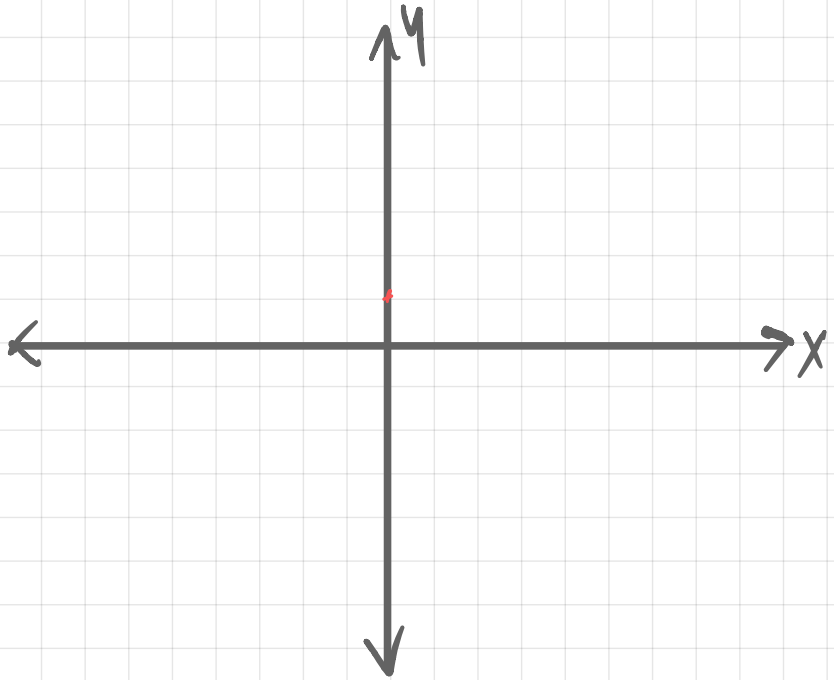




Graph Transformations.

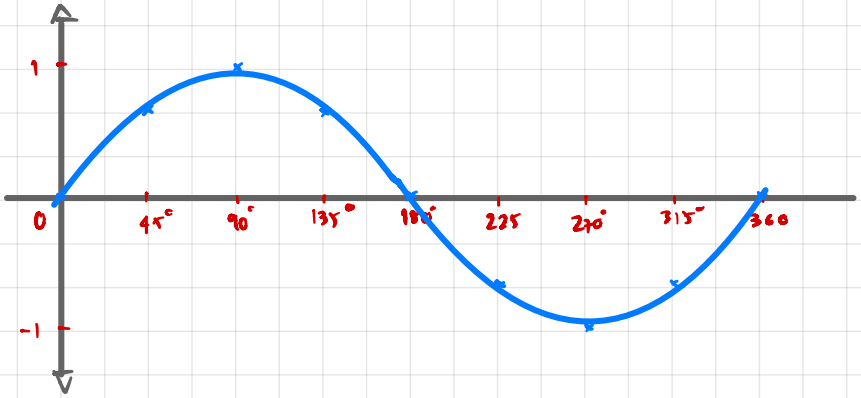
Example: Sketch $y = f(x) - 2$



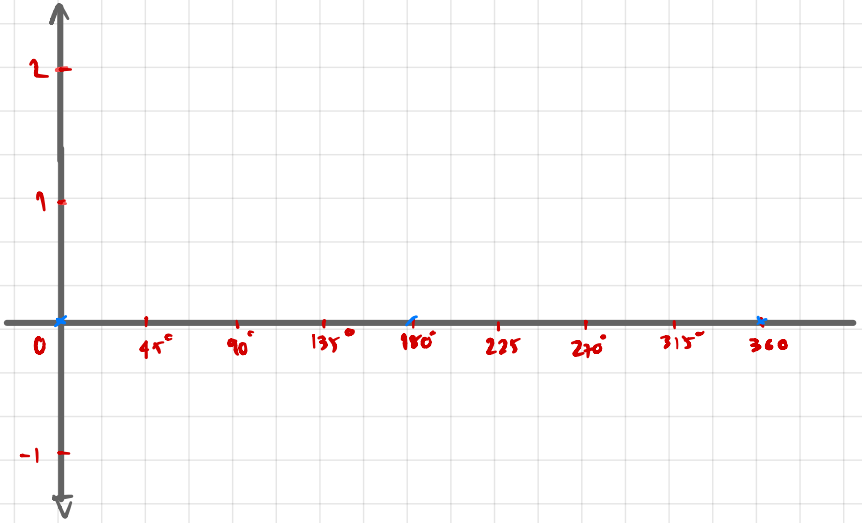


Graph Transformations.

Example: $y = \sin(x)$



$y = 1 + \sin(x)$



Graph Transformations.

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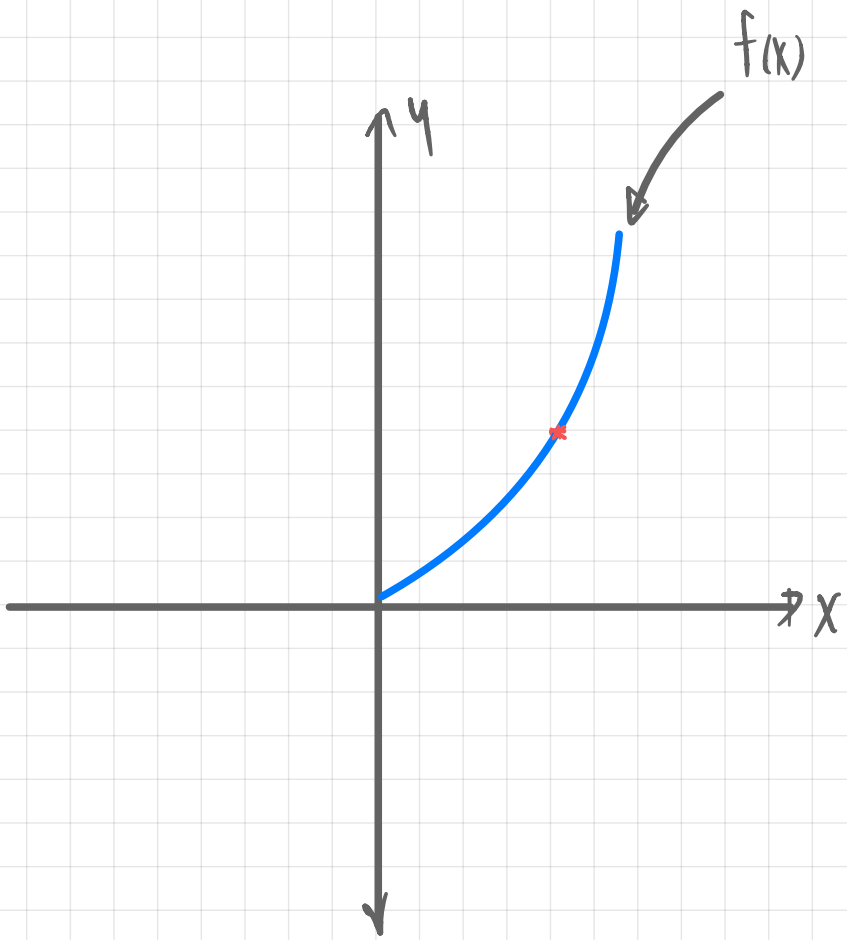
f(ax)

(x,y) >>>>> (x/a,y) >>>>> Stretches in the x direction,scale factor 1/a



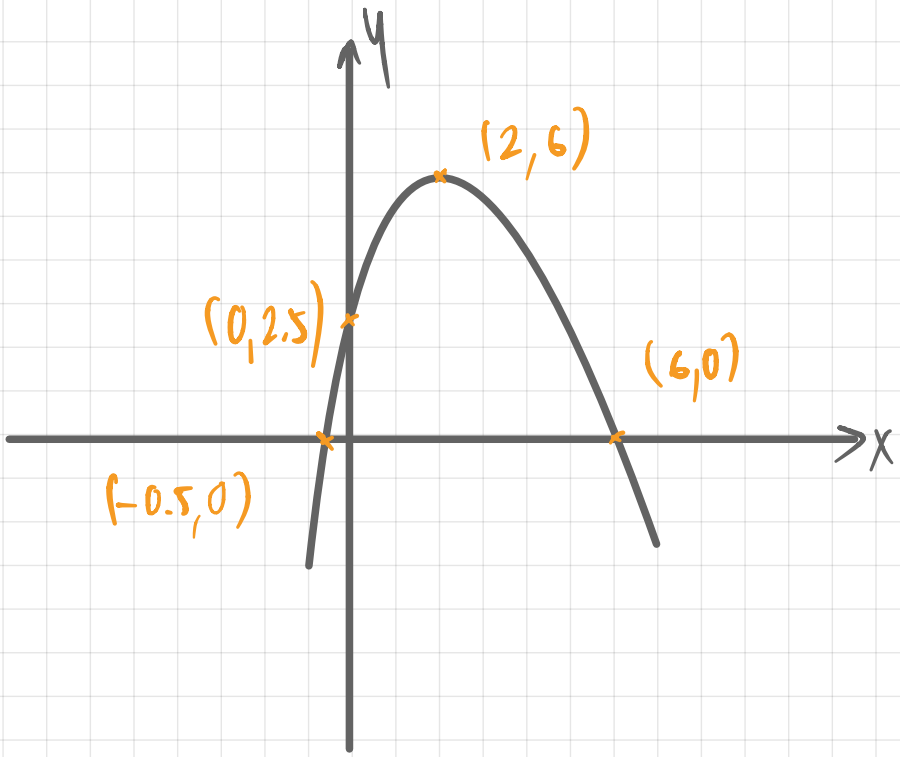
f(x) = (4,4)

f(2x)

$f(\frac{1}{2}x)$

Graph Transformations.

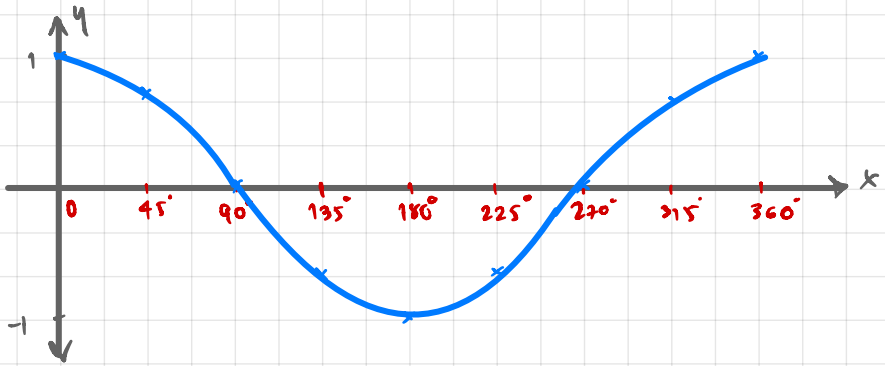
Example: Sketch $y = f(\frac{1}{2}x)$



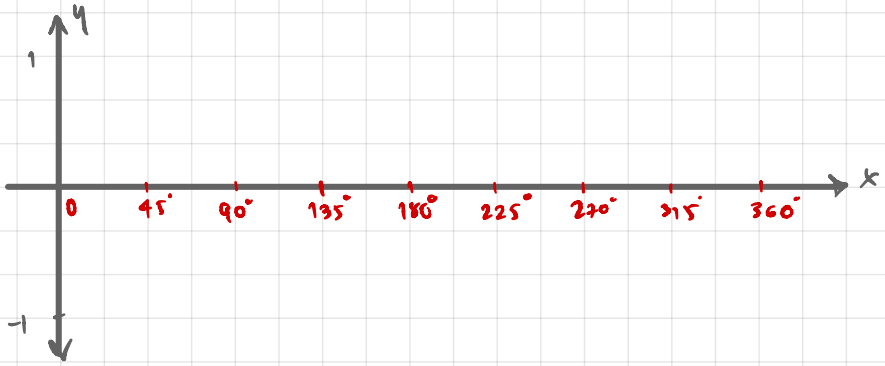
Graph Transformations.

Example:

$y = \cos(x)$



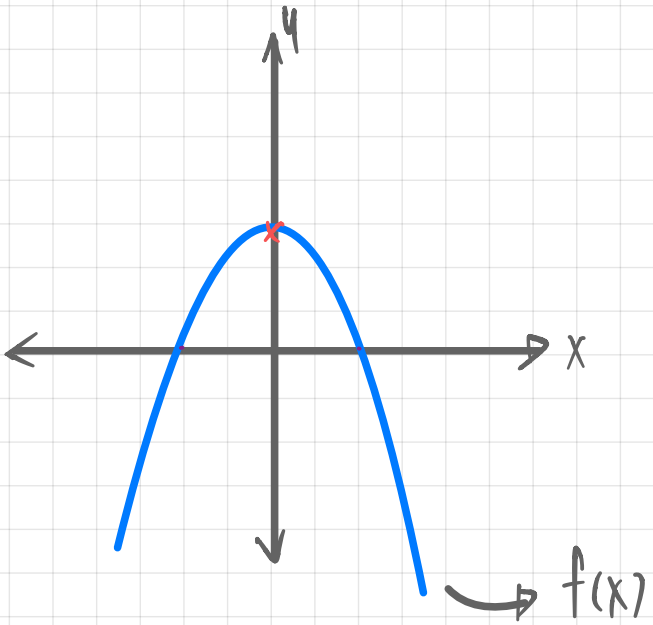
$y = \cos(2x)$



Graph Transformations.

af(x)

(x,y) >>>>> (x,ay) >>>>> Stretches in the y-direction,scale factor a.



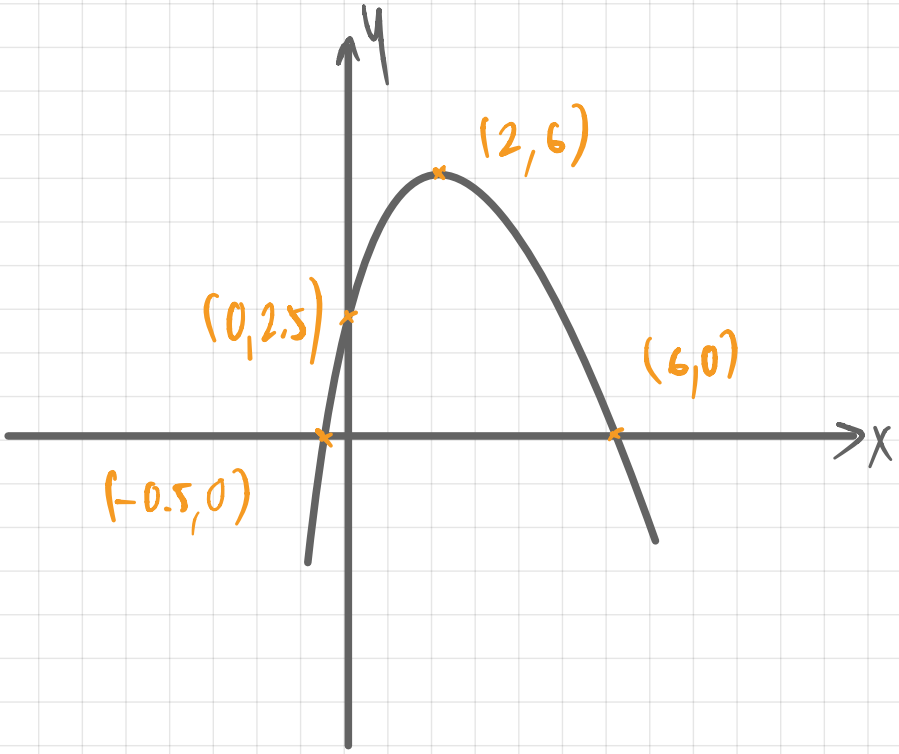
f(x) = (0,3)

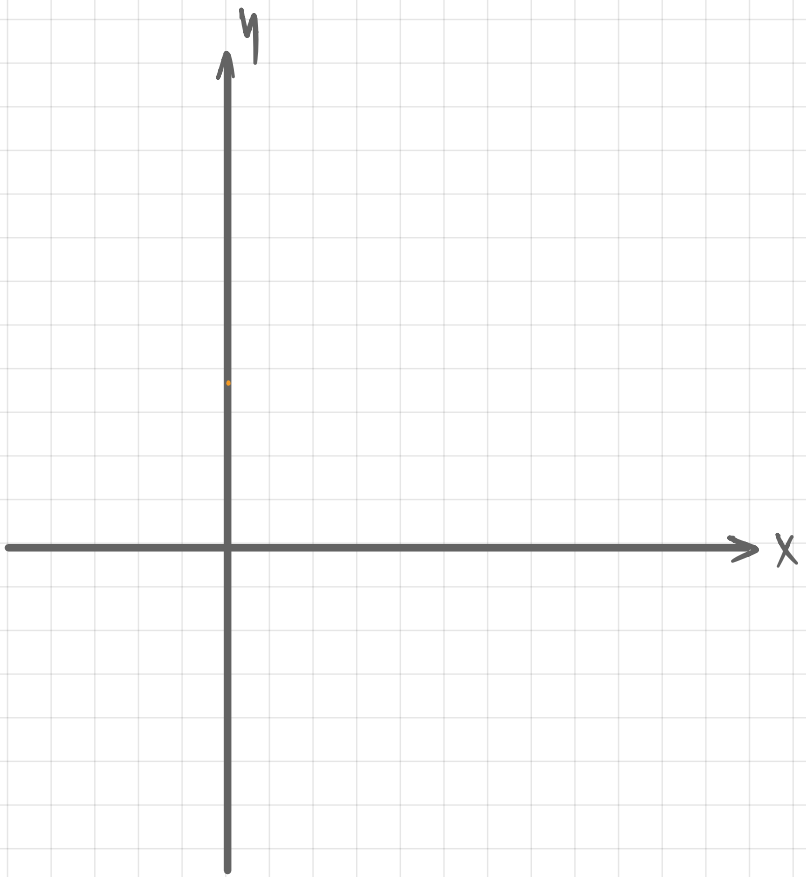
$\frac{1}{3}f(x)$

$2f(x)$

Graph Transformations.

Example: Sketch $y = \frac{3}{2} f(x)$

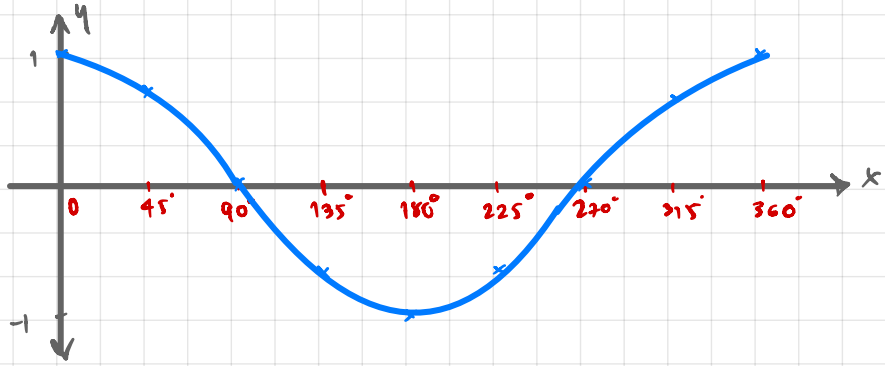




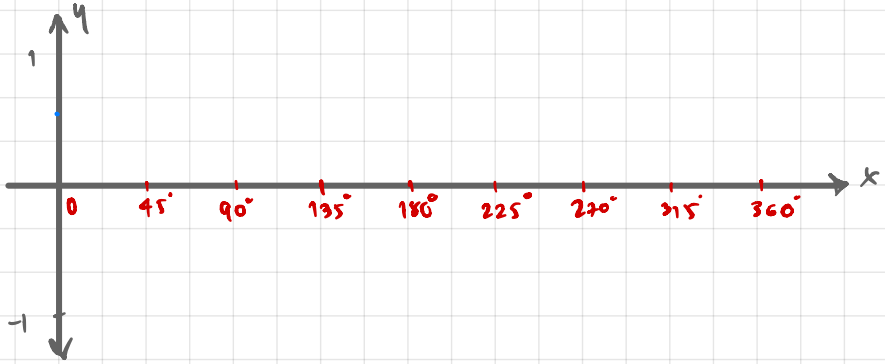
Graph Transformations.

Example:

$y = \cos(x)$ for $0 \leq x \leq 360$



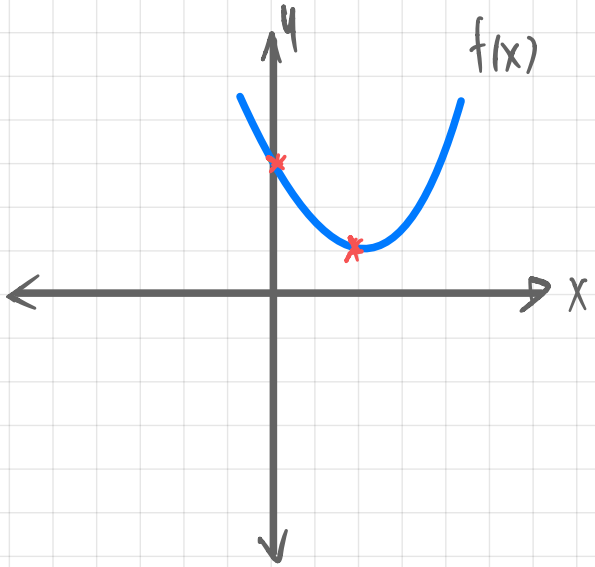
$y = \frac{1}{2}\cos(x)$ for $0 \leq x \leq 360$



Graph Transformations.

f(-x)

(x,y) >>>>> (-x,y) >>>>> Reflect in y-axis

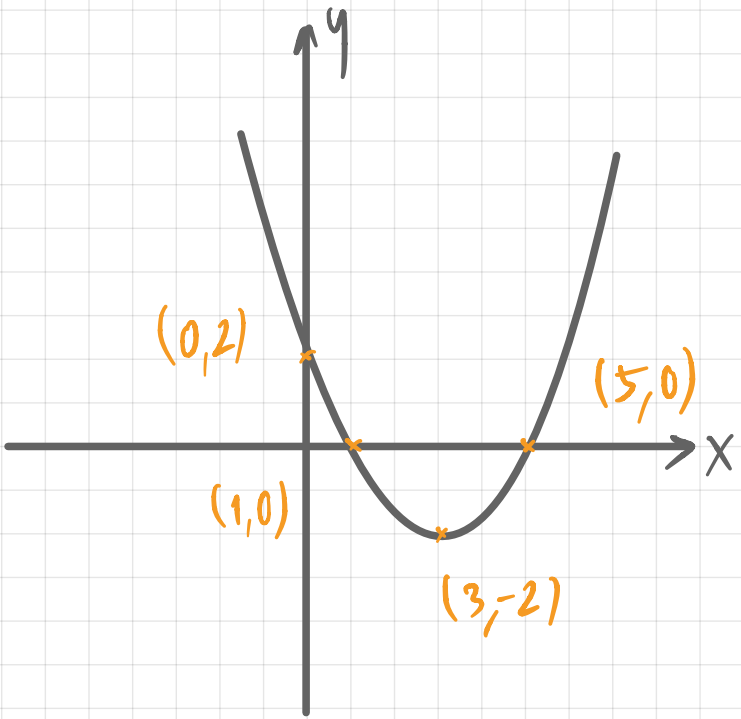


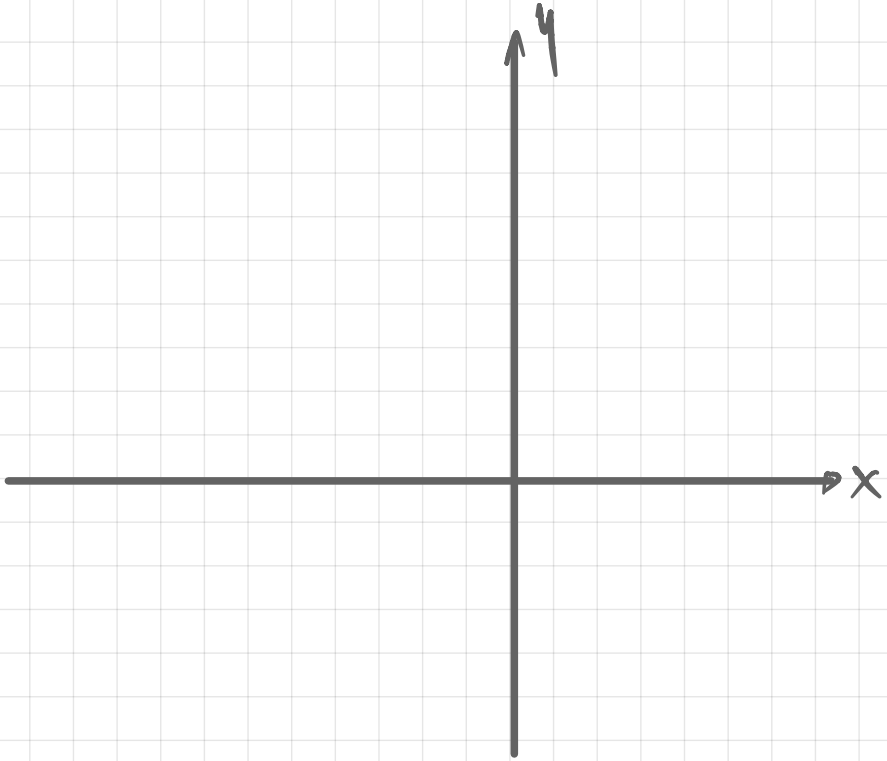
f(x) = (2,1)

f(x) = (0,3)

Graph Transformations.

Example: Sketch $y = f(-x)$

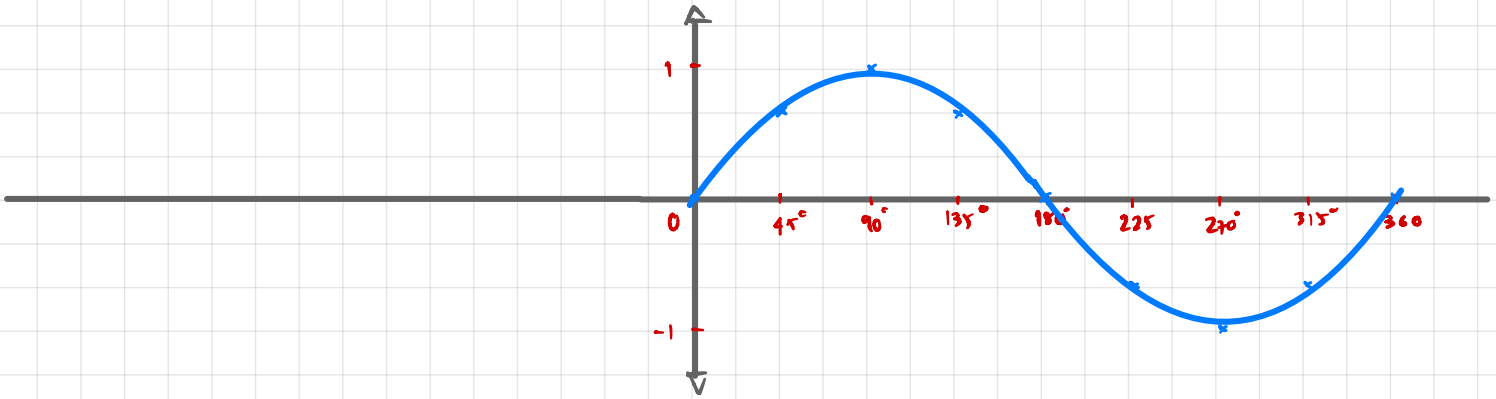




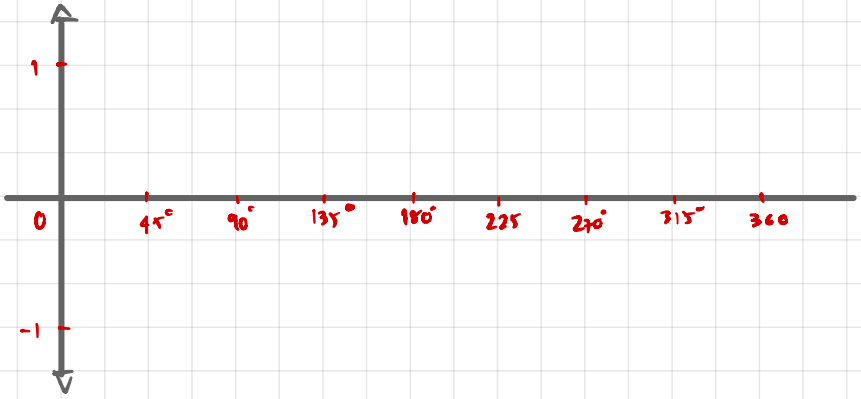
Graph Transformations.

Example:

$y = \sin(x)$ for $0 \leq x \leq 360$



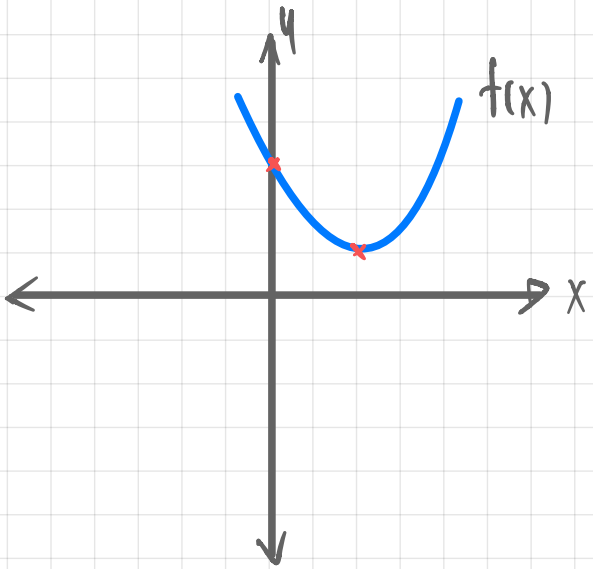
$y = \sin(-x)$ for $0 \leq x \leq 360$



Graph Transformations.

-f(x)

(x,y) >>>>> (x,-y) >>>>> Reflect in x-axis

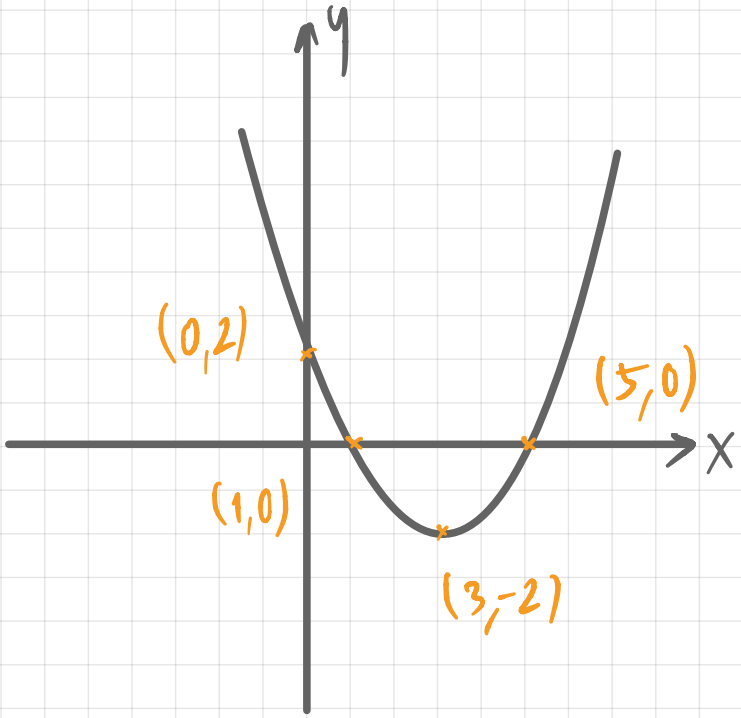


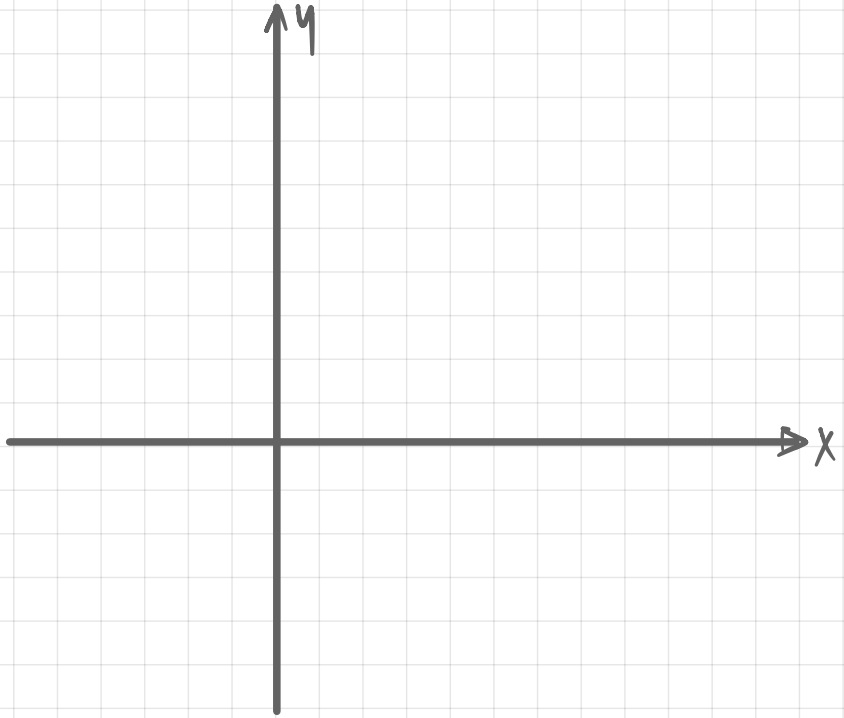
f(x) = (2,1)

f(x) = (0,3)

Graph Transformations.

Example: Sketch $y = -f(x)$

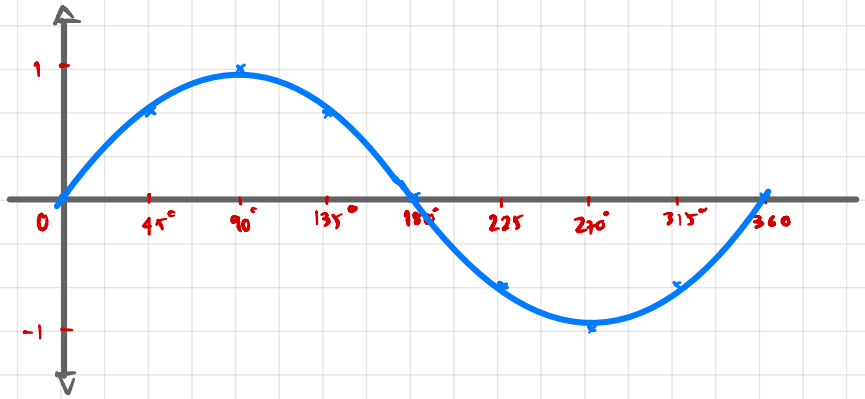




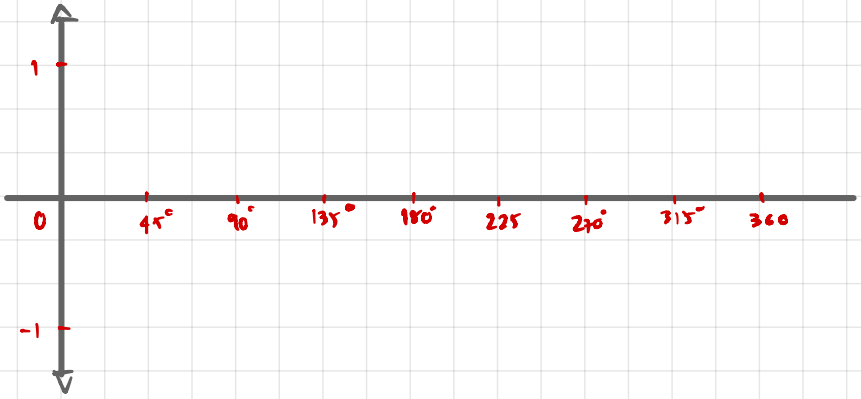
Graph Transformations.

Example:

$y = \sin(x)$ for $0 \leq x \leq 360$



$y = -\sin(x)$ for $0 \leq x \leq 360$



Graph Transformations.

Example:
The coordinates of the maximum point of the graph y=f(x) are (-2,5).
Give the coordinates of maximum point of the graph with equation:

1) y=f(x/2)

2) y=f(2x)

3) y=f(x+3)

Graph Transformations.

4) $y=-f(x)$

5) $y=5f(x)$

6) $y=f(x)+3$

7) $y=f(x-4)$

Graph Transformations.

8) $y=2f(x)+3$

9) $y=f(-x)+2$

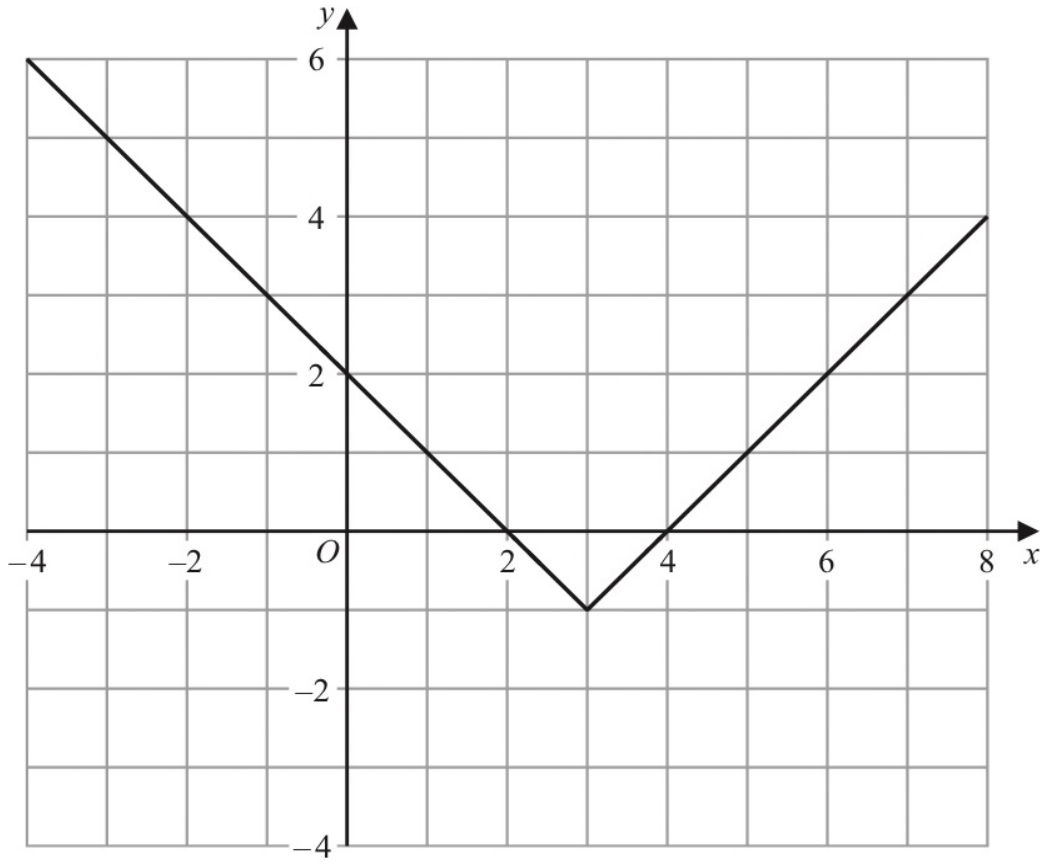
10) $y=f(0.5x)-4$

Graph Transformations.

Example:

g(x) 2g(x)

The graph of $y = g(x)$ is shown on the grid below.



On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leqslant x \leqslant 7$

(2)

Graph Transformations.

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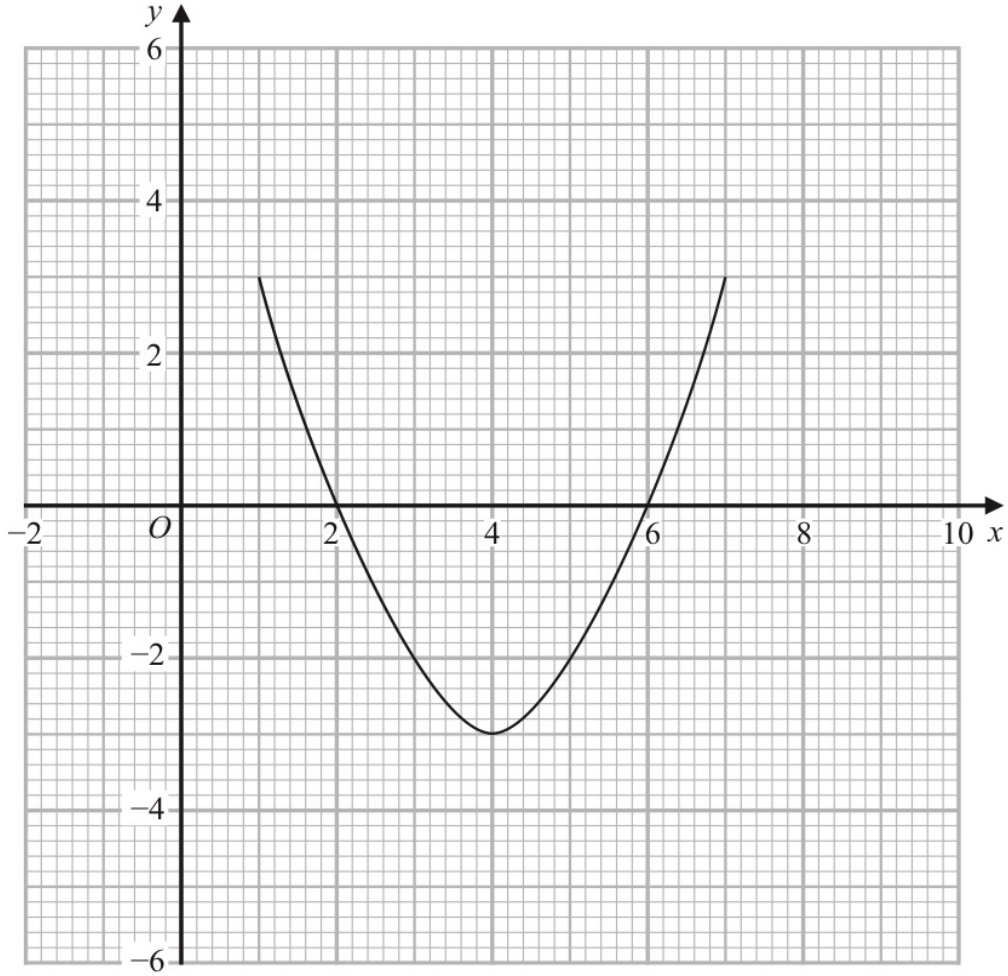
Example:

$f(x - 1)$ $\frac{1}{2}f(x - 1)$

The diagram shows the graph of $y = f(x)$ $\longrightarrow$ $Y = \frac{1}{2}f(X - 1)$



Graph Transformations.

Extra

$f(x) = x^2$

$f(x) - 5 \longrightarrow$

$f(x+2) \longrightarrow$

$3f(x) \longrightarrow$

$2f(x) + 3 \longrightarrow$

$5f(x-3) \longrightarrow$

$3f(x+2) - 5 \longrightarrow$

$-f(x) \longrightarrow$

$-f(x) + 5 \longrightarrow$

$-2f(x-3) \longrightarrow$

$-f(x+2) + 3 \longrightarrow$

Graph Transformations.

Example: $f(x) = x^2$

1) $g(x) = x^2 + 5$

Graph Transformations.

Example: $f(x) = x^2$

2) $h(x) = -(x-3)^2$

Graph Transformations.

Example: $f(x) = x^2$

3) $g(x) = 3(x-1)^2 - 2$

Graph Transformations.

Graph Transformations.

Example: The curve C has equation $y = f(x)$ where $f(x) = x^2$

The curve S has equation $y = g(x)$ where $g(x) = 2x^2 - 8x + 5$

By writing $g(x)$ in the form $a(x-b)^2 - c$.

Describe fully a series of transformations that map the curve C onto the curve S

Graph Transformations.

Example: The curve c has equation $y = f(x)$ where $f(x) = x^2$

The curve s has equation $y = g(x)$ where $g(x) = 2x^2 - 8x + 5$

By writing $g(x)$ in the form $a(x-b)^2 - c$.

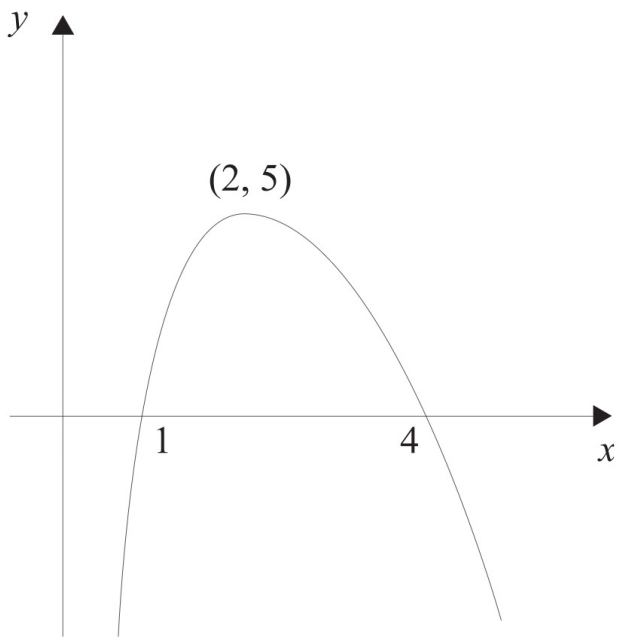
Describe fully a series of transformations that map the curve c onto the curve s

Graph Transformations.

Example:

The maximum point on the curve with equation $y = f(x + a)$ is on the y-axis.

Write down the value of the constant a.



Graph Transformations.

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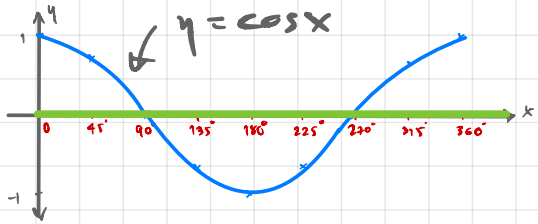
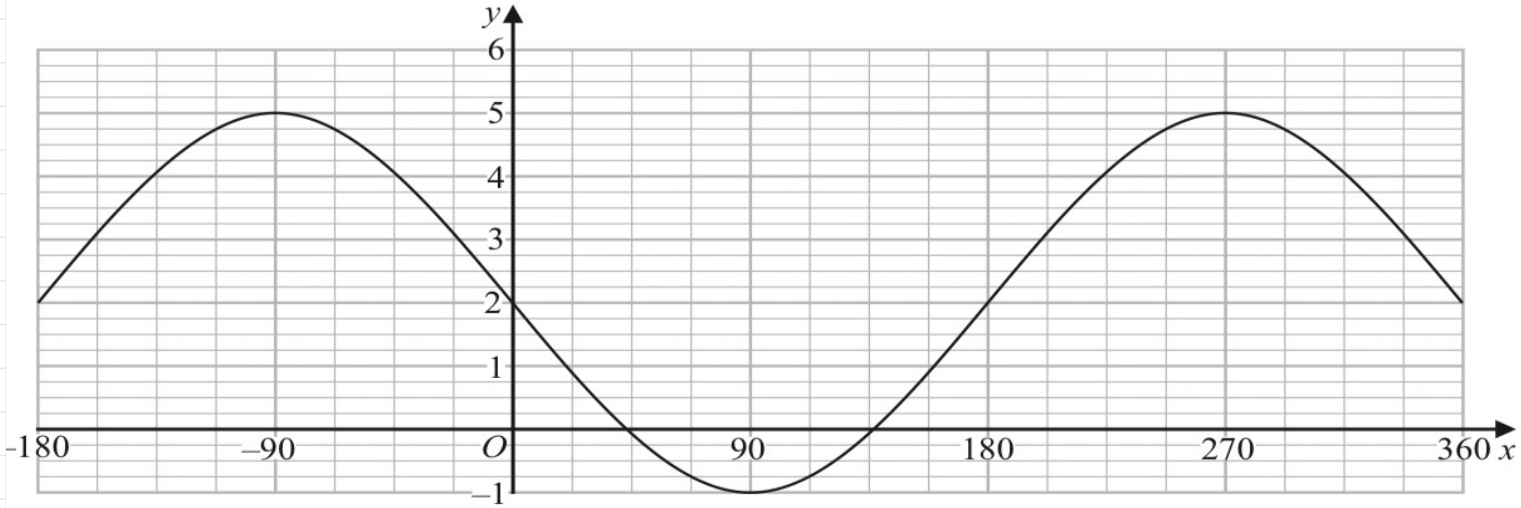
Example:



The graph of $y = a\cos(x - b)° + c$ for $-180 \leqslant x \leqslant 360$ is drawn on the grid below.



(d) Find the value of a, the value of b and the value of c.

$a =$ .....

$b =$ .....

$c =$ .....