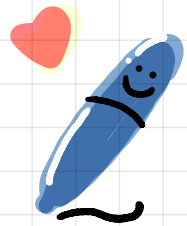
3.7) Differentiation



Differentiation.

Review:Indices

ax^n

$a = ax^0$

$ax = ax^1$

$(x^m)(x^n) = x^{m+n}$

$\frac{x^m}{x^n} = x^{m-n}$

$\frac{1}{x^m} = 1x^{-m}$

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[3]{x} = x^{\frac{1}{3}}$

Differentiation.

Simplify ax^n [Before Differentiation]

Example

1) $y = 4x^3 - 2x^{-1} + 7x + 5$

2) $f(x) = (2x+5)(x-3)$

Differentiation.

3) $y = \dfrac{5x^3 + 4}{2x}$

4) $f(x) = 4x\sqrt{x} - 5\sqrt[3]{x}$

Differentiation.

5) $y = (3x-2)(4x-\frac{1}{x})$

Differentiation.

Differentiation:

$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots$

$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots$

$ax^n \xrightarrow{D} a(n)x^{n-1}$

$3x^5 \xrightarrow{D}$

$-4x^{\frac{1}{2}} \xrightarrow{D}$

$2x^{-3} \xrightarrow{D}$

Differentiation.

$ax \xrightarrow{D} a$

Prove! $ax = ax^1 \xrightarrow{D} a(1)x^{1-1}$

$ax^0 = a$

$5x \xrightarrow{D}$

$-100x \xrightarrow{D}$

$\frac{2}{3}x \xrightarrow{D}$

Differentiation.

$a \xrightarrow{D} 0$

$ax^n \xrightarrow{D} a(n)x^{n-1}$

Prove! $a = ax^0 \xrightarrow{D} a(0)x^{0-1}$

$= 0$

$5 \xrightarrow{D}$

$-100 \xrightarrow{D}$

$\frac{3}{2} \xrightarrow{D}$

Differentiation.

Example:Differentiation

$$y = 3x^4 + 5x - 4x^{\frac{1}{2}} + 1 - 2x^{-1}$$

Differentiation.

Example:Differentiation

$$f(x) = (3x-5)^2$$

Differentiation.

Example:Differentiation

$$y = (3x+1)(x-1)(x+2)$$

Differentiation.

Example:Differentiation

$$f(x) = 4\sqrt{x} + \frac{5}{2x}$$

Differentiation.

Example:Differentiation

$$y = \frac{3x^2 + 4x - 6}{2x^3}$$

Differentiation.

Variables

$y = 7x^2 + 3x \xrightarrow{D}$

$A = x^3 + 5 \xrightarrow{D}$

$x = 4y - 1 \xrightarrow{D}$

$A = \pi r^2 \xrightarrow{D}$

$s = 3t^2 + 5t - 2 \xrightarrow{D}$

Differentiation.

Second order derivative

$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots \xrightarrow{D} \frac{d^2y}{dx^2} = \ldots$

$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots \xrightarrow{D} f''(x) = \ldots$

$s = t\ldots \xrightarrow{D} \frac{ds}{dt} = \ldots \xrightarrow{D} \frac{d^2s}{dt^2} = \ldots$

Differentiation.

Example: Find second order derivative.

1) $y = 5x^2 + 3x - 4$

Differentiation.

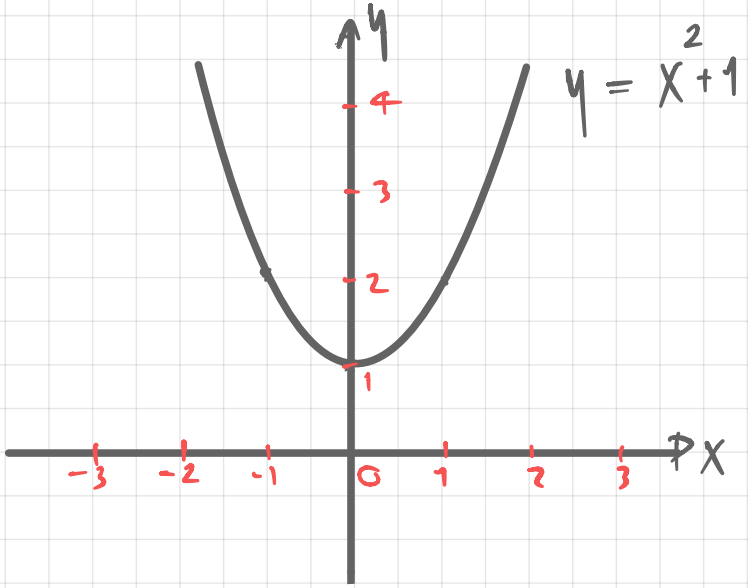
2) $f(x) = \frac{2}{x} + \sqrt{x}$

Differentiation.

3) $s = 3t^2 + \frac{2}{t}$

Differentiation.

Differentiation to find a Gradient.



1) Differentiating the equation of curve.

2) Find the gradient of the curve at any point by substituting the value for x into 1)...

Find the gradient of curve at $x = 1$

Differentiation.

Example: Find the gradient of the graph $y = x^3 - 12x + 1$ at .....

1) $x = -1$

2) $x = 2$

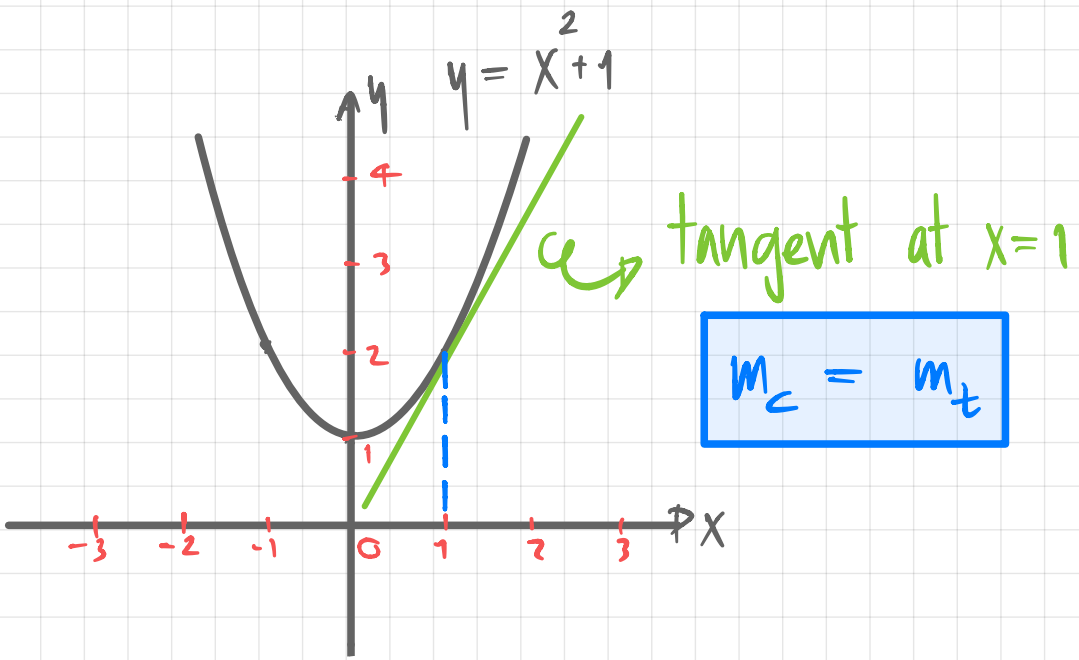
3) $x = \frac{5}{2}$

Differentiation.

Example Find coordinate of point of curve $y = x^2 + 1$ has gradient is -4.

Differentiation.

Gradient of the curve = Gradient of the tangent.



Find equation of tangent to curve

$y = x^2 + 1$ at point (1,2)

Differentiation.

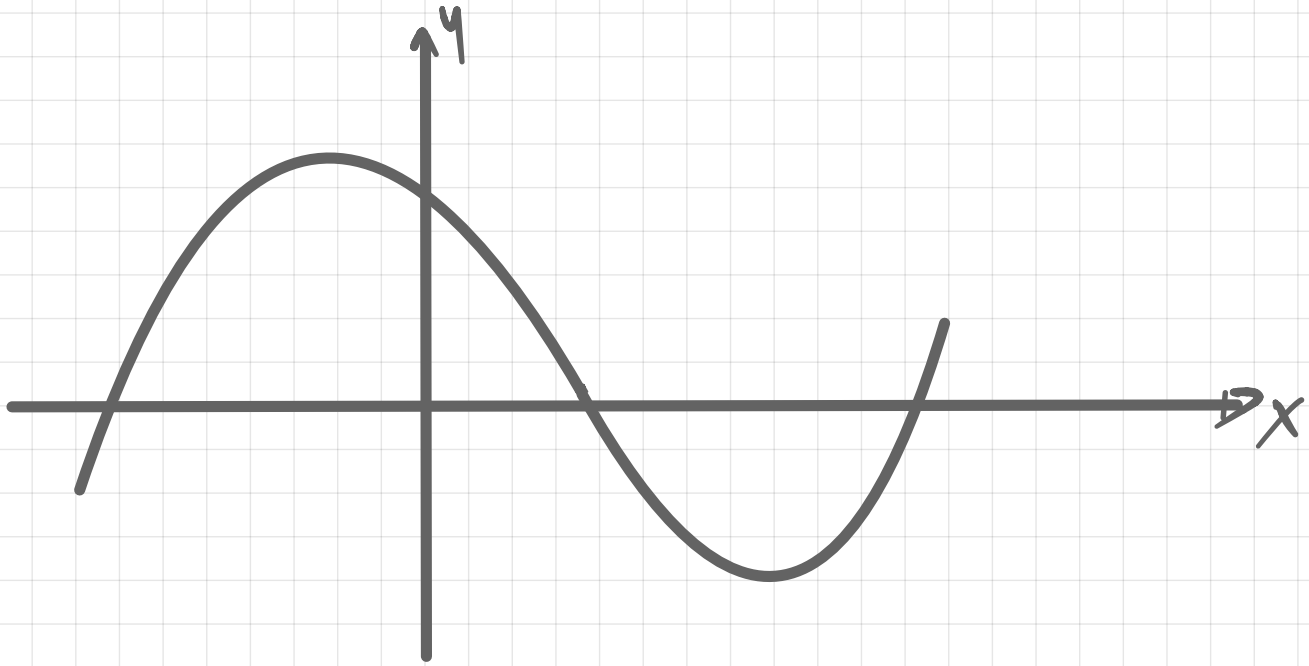
Example: Find equation of tangent to curve $y = x^3 - 3x^2 + 2x - 1$ at $(3, 5)$

Differentiation.

Example: Find equation of tangent to curve $y = \frac{4}{x} + 1$ at $x = 2$

Differentiation.

Increase function and Decrease function



Differentiation.

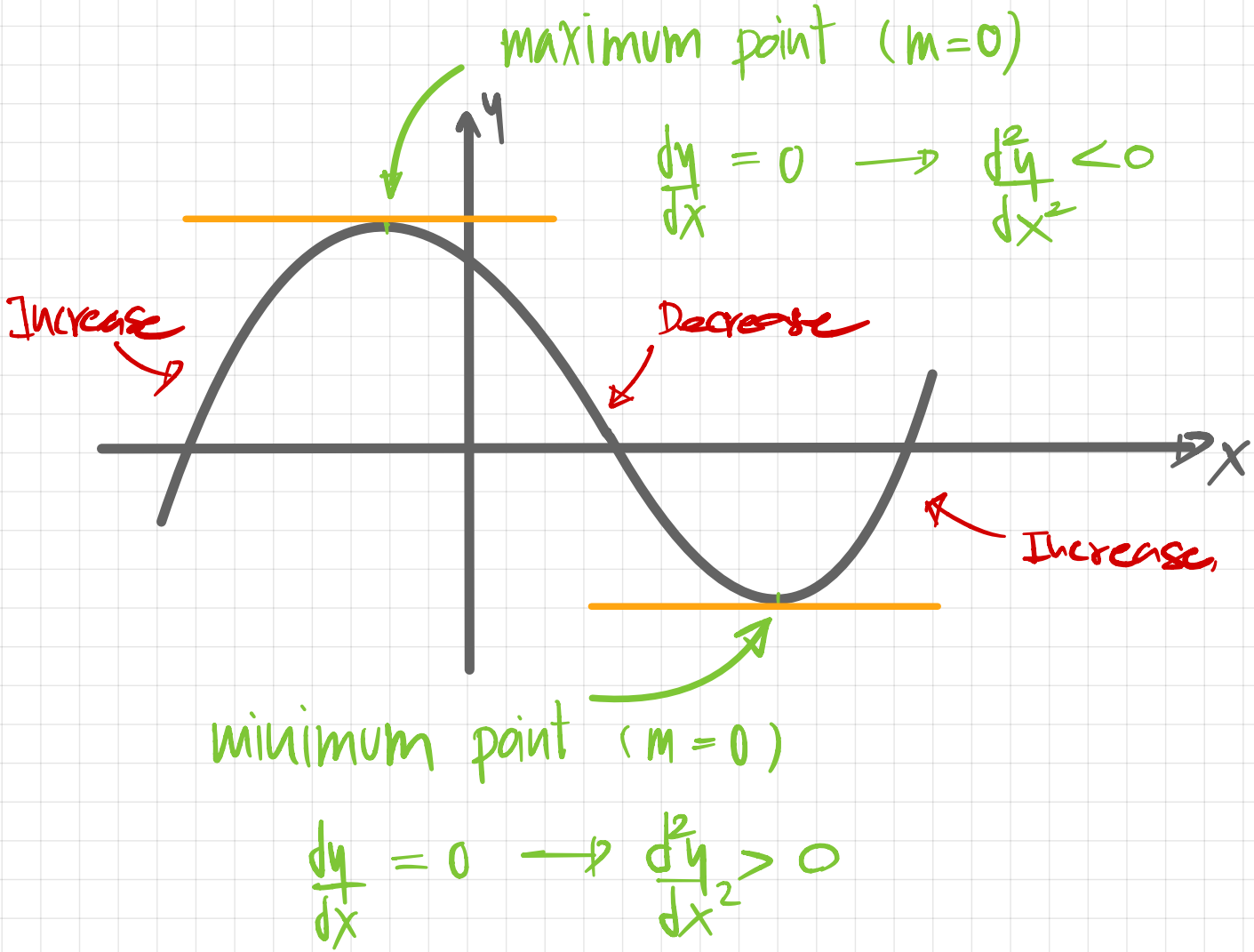
Example: Find values of x for $y = x^2 + 3x - 1$ is increase function.

Differentiation.

Example: Find values of x for $y = (3x-1)(x+5)$ is decrease function.

Differentiation.

Differentiation for Stationary points (Turning points)



Differentiation.

Example Find the stationary point of curve $y = x^2 + 4x - 3$ and determine.

1)Differentiate

2)Differentiate=0 ; Solve for x

3)Substitute x to Original equation to find y for coordinates

Differentiation.

4)Second order derivative for Max or Min

Differentiation.

Example Find the stationary point of curve $y = -x^2 - 4x + 1$ and determine.

1)Differentiate

2)Differentiate=0 ; Solve for x

3)Substitute x to Original equation to find y for coordinates

Differentiation.

4)Second order derivative for Max or Min

Differentiation.

Example Find the stationary point of curve $y = x^3 - x^2 - x$ and determine.

1)Differentiate

2)Differentiate=0 ; Solve for x

Differentiation.

3)Substitute x to Original equation to find y for coordinates

$y = x^3 - x^2 - x$

Differentiation.

4)Second order derivative for Max or Min

Differentiation.

Problem solving

The diagram shows a rectangular photo frame of area A cm^2.
The width of the photo frame is x cm.
The height of the photo frame is y cm.
The perimeter of the photo frame is 72 cm.

(a) Show that $A = 36x - x^2$

(b) Find $\frac{dA}{dx}$

(c) Find the maximum value of A.

Differentiation.

Example

A rectangular garden is fenced on three sides, and the house forms the fourth side of the rectangle.

a Given that the total length of the fence is 80 m, show that the area, A, of the garden is given by the formula $A = y(80 - 2y)$, where y is the distance from the house to the end of the garden.

Differentiation.

b Given that the area is a maximum for this length of fence, find the dimensions of the enclosed garden, and the area which is enclosed.

Differentiation.

Rate of change

→ **Time** $(t \geq 0)$ hr or min or sec

→ **Unit**

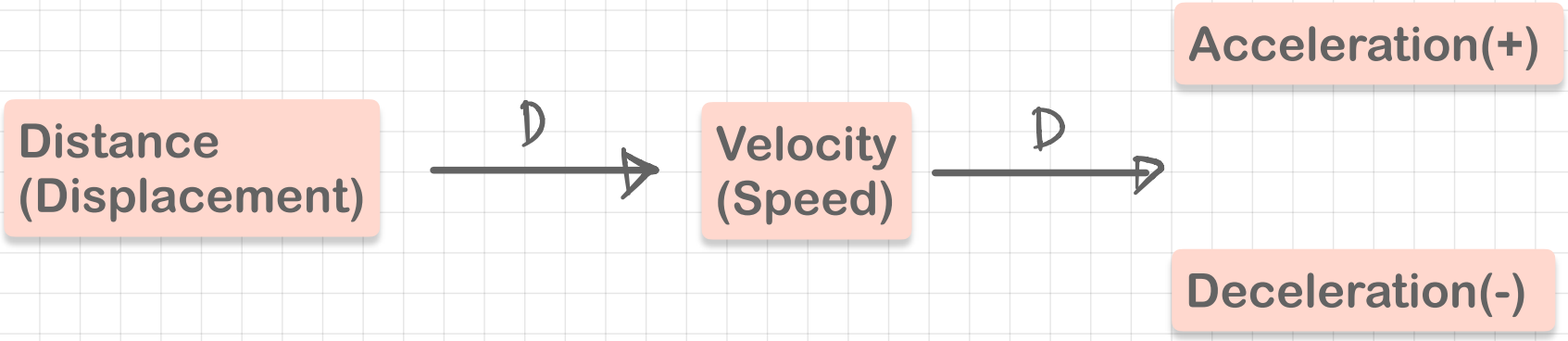
m/s
km/hr
} $x = t\ldots$

cm^2/min $A = t\ldots$

°c/min $T = t\ldots$

Differentiation.

Rate of change
(Motion of a particle in a straight line)



$s = t_{ooo}$

$x = t_{ooo}$

$v = \frac{ds}{dt}$

$A = \frac{dv}{dt}$

Differentiation.

Example

A particle moves in a straight line through a fixed point O.
The displacement, s metres, of the particle from O at time t seconds is given by

$$s = t^3 - 5t^2 + 8$$

Find $\frac{ds}{dt}$

Find an expression for the velocity, v m/s, of the particle after t seconds.

Differentiation.

Find the velocity of the particle when $t = 5$

Find the acceleration of the particle.

Find the time at which the acceleration of the particle is $20\,\text{m/s}^2$.

Differentiation.

The direction of motion of particle reverses

Moving in the same direction

Maximum or Minimum velocity

Maximum or Minimum of h