Probability

Recall that:

$$P(event) = \frac{matching\ outcomes}{total\ possible\ outcomes}$$

1 If I toss a coin twice, I see a Heads and a Tails (in either order).

2 If I toss a coin three times, I see a 2 Heads and 1 Tail.

3 In 3 throws of a coin, a Heads never follows a Tails.

Recall that:

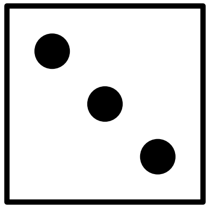
$$P(event) = \frac{matching\ outcomes}{total\ possible\ outcomes}$$

A Throwing three square numbers on a die in a row.

B Seeing exactly two heads in four throws of a coin.

How can we find the probability of an event?

1. We might just know!



For a **fair** die, we **<u>know</u>** that the probability of each outcome is $\frac{1}{6}$, by definition of it being a fair die.

This is known as a:

Theoretical Probability

2. We can do an experiment and count outcomes

We could throw the dice **100 times** for example, and count how many times we see each outcome.

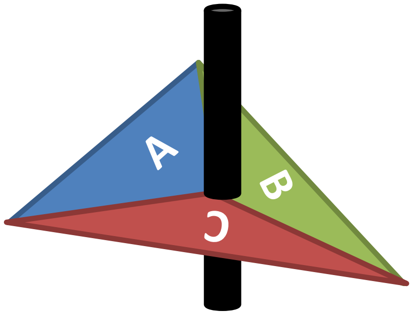
Outcome	1	2	3	4	5	6
Count	27	13	10	30	15	5
R.F.						

This is known as an:

Experimental Probability

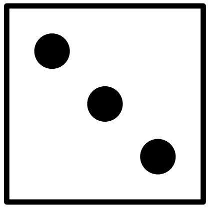
Estimating counts and probabilities

A spinner has the letters A, B and C on it. I spin the spinner 50 times, and see A 12 times. What is the experimental probability for P(A)?



Answer:

Answer:



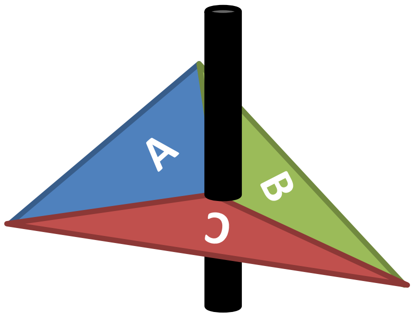
The probability of getting a 6 on an unfair die is 0.3. I throw the die 200 times. How many sixes might you expect to get?

Estimating counts and probabilities

A

The table below shows the probabilities for spinning an A, B and C on a spinner. If I spin the spinner 150 times, estimate the number of Cs I will see.

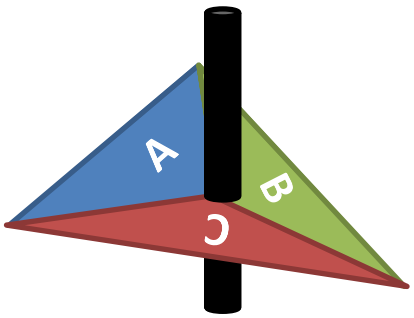
Outcome	A	B	C
Probability	0.12	0.34	



B

I spin another spinner 120 times and see the following counts:

Outcome	A	B	C
Count	30	45	45



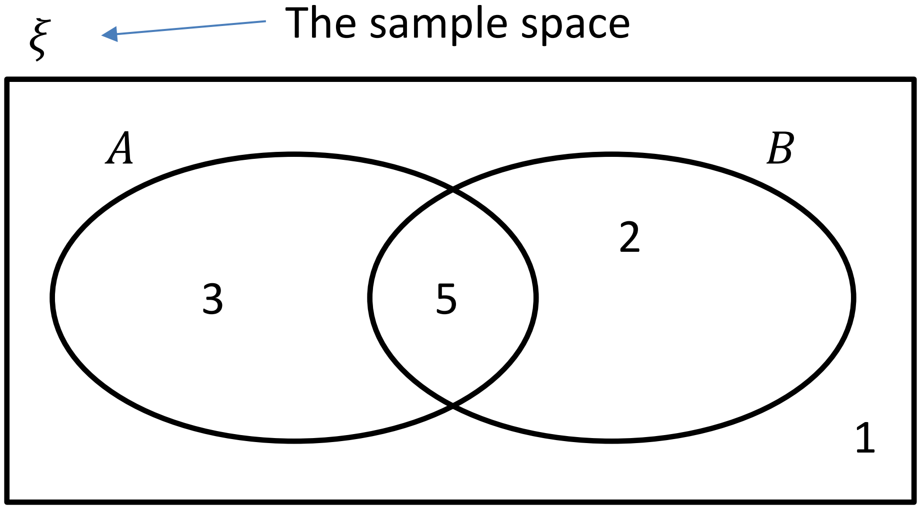
What is the relative frequency of B?

Examples of events:
Throwing a 6, throwing an odd number, tossing a heads, a randomly chosen person having a height above 1.5m.

- The sample space is the
- An event is

$$P(A) = \frac{1}{3}$$

We often use capital letters to represent an event, then use $P(A)$ to mean the probability of it.



From Year 7 you should be familiar with representing sets using a Venn Diagram, although you won't need to at GCSE.

Independent Events

When a fair coin is thrown, what's the probability of:

$P(H) =$

And when 3 fair coins are thrown:

p(1^{st} coin H and 2^{nd} coin H and 3^{rd} coin H) =

Therefore in this particular case we found the following relationship between these probabilities:

P(event_1 **and** event_2 **and** event_3)

=

Mutually Exclusive Events

If A and B are mutually exclusive events, they can’t happen at the same time. Then:

P(A **or** B) = P(A) + P(B)

Independent Events

If A and B are independent events, then the outcome of one doesn’t affect the other. Then:

P(A **and** B) = P(A) $\times$ P(B)

But be careful...

1 2 3 4 5 6 7 8

P(num divisible by 2) =

P(num divisible by 4) =

P(num divisible by 2 **and** by 4) =

Why would it have been wrong to multiply the probabilities?

Dave and Bob both come into school by bus from Hounslow.
The probability that Dave is late to school is 0.7.
The probability that Bob is late to school is 0.4.

Sheila claims that the probability Dave is late to school **and** Bob is late to school is $0.7 \times 0.4 = 0.28$
Sheila is wrong. Explain why this might be.

Add or multiply probabilities?

Getting a 6 on a die and a T on a coin.	+	×
Hitting a bullseye or a triple 20.	+	×
Getting a HHT or a THT after three throws of an unfair coin (presuming we've already worked out P(HHT) and P(THT).	+	×
Getting 3 on the first throw of a die and a 4 on the second.	+	×
Bart's favourite colour being red and Pablo's being blue.	+	×
Shaan's favourite colour being red or blue.	+	×

Independent?

TOP Maths IGCSE

Event 1	Event 2		
Throwing a heads on the first flip.	Throwing a heads on the second flip.	No	Yes
It rains tomorrow.	It rains the day after.	No	Yes
That I will choose maths at A Level.	That I will choose Physics at A Level.	No	Yes

Example

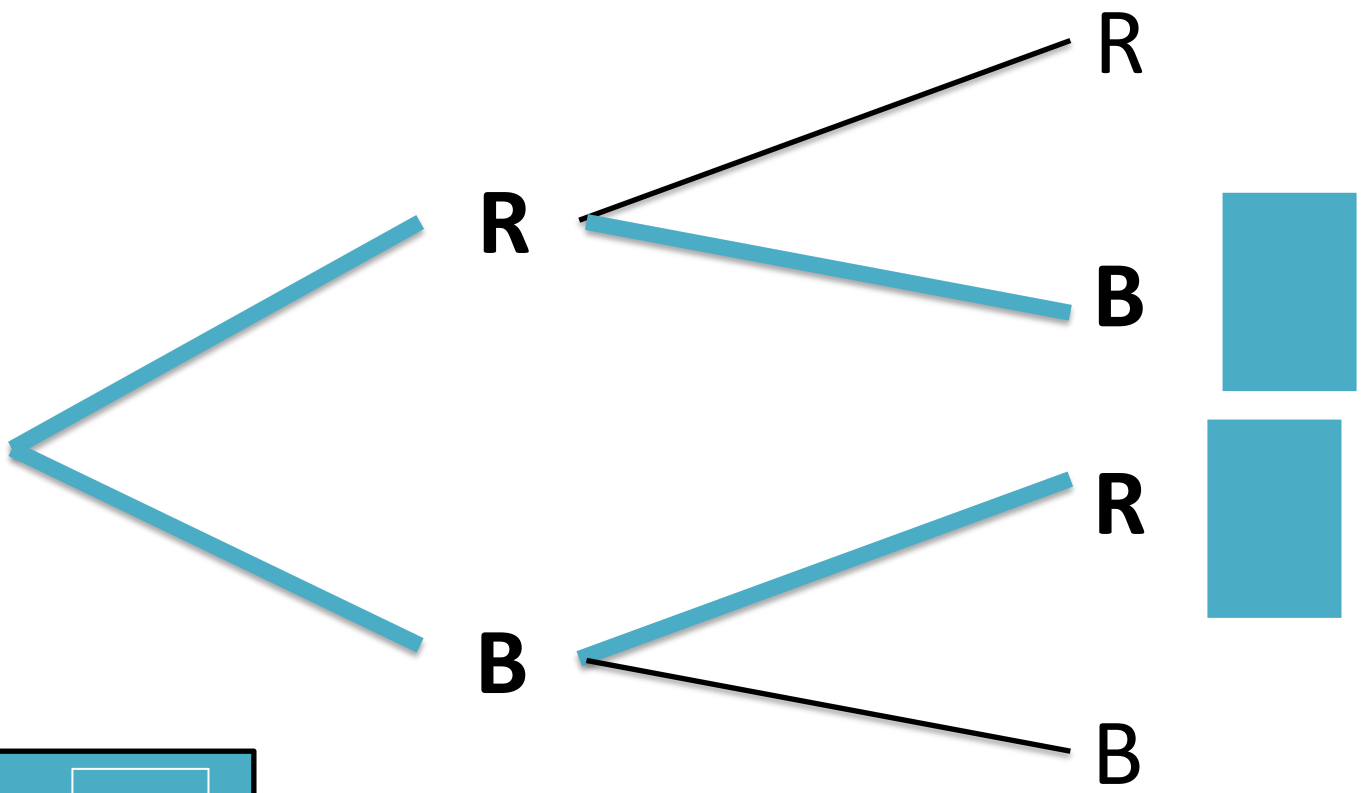
TOP Maths IGCSE

a I pick two cards from the following. What is the probability the first number is a 1 and the second number a 2?

1	2	2	3

b I throw 100 dice and 50 coins. What's the probability I get all sixes and all heads?

Question: Give there's 5 red balls and 2 blue balls. What's the probability that after two picks we have **a red ball and a blue ball**?



P(red and blue) =

Summary

...with replacement:

The item is returned before another is chosen.
The probability of each event on each trial is fixed.

...without replacement:

The item is not returned.

- Total balls decreases by 1 each time.
- Number of items of this type decreases by 1.

Note that if the question doesn't specify which, e.g. "You pick two balls from a bag", then PRESUME WITHOUT REPLACEMENT.

Algebraic Probability Questions

There are n sweets in a bag.
6 of the sweets are orange.
The rest of the sweets are yellow.

Hannah takes at random a sweet from the bag.
She eats the sweet.

Hannah then takes at random another sweet from the bag.
She eats the sweet.

The probability that Hannah eats two orange sweets is $\frac{1}{3}$

(a) Show that $n^2 - n - 90 = 0$

b) Solve $n^2 - n - 90 = 0$ to find the value of n.

Algebraic Probability Questions

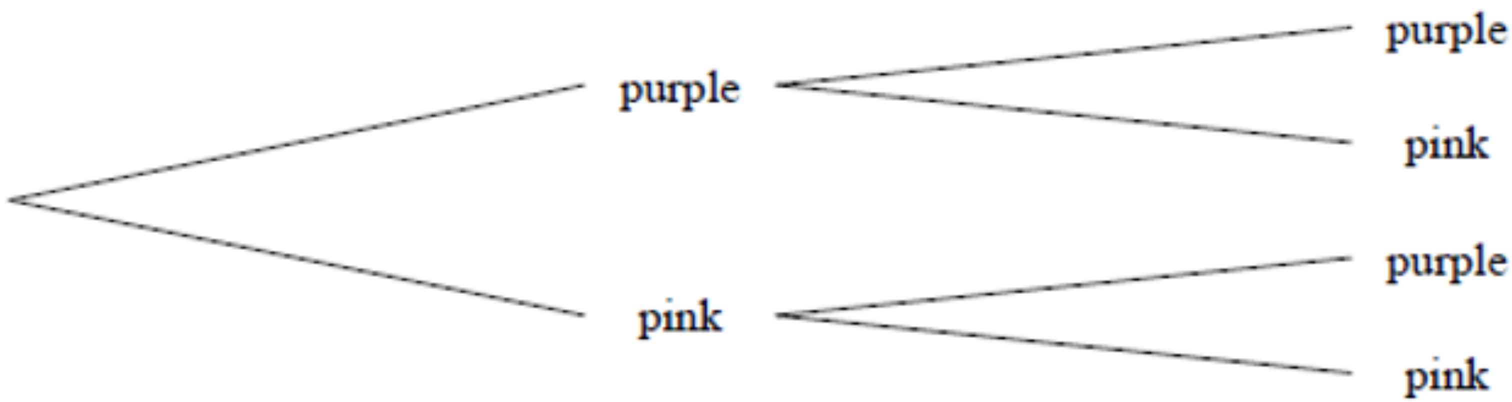
I have 5 sweets, n of which are blue. I take a sweet, eat it, then take another.

The probability both of my sweets are blue is $\frac{3}{10}$

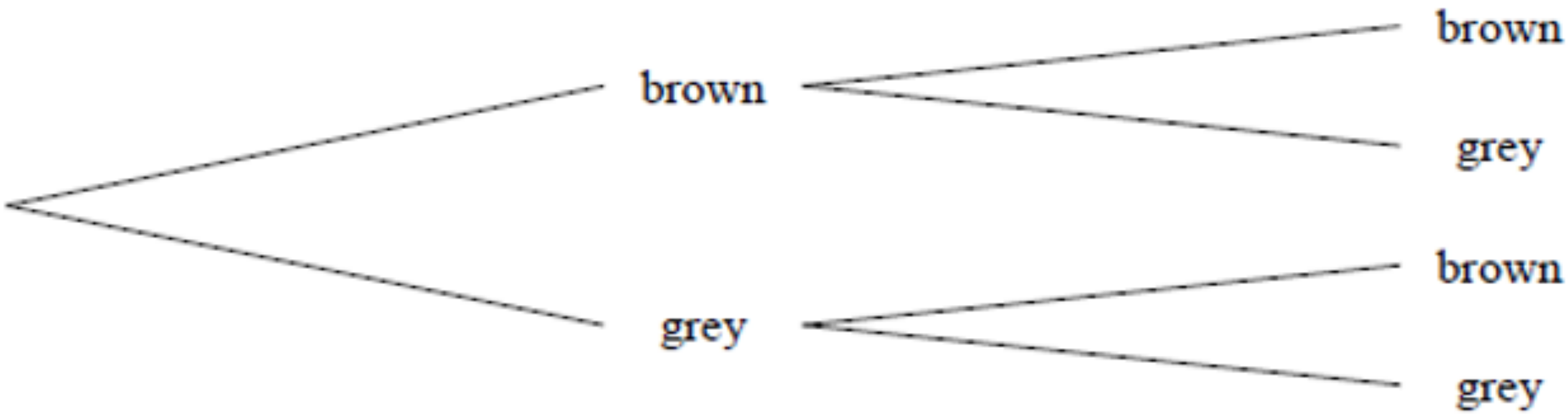
Use the information to form an appropriate quadratic equation $n^2 + an + b = 0$, specifying the constants a and b.

$a =$ ______ , $b =$ ______

A box contains a total of 4 balls; some are purple and the rest pink. A ball is chosen at random, replaced before choosing again. If $P(two\ purple) = \frac{1}{16}$ then determine the number of purple balls.



A bag contains a total of 10 sweets; some are brown and the rest are grey. A sweet is chosen at random and not replaced before choosing another one. If $P(two\ brown) = \frac{28}{45}$ then find $P(two\ grey)$.



Doing without a tree: Listing outcomes

There are 17 girls and 14 boys in Mr. Taylor's class.

Mr. Taylor is going to choose at random 3 children from his class.

Work out the probability that he will choose exactly 2 girls and 1 boy.

Answer =

Doing without a tree: Listing

Q

I have a bag consisting of 6 red balls, 4 blue and 3 green. I take three balls out of the bag at random. Find the probability that the balls are the same colour.