Quartiles, Cumulative Frequency and Box Plots

Measures of Location

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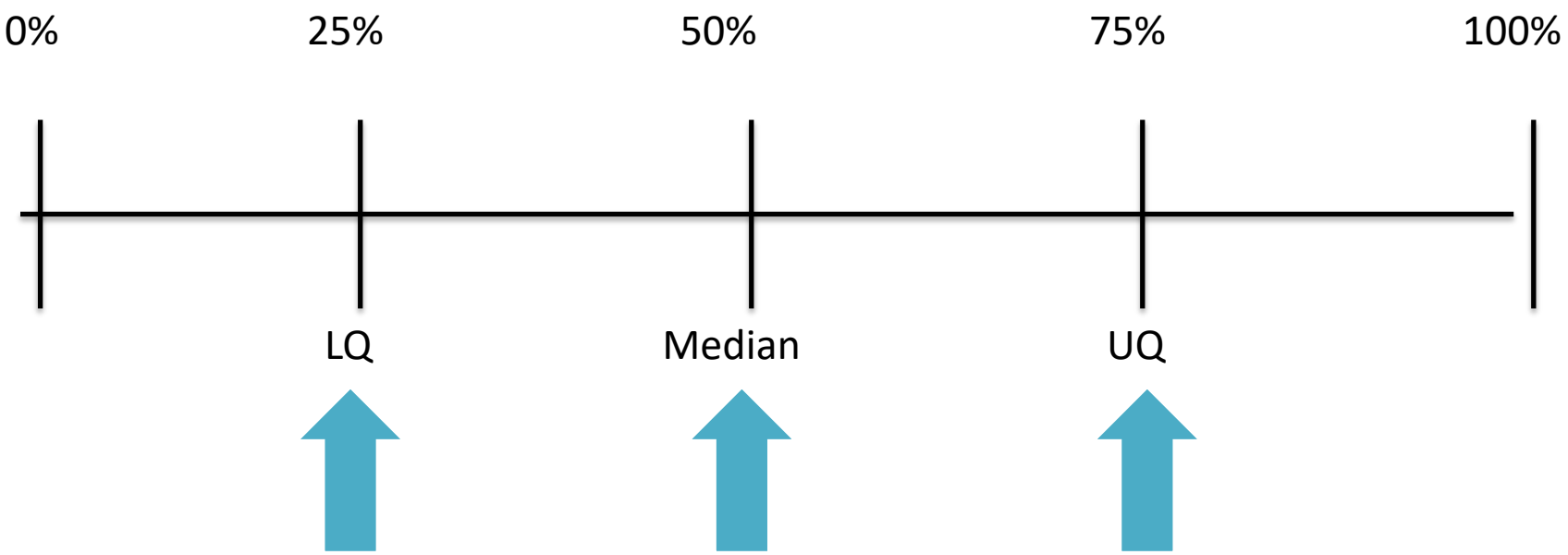
Suppose that we line up everyone in the school in order of height.



The height of the person 25% along the line is known as the: _____

We already know that the **median** would be the middle person's height.

50% of the people in the school would have a height less than them.

The **upper quartile** is the height of the person 75% along the data.

Test Your Understanding so far…

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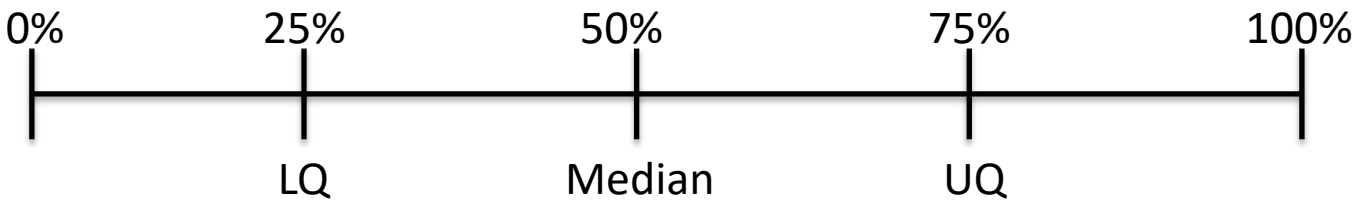
50% of the data has a value **more** than the

75% of the data has a value **less** than the

25% of the data has a value **more** than the

75% of the data has a value **more** than the



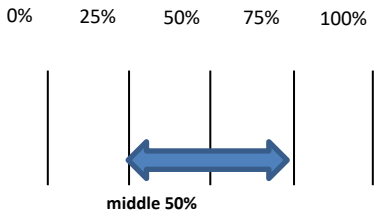
Determining quartiles from listed data

Here are the ages of 10 people at Pablo's party. Choose the correct value.

12, 13, 14, 14, 15, 16, 16, 17, 19, 24

Median:	15	15.5	16
LQ:	13	13.5	14
UQ:	17	18	19

Interquartile Range:



Rule for lower quartile:

- **Even** num of items: find median of bottom half.
- **Odd** num of items: throw away middle item, find median of remaining bottom half.

	LQ	Median	UQ
1, 2, 3			
1, 2, 3, 4			
1, 2, 3, 4, 5			
1, 2, 3, 4, 5, 6			

What if there's lots of items?

There are 31 items, in order of value. What items should we use for the median and lower/upper quartiles?

0 1 1 2 4 5 5 6 7 8 10 11 14 15 16 17 18 19 19
29 31 31 37 38 39 39 40 40 41 43 44

LQ Use the ☐ item ➡ ☐

Median Use the ☐ item ➡ ☐

UQ Use the ☐ item ➡ ☐

What item numbers should you use?

Num items	LQ	Median	UQ
15			
23			
39			
47			

1 Here are the ages of 7 students:

7 9 10 12 14 16 17

(a) Determine the lower quartile.

(b) Determine the interquartile range.

2 Here are the marks that James scored in eleven maths tests.

16 12 19 18 17 13 13 20 11 19 17

(a) Determine the interquartile range of these marks.

Sunil did the same eleven maths tests.
The median mark Sunil scored in his tests is 17.
The interquartile range is 8.

(b) Which one of Sunil or James has the more consistent marks?

Median/quartiles from a frequency table

Age (years)	Frequency
18	8
19	7
20	1
21	4
22	7

Total frequency:
Position of item for median:
Position of item for LQ:
Position of item for UQ:

Median:
LQ:
UQ:
IQR:

Time taken (s)	Freq
$0 \leq t < 3$	4
$3 \leq t < 5$	9
$5 \leq t < 10$	2
$10 \leq t < 20$	11
$20 \leq t < 50$	13

Total frequency:
Position of item for median:
Position of item for LQ:
Position of item for UQ:

Median class interval:
LQ class interval:
UQ class interval:

Exercises

1 Determine the Lower Quartile, Median, Upper Quartile and Interquartile Range of each list of numbers.

a 1, 7, 9, 10, 14, 15, 20

b 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

2

Weight of cat (kg)	Freq
$2 \leq w < 6$	6
$6 \leq w < 8$	8
$8 \leq w < 20$	5
$20 \leq w < 30$	3
$30 \leq w < 32$	1

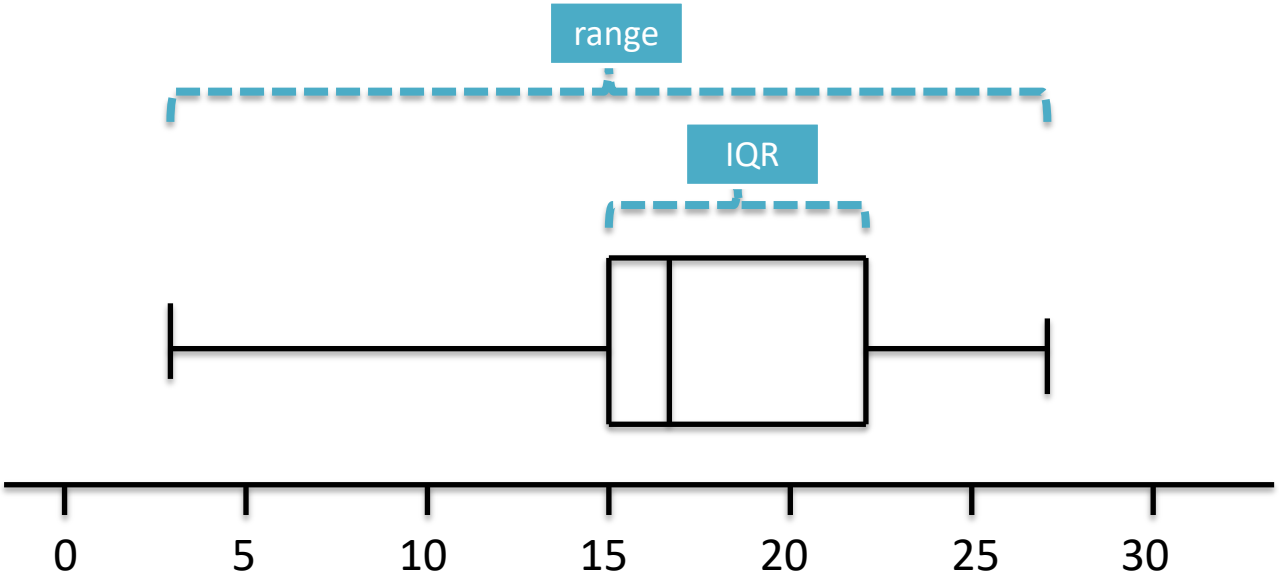
Mean class interval:

Modal class interval:

Box Plots

Box Plots allow us to visually represent the distribution of the data.

Minimum	Lower Quartile	Median	Upper Quartile	Maximum
3	15	17	22	27



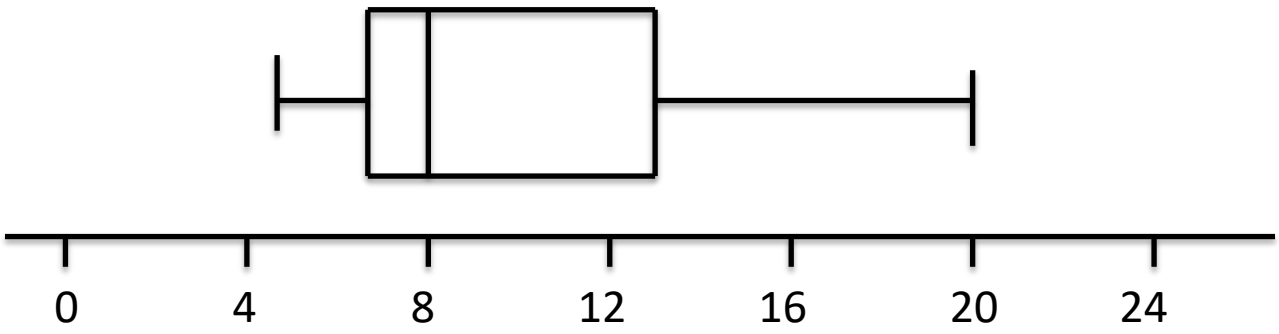
How is the **IQR** represented in this diagram?

How is the **range** represented in this diagram?

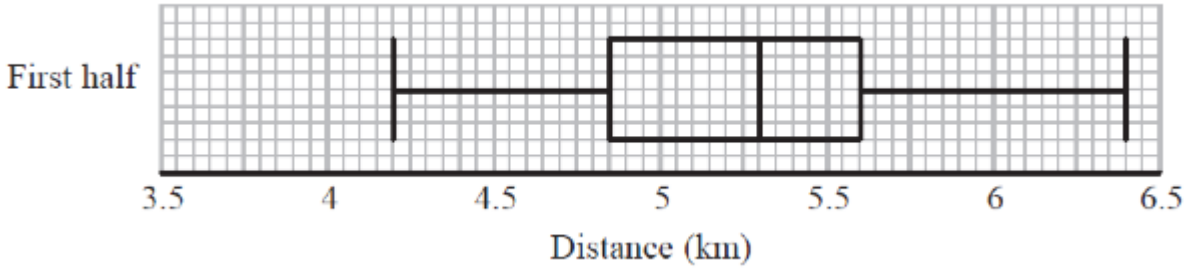
Sketch a box plot to represent the given weights of cats:

5lb, 6lb, 7.5lb, 8lb, 8lb, 9lb, 12lb, 14lb, 20lb

Minimum	Maximum	Median	Lower Quartile	Upper Quartile



Colin took a sample of 80 football players.
He recorded the total distance, in kilometres, each player ran in the first half of their matches on Saturday.
Colin drew this box plot for his results.
There were 80 players in Colin's sample.
Work out the number of players who ran a distance of more than 5.6 km.



Comparing Box Plots

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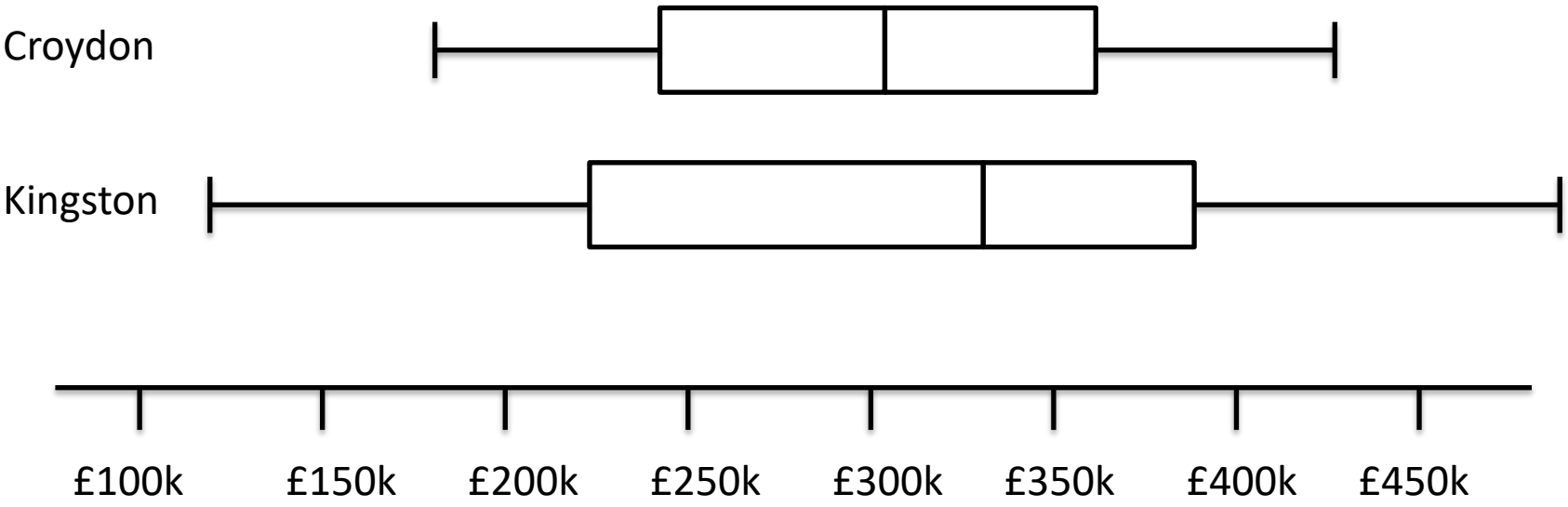
Box Plot comparing house prices of Croydon and Kingston-upon-Thames.



"Compare the prices of houses in Croydon with those in Kingston". (2 marks)

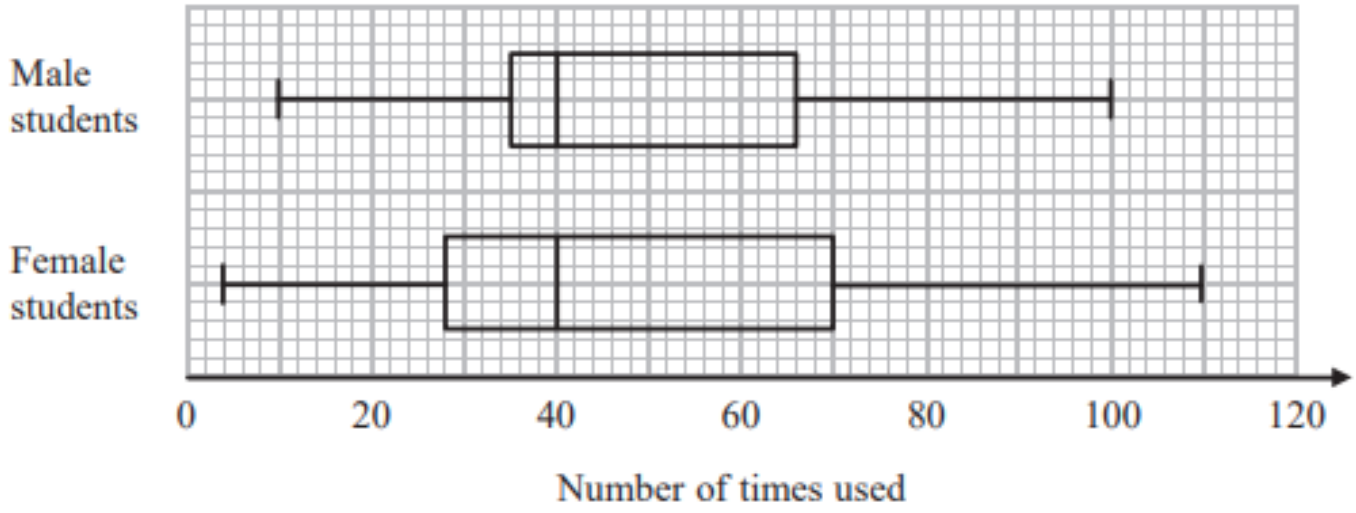
For 1 mark, one of:

For 1 mark:

Example

Some students were asked how many times they each used their mobile phones last week.

The box plots give information about the male students' answers and about the female students' answers.

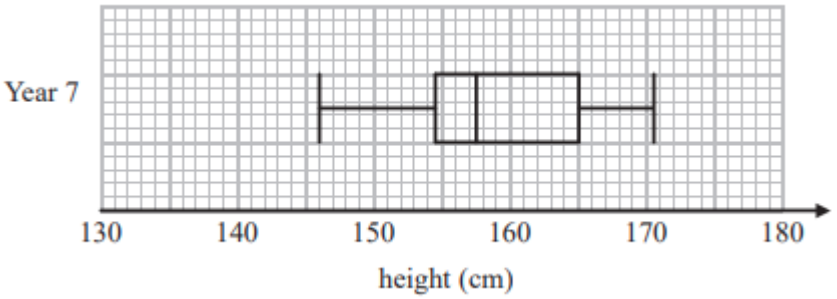


Example

The table shows information about the heights, in cm, of a group of Year 11 girls.

	height (cm)
least height	154
median	165
lower quartile	161
interquartile range	7
range	20

The box plot below shows information about the heights, in cm, of a group of Year 7 girls.



Compare the distribution of heights of the Year 7 girls with the distribution of heights of the Year 11 girls.

Cumulative Frequency

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"Cumulative" means **running total**. It allows us to say the count **up to** a particular value.

Number of certificates	Number of students
0	4
1	9
2	7
3	1
4	6
5	3

Convert to cumulative frequency table.

Number of certificates	Cumulative Frequency
0	
1	
2	
3	
4	
5	

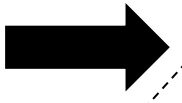
How many students had "**up to**" 2 certificates?

How many students had "**up to**" 3 certificates?

Cumulative Frequency Tables for Grouped Data

Height (h cm)	Frequency
$130 < h \leqslant 140$	4
$140 < h \leqslant 150$	11
$150 < h \leqslant 160$	24
$160 < h \leqslant 170$	22
$170 < h \leqslant 180$	19

Convert to cumulative frequency table.



Height (h cm)	Cumulative Freq
$130 < h \leqslant 140$	

This updated class interval should reflect 'everything **up to** 150' (i.e. not just between 140 and 150).

100m times at the 2012 London Olympics

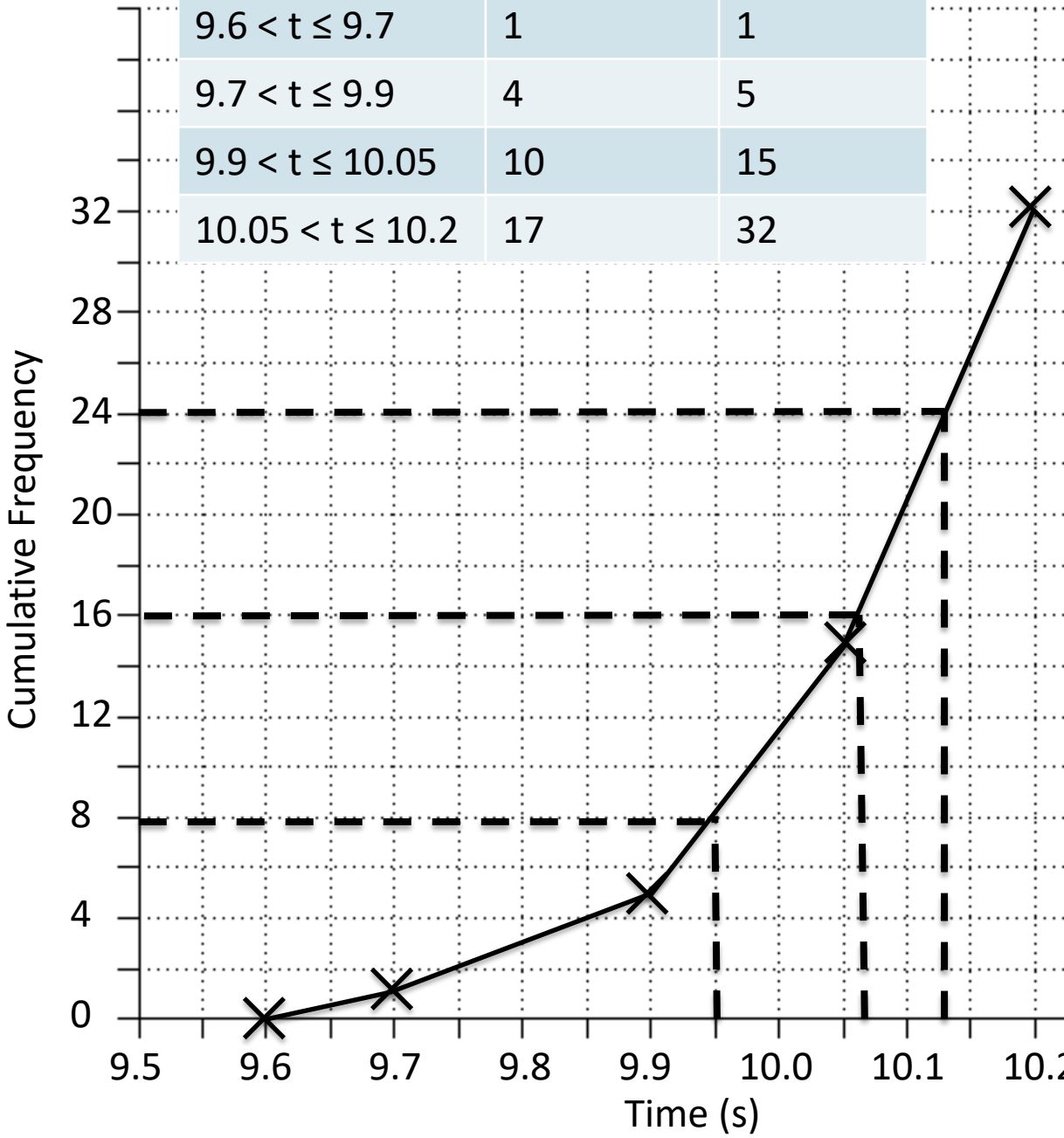
Time (s)	Frequency	CF
$9.6 < t \leq 9.7$	1	
$9.7 < t \leq 9.9$	4	
$9.9 < t \leq 10.05$	10	
$10.05 < t \leq 10.2$	17	

Let's use this data to plot something called a **cumulative frequency graph**.

To save the hassle of having to write a separate cumulative frequency table, we can just add an extra cumulative frequency column.

Cumulative Frequency Graphs

Time (s)	Frequency	Cum Freq
$9.6 < t \leq 9.7$	1	1
$9.7 < t \leq 9.9$	4	5
$9.9 < t \leq 10.05$	10	15
$10.05 < t \leq 10.2$	17	32



This type of graph is incredibly useful in estimating how many people had a particular value of less, e.g. "*Estimate how many runners had a time of up to 10 secs*". It also allows us to estimate the median/quartiles.

Median =

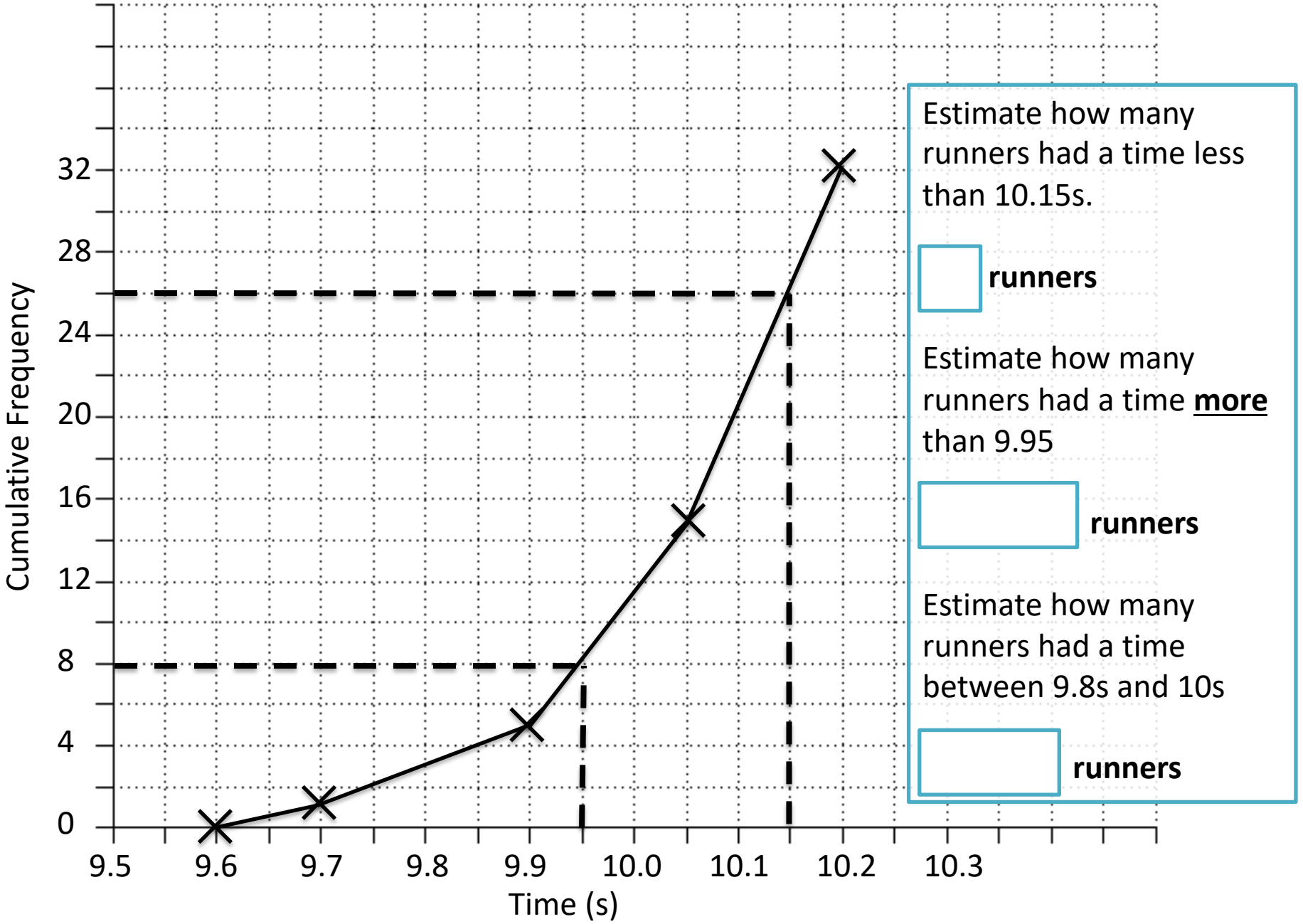
Lower Quartile =

Upper Quartile =

Interquartile Range =

Cumulative Frequency Graphs

A Cumulative Frequency Graph is very useful for finding the number of values greater/smaller than some value, or within a range.



Estimate how many runners had a time less than 10.15s.

_____ **runners**

Estimate how many runners had a time **more** than 9.95

_____ **runners**

Estimate how many runners had a time between 9.8s and 10s

_____ **runners**

Example

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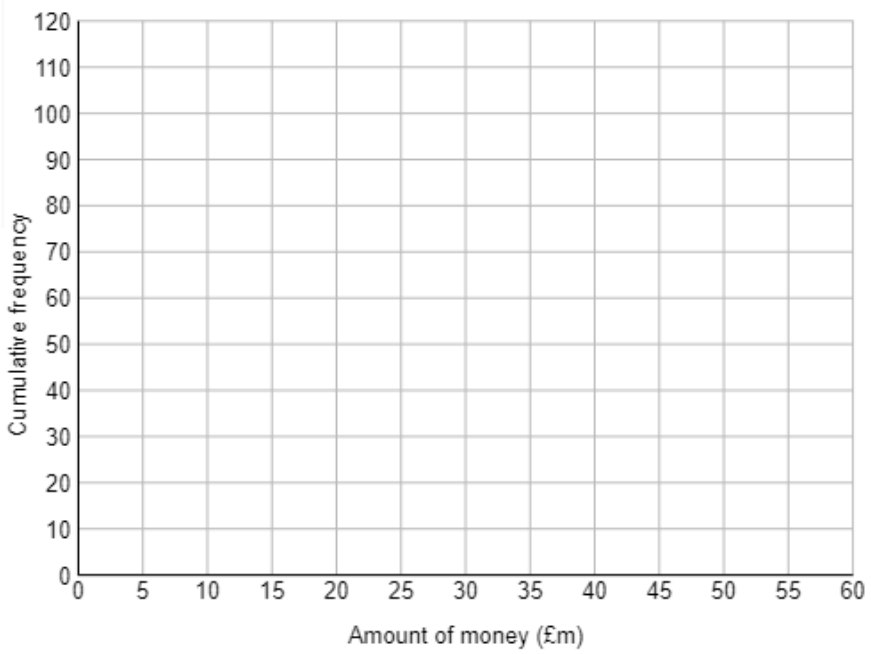
The table shows information about the amount of money that 120 people spent in a shop.

Amount of money (£m)	Frequency
$0 < m \leqslant 10$	8
$10 < m \leqslant 20$	17
$20 < m \leqslant 30$	25
$30 < m \leqslant 40$	40
$40 < m \leqslant 50$	22
$50 < m \leqslant 60$	8

a Complete the cumulative frequency table.

Amount of money (£m)	Cumulative frequency
$0 < m \leq 10$	
$0 < m \leq 20$	
$0 < m \leq 30$	
$0 < m \leq 40$	
$0 < m \leq 50$	
$0 < m \leq 60$	

b On the grid, draw a cumulative frequency graph for the table.



c Use the graph to find an estimate for the median amount of money spent in the shop by these people.

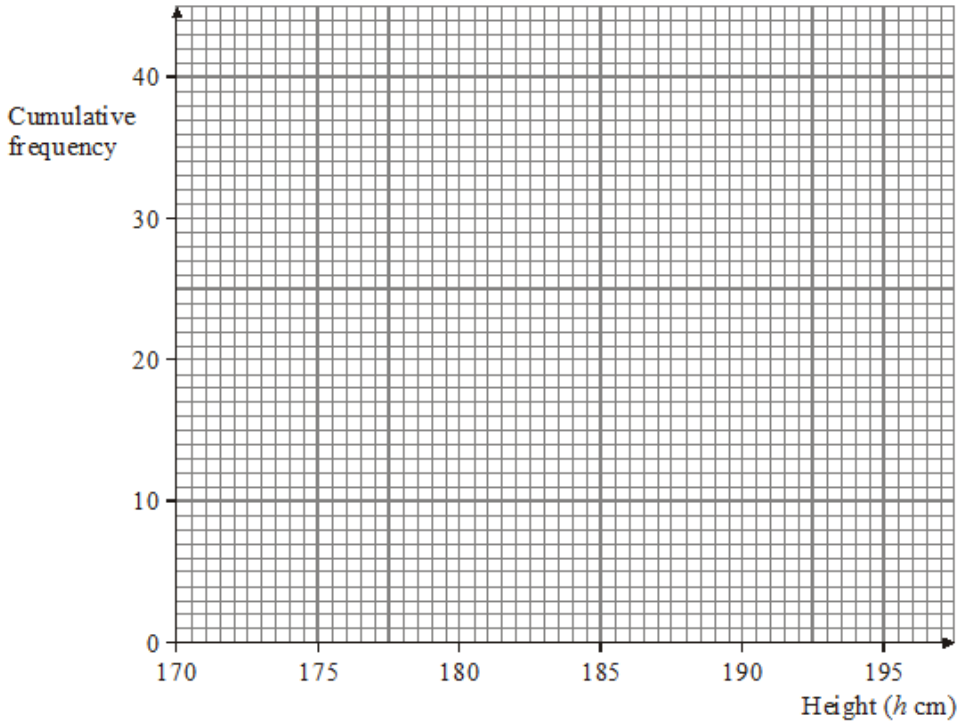
1 The table shows information about the heights of 40 bushes.

Height (h cm)	Frequency
$170 \leq h < 175$	5
$175 \leq h < 180$	18
$180 \leq h < 185$	12
$185 \leq h < 190$	4
$190 \leq h < 195$	1

(a) Complete the cumulative frequency table.

Height (h cm)	Cumulative Frequency
$170 \leq h < 175$	
$170 \leq h < 180$	
$170 \leq h < 185$	
$170 \leq h < 190$	
$170 \leq h < 195$	

(b) On the grid, draw a cumulative frequency graph for your table.



(c) Use the graph to find an estimate for the median height of the bushes.

.... cm

2 The table gives information about the ages of 160 employees of an IT company.

Age (A) in years	Frequency
$15 < A \leq 25$	44
$25 < A \leq 35$	56
$35 < A \leq 45$	34
$45 < A \leq 55$	19
$55 < A \leq 65$	7

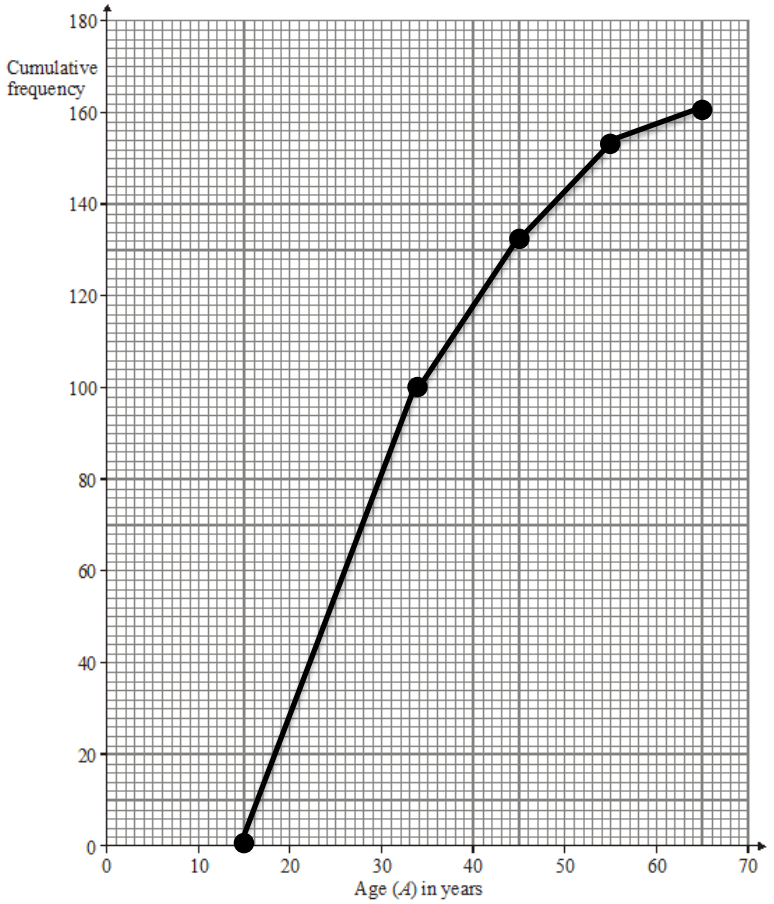
(a) Write down the modal class interval.

.....

(b) Complete the cumulative frequency table.

Age (A) in years	Cumulative Frequency
$15 < A \leq 25$	
$15 < A \leq 35$	
$15 < A \leq 45$	
$15 < A \leq 55$	
$15 < A \leq 65$	

(c) On the grid below, draw a cumulative frequency graph for your table.



(d) Use your graph to find an estimate for

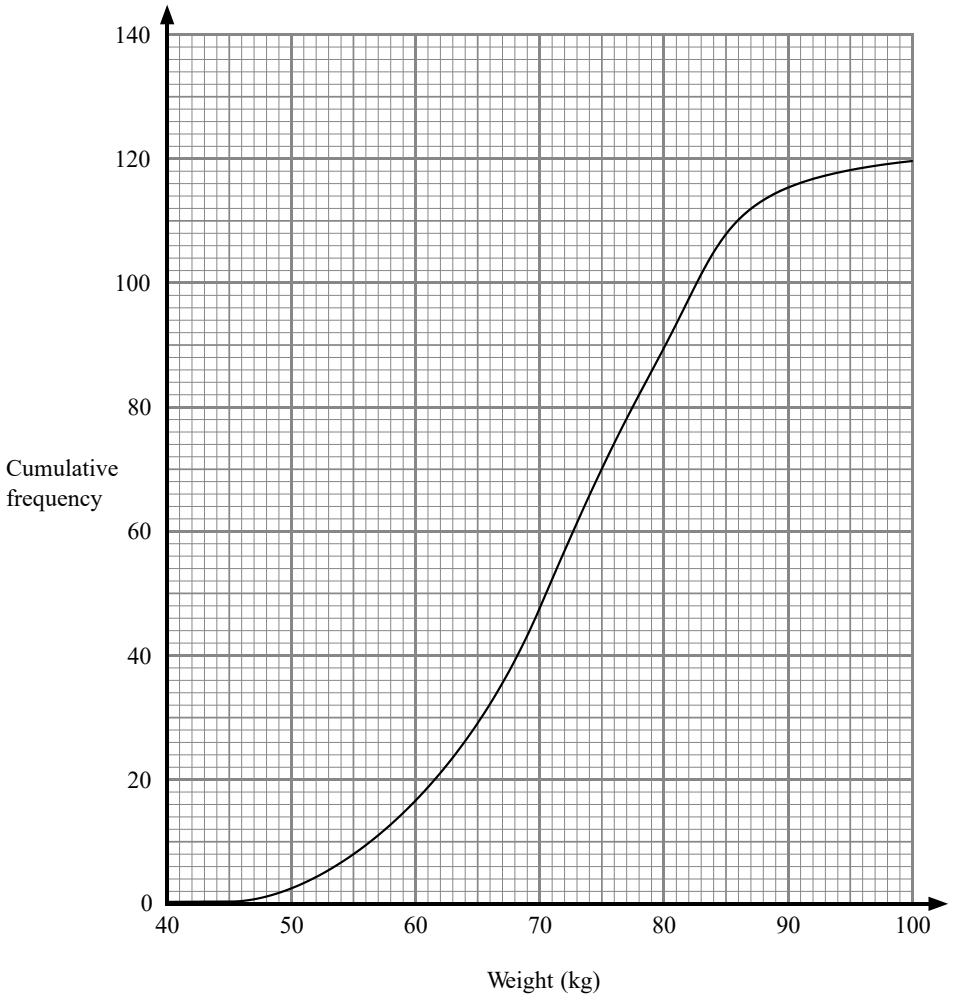
(i) the median age of the employees,

..... years

(i) the interquartile range of the ages of the employees.

..... years

3 Here is the cumulative frequency curve of the weights of 120 girls at Mayfield Secondary School.



Use the cumulative frequency curve to find an estimate for the

(i) median weight,

(ii) interquartile range of the weights.