

# P1 Chapter 1: Algebraic Expressions



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## 1: Basic Index Laws

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$

## 2: Expand brackets

Expand and simplify  
 $(x - 3)^2(x + 1)$

## 3: Factorise quadratics/cubics

Factorise fully:  
 $x^3 - 16x$

## 4: Fractional/Negative Powers

Evaluate:

$$\left(\frac{27}{8}\right)^{-\frac{2}{3}}$$

## 5: Surds

Simplify  
 $\sqrt{12} \times 2 = 6\sqrt{3}$

## 6: Rationalising denominators

Rationalise the denominator of

$$\frac{1}{3 + \sqrt{2}}$$

# 1: Index Laws

base  $\rightarrow$   $3^5$   $\leftarrow$  exponent or power or index (plural: indices)



$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$

Simplify  $(a^3)^2 \times 2a^2$

Simplify  $(4x^3y)^3$

Simplify  $2x^2(3 + 5x) - x(4 - x^2)$

Simplify  $\frac{x^3 - 2x}{3x^2}$

1

Simplify  $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2

Simplify  $\frac{2x+x^5}{4x^3}$

3

Expand and simplify

$$2x(3 - x^2) - 4x^3(3 - x)$$

4

Simplify  $2^x \times 3^x$



## Non-Calculator Questions

1 Which of the following numbers is largest?

- $\left(\left(2^3\right)^2\right)^3$
- $\left(2^3\right)^{\left(2^3\right)}$
- $2\left(\left(3^2\right)^3\right)$
- $2\left(3^{\left(2^3\right)}\right)$

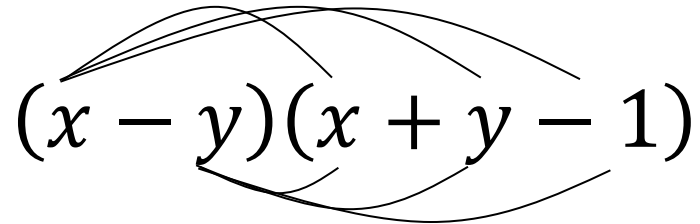
2 Let  $N = 2^k \times 4^m \times 8^n$  where  $k, m, n$  are positive whole numbers.

Then  $N$  will definitely be a square number whenever:

- $k$  is even;
- $k + n$  is odd;
- $k$  is odd but  $m + n$  is even;
- $k + n$  is even.

## 2 : Expanding Brackets

Multiply each term in the first bracket by each term in the second.

$$(x - y)(x + y - 1)$$


$$(x + 1)(x + 2)(x + 3)$$

1 Expand and simplify  
 $(x + 5)(x - 2)(x + 1)$

2 Expand and simplify:  
 $2(x - 3)(x - 4)$

3 Expand and simplify:  
 $(2x - 1)^3$



## Non-Calculator Questions

1

Of the following three alleged algebraic identities, at least one is wrong.

$$\begin{aligned} \text{(i)} \quad &yz(z-y) + zx(x-z) + xy(y-x) \\ &= (z-y)(x-z)(y-x) \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad &yz(z-y) + zx(x-z) + xy(y-x) \\ &= (z-y)(z-x)(y-x) \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad &yz(x+y) + zx(z+x) + xy(y+x) \\ &= (z+y)(z+x)(y+x) \end{aligned}$$

Which of the following statements are correct? Tick all that apply.

- (i)
- (ii)
- (iii)

2

If  $x$  and  $n$  are integers then

$$(1-x)^n(2-x)^{2n}(3-x)^{3n}(4-x)^{4n}(5-x)^{5n}$$

is:

- negative when  $n > 5$  and  $x < 5$
- negative when  $n$  is odd and  $x > 5$
- negative when  $n$  is a multiple of 3 and  $x > 5$
- negative when  $n$  is even and  $x < 5$

# 3 : Factorising

Informally, factorising is the opposite of expanding brackets.

More formally, a factorised expression is one which is expressed as a **product of expressions**.

$$x(x + 1)(x + 2)$$



Factorised as it is the product of 3 linear factors,  $x$ ,  $x + 1$  and  $x + 2$ .

$$x(x + 1) + (x - 1)(x + 1)$$



Not factorised because the outer-most operation is a sum, not a product.

Basic Examples:

$$x^3 + x^2 =$$

$$4x - 8xy =$$

# Factorising Quadratics

Recap:

$$x^2 \oplus -5x \ominus 14 =$$

We find two numbers which multiply to give the coefficient of  $x$  and multiply to give the constant term.

But what if the coefficient of  $x^2$  is not 1?

$$2x^2 + 5x - 12 =$$

$$2x^2 + 5x - 12 \quad \begin{array}{l} \oplus \\ \ominus \end{array} \begin{array}{|c|} \hline \\ \hline \\ \hline \end{array}$$

**STEP 1:** Find two numbers which add to give the middle number and multiply to give the first times last.

**STEP 2:** Split the middle term.

**STEP 3:** Factorise first half and second half ensuring bracket is duplicated.

**STEP 4:** Factorise out bracket.

Difference of two squares:

$$4x^2 - 9 =$$

Using multiple factorisations:

$$x^3 - x$$

=

=

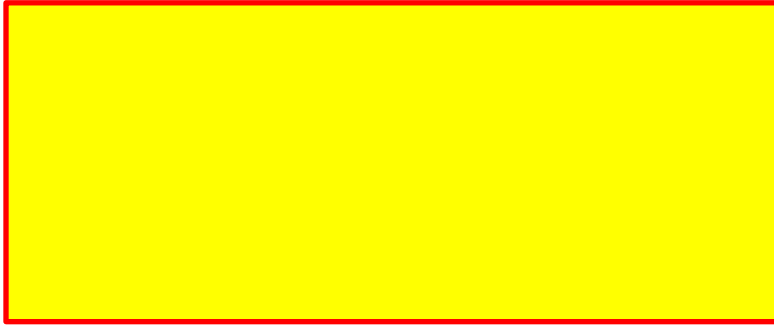
$$x^3 + 3x^2 + 2x$$

=

=

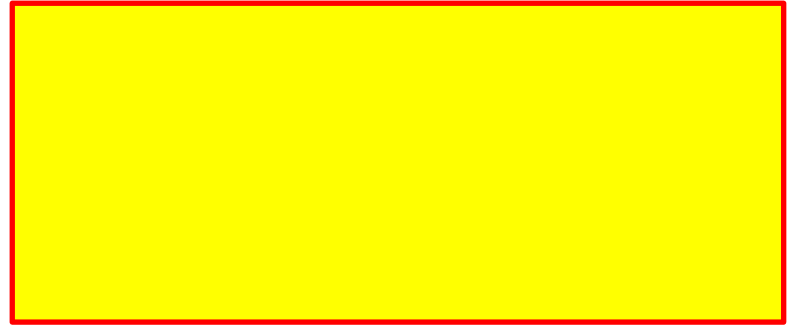
1 Factorise completely:

$$6x^2 + x - 2$$



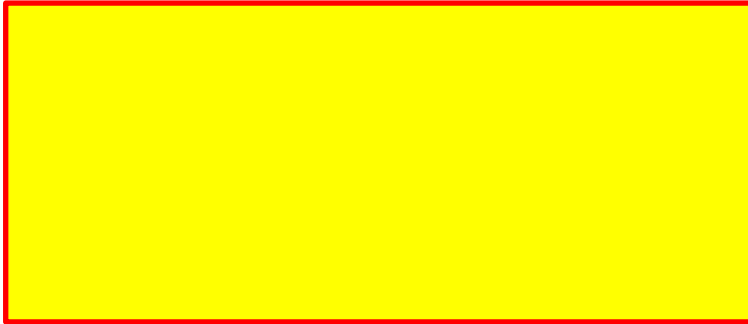
2 Factorise completely:

$$x^3 - 7x^2 + 12x$$



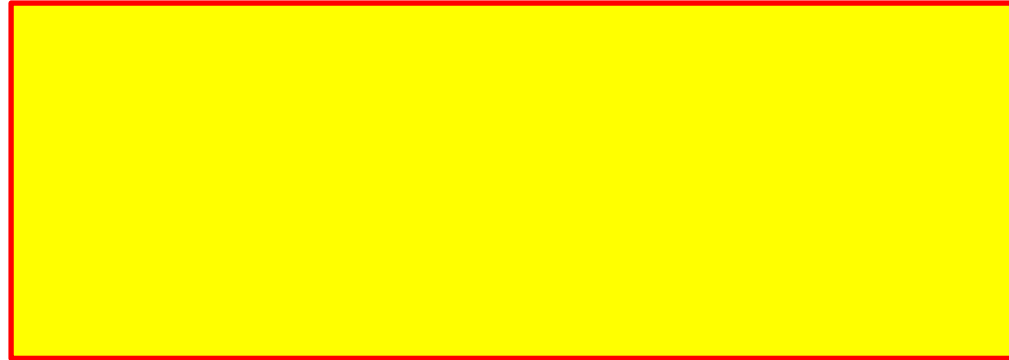
3 Factorise completely:

$$x^4 - 1$$



4 Factorise completely:

$$x^3 - 1$$



# 4 : Negative and Fractional Indices

$$a^0 = 1$$

$$a^{\frac{1}{m}} = \sqrt[m]{a}$$

$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$

$$a^{-m} = \frac{1}{a^m}$$

**Note:**  $\sqrt{9}$  only means the positive square root of 9, i.e. 3 not -3.

Otherwise, what would be the point of the  $\pm$  in the quadratic formula before the  $\sqrt{b^2 - 4ac}$ ?

Prove that  $x^{\frac{1}{2}} = \sqrt{x}$

Evaluate  $27^{-\frac{1}{3}}$

Evaluate  $32^{\frac{2}{5}}$

# 4 : Negative and Fractional Indices

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**Note:**  $\sqrt{9}$  only means the positive square root of 9, i.e. 3 not -3.

Otherwise, what would be the point of the  $\pm$  in the quadratic formula before the  $\sqrt{b^2 - 4ac}$ ?

Simplify  $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Evaluate  $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

If  $b = \frac{1}{9}a^2$ , determine  $3b^{-2}$  in the form  $ka^n$  where  $k, n$  are constants.



## Non-Calculator Questions

Let  $r$  and  $s$  be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

## Recap:

A surd is a root of a number that does not simplify to a rational number.

Laws:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

A *rational* number is any which can be expressed as  $\frac{a}{b}$  where  $a, b \neq 0$  are integers.  $\frac{2}{3}$  and  $\frac{4}{1} = 4$  are rational numbers, but  $\pi$  and  $\sqrt{2}$  are not.

$$\sqrt{3} \times 2$$

=

$$3\sqrt{5} \times 2\sqrt{5}$$

=

$$\sqrt{8} = \sqrt{4}\sqrt{2}$$

=

$$\sqrt{12} + \sqrt{27}$$

=

$$(\sqrt{8} + 1)(\sqrt{2} - 3)$$

=

=

=



## Non-Calculator Questions

**1** [SMC 2014 Q24] Which of the following is smallest?

- $10 - 3\sqrt{11}$
- $8 - 3\sqrt{7}$
- $5 - 2\sqrt{6}$
- $9 - 4\sqrt{5}$
- $7 - 4\sqrt{3}$

**2** [SMC 2012 Q21] Which of the following numbers does *not* have a square root in the form  $x + y\sqrt{2}$ , where  $x$  and  $y$  are positive integers?

- $17 + 12\sqrt{2}$
- $22 + 12\sqrt{2}$
- $38 + 12\sqrt{2}$
- $54 + 12\sqrt{2}$
- $73 + 12\sqrt{2}$

# 6 : Rationalising The Denominator

Here's a surd. What could we multiply it by such that it's no longer an irrational number?

$$\sqrt{5} \times \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

$$\frac{1}{\sqrt{2}} \times \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

$$\frac{3}{\sqrt{2}} = \boxed{\phantom{000}}$$

$$\frac{12}{\sqrt{3}} = \boxed{\phantom{000}}$$

$$\frac{6}{\sqrt{3}} = \boxed{\phantom{00000}}$$

$$\frac{2}{\sqrt{6}} = \boxed{\phantom{000}}$$

$$\frac{7}{\sqrt{7}} = \boxed{\phantom{000000}}$$

$$\frac{4\sqrt{2}}{\sqrt{8}} = \boxed{\phantom{000}}$$

$$\frac{15}{\sqrt{5}} + \sqrt{5} = \boxed{\phantom{0000000}}$$





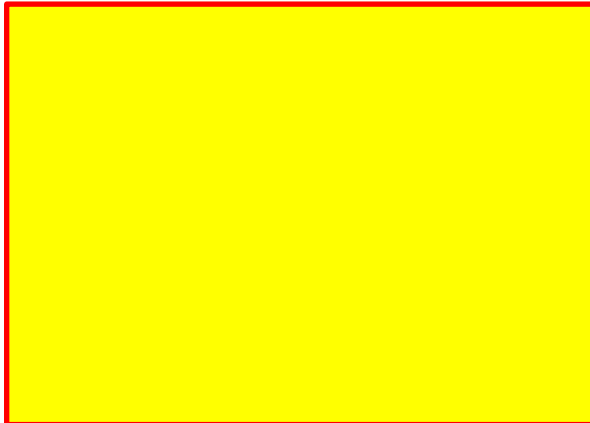
Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5} - 2}$$



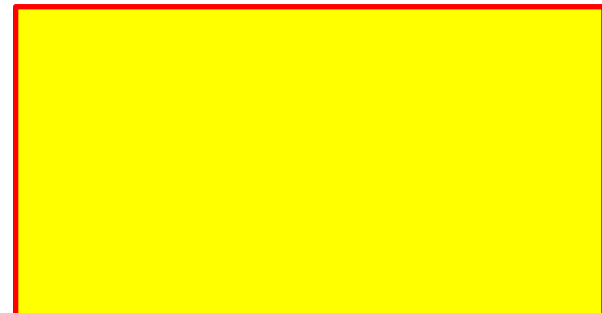
Rationalise the denominator and simplify

$$\frac{2\sqrt{3} - 1}{3\sqrt{3} + 1}$$



Solve  $y(\sqrt{3} - 1) = 8$

Give your answer in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers.



1 Expand and simplify:

$$(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1)$$

2 Rationalise the denominator, giving your answer in the form  $a + b\sqrt{3}$ .

$$\frac{3\sqrt{3} + 7}{3\sqrt{3} - 5}$$

3 Solve  $y(1 + \sqrt{2}) - \sqrt{2} = 3$

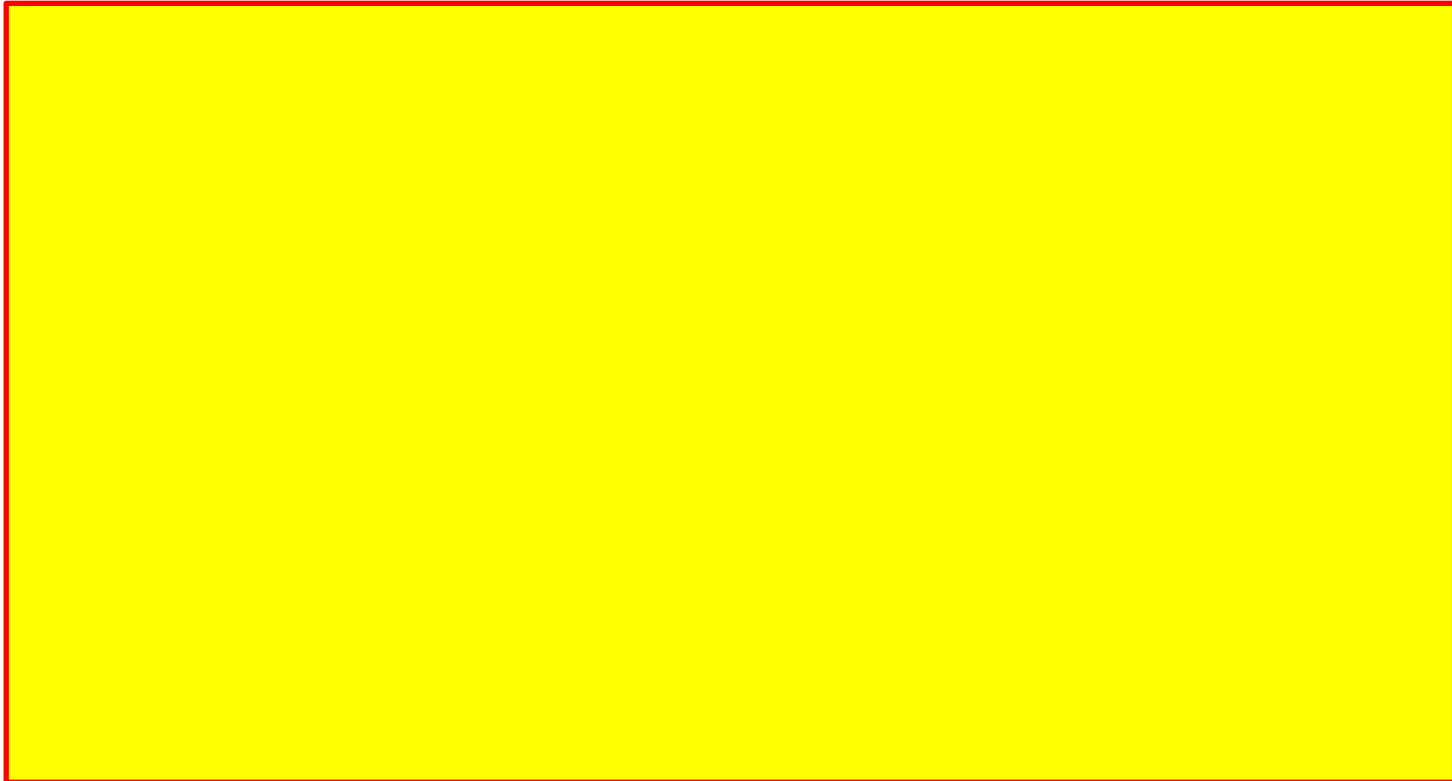
4 Simplify:

$$\frac{\sqrt{a+1} - \sqrt{a}}{\sqrt{a+1} + \sqrt{a}}$$



Non-Calculator Question

$$\text{Solve } \frac{\sqrt[4]{9}}{\sqrt[5]{27}} = \sqrt[x]{3}$$



1

Answer this question without a calculator, showing all your working and giving your answers in their simplest form.

(i) Solve the equation

$$4^{2x+1} = 8^{4x} \quad (3)$$

(ii) (a) Express

$$3\sqrt{18} - \sqrt{32}$$

in the form  $k\sqrt{2}$ , where  $k$  is an integer. (2)

(b) Hence, or otherwise, solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n} \quad (2)$$

**Answer this question without the use of a calculator and show your method clearly.**

(i) Show that

$$\sqrt{45} - \frac{20}{\sqrt{5}} + \sqrt{6}\sqrt{30} = 5\sqrt{5} \quad (2)$$

(ii) Show that

$$\frac{17\sqrt{2}}{\sqrt{2} + 6} = 3\sqrt{2} - 1 \quad (3)$$

3

Simplify the following expressions fully.

(a)  $\left(\frac{1}{9}x^4\right)^{0.5}$  (1)

(b)  $\left(\frac{x}{\sqrt{2}}\right)^{-2}$  (1)

(c)  $x\sqrt{3} \div \sqrt{\frac{48}{x^4}}$  (2)