

Equations and Inequalities



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Equations and Inequalities

3.1) Linear simultaneous equations

Substitution

$$X - 2Y = 12$$

$$2X + 3Y = -4$$

Equations and Inequalities

Elimination

$$5X + 2Y = 8$$

$$2X - 3Y = 7$$

$$5X + 2Y = 8$$

$$2X - 3Y = 7$$

Equations and Inequalities

$$X - 2Y = 12$$

Substitution

$$2X + 3Y = -4$$

$$5X + 2Y = 8$$

Elimination

$$2X - 3Y = 7$$

Equations and Inequalities

3.2) Quadratic simultaneous equation

Example

$$X + 2Y = 1$$

$$2X^2 + XY - Y^2 = 36$$

Equations and Inequalities

Example

$$X + Y = 6$$

$$Y = 6X - 2X^2 + 1$$

Equations and Inequalities

Example

$$Y + 4X = 2$$

$$3X^2 + XY + 11 = 0$$

Equations and Inequalities

3.3) Simultaneous equations on graphs

Solutions of simultaneous equations = Points of intersection

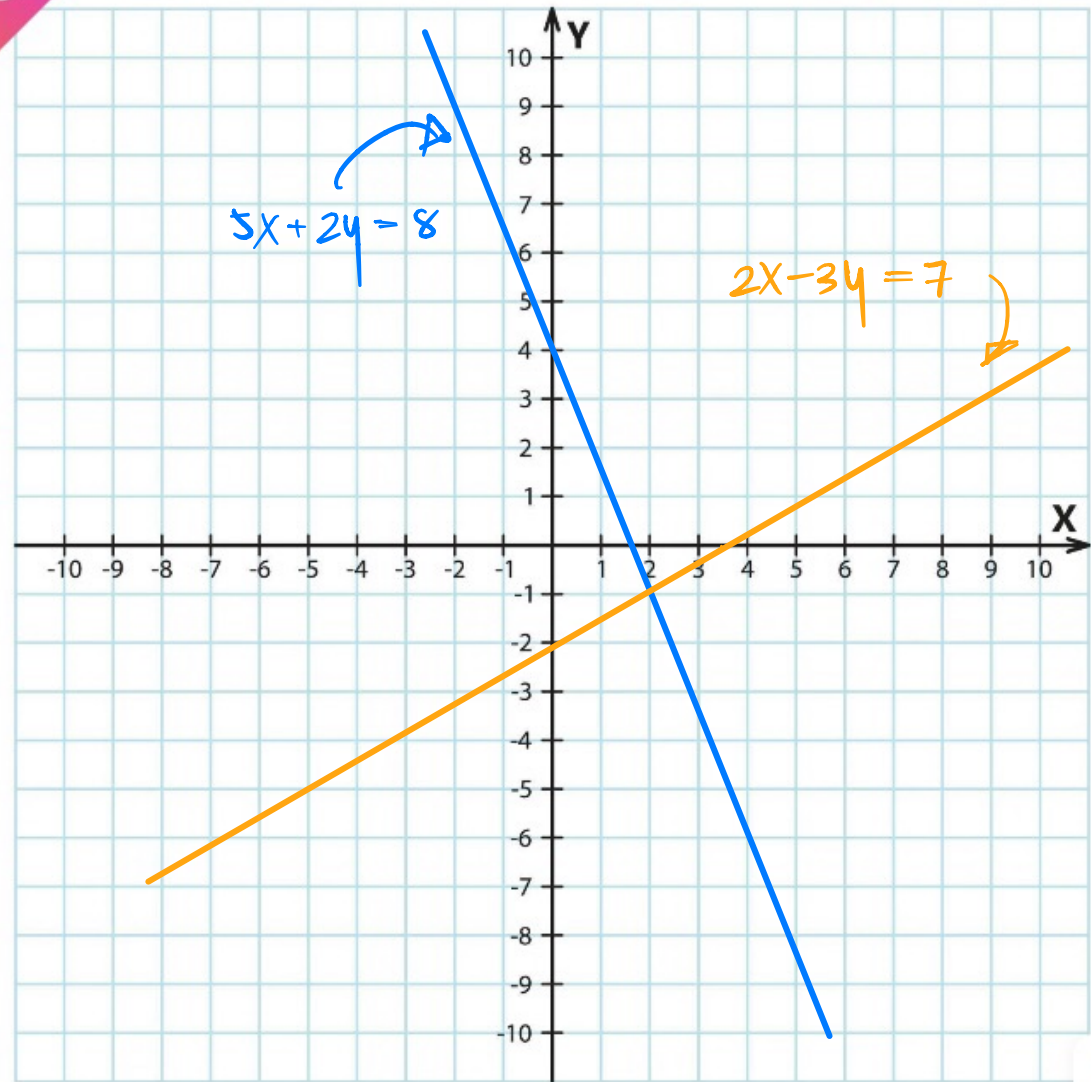
Equations and Inequalities

Linear simultaneous equations

$$5X + 2Y = 8$$

$$2X - 3Y = 7$$

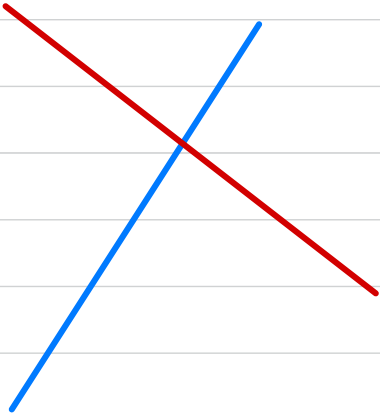
$(2, -1)$



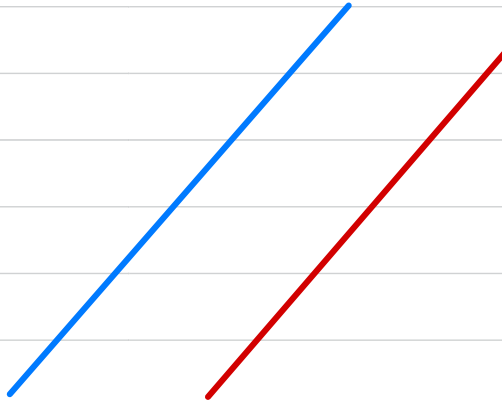
Equations and Inequalities

Intersection points for Linear simultaneous equations

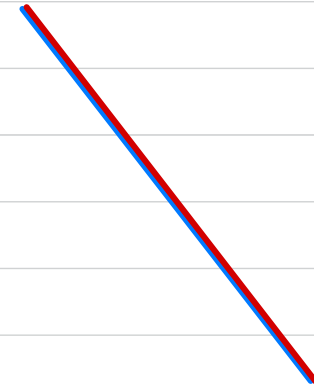
One solution



No solution



Many solutions



Equations and Inequalities

No solution

$$AX + BY = C$$

$$DX + EY = F$$

Parallel lines $\rightarrow \frac{A}{D} = \frac{B}{E} \neq \frac{C}{F}$

$$2x + 3y = 7$$

$$4x + 6y = 10$$

$$y = 4x - 10$$

$$3y - 12x = 5$$

Equations and Inequalities

Many solutions

$$AX + BY = C$$

$$DX + EY = F$$

$$\text{Same lines} \rightarrow \frac{A}{D} = \frac{B}{E} = \frac{C}{F}$$

$$3x - 6y = 12$$

$$x - 2y = 4$$

$$2x = 1 + 3y$$

$$6y = 4x - 2$$

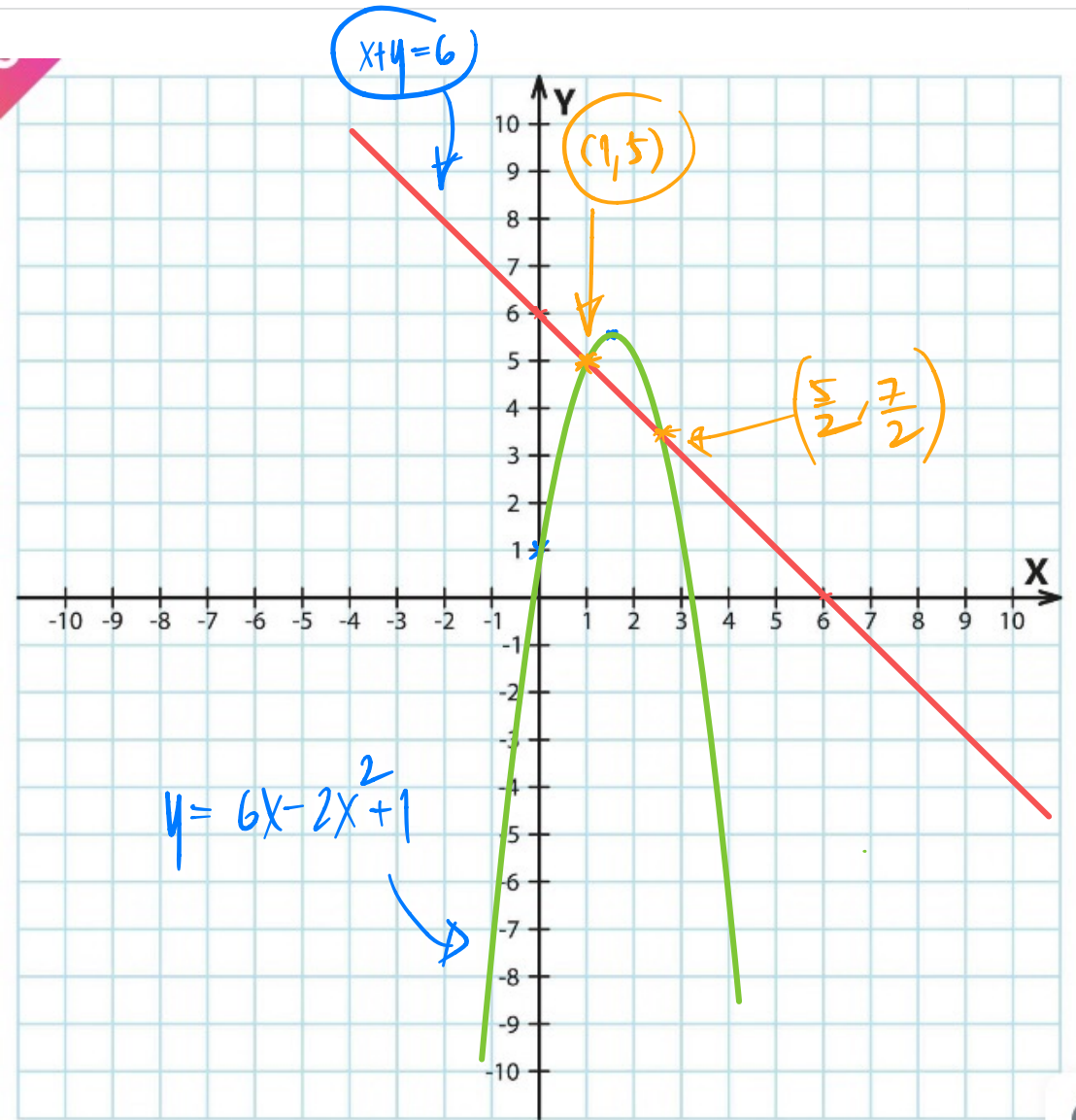
Equations and Inequalities

Quadratic simultaneous equation

$$X + Y = 6$$

$$Y = 6X - 2X^2 + 1$$

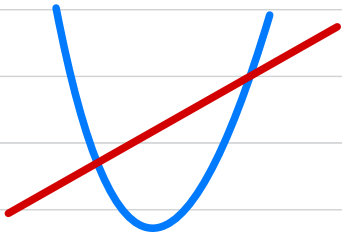
$$\left\{ \left(\frac{5}{2}, \frac{7}{2} \right) \text{ and } (1, 5) \right.$$



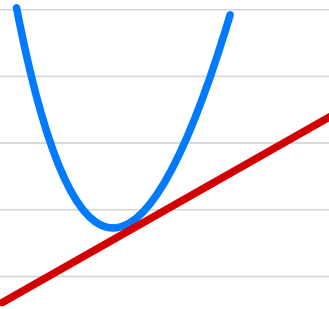
Equations and Inequalities

Intersection points for Quadratic simultaneous equations

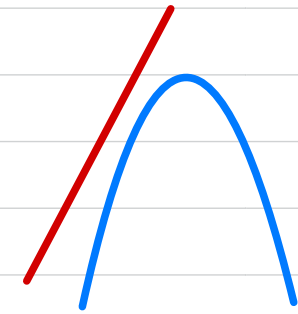
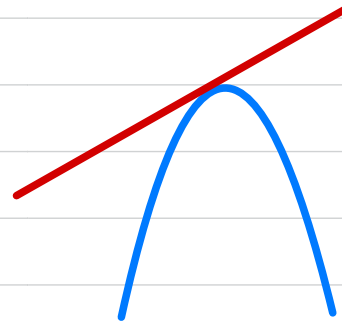
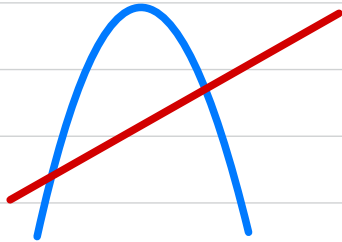
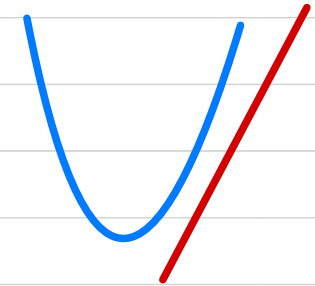
Two solutions



One solution



No solution



$$b^2 - 4ac > 0$$

$$b^2 - 4ac = 0$$

$$b^2 - 4ac < 0$$

Equations and Inequalities

Example: The curve C has equation $Y = 3X^2 - 4X + 2$

The line has equation $kX + 2Y - 3 = 0$ where k is a constant.

Find the two exact values of k for which line is a tangent to C.

Equations and Inequalities

Example: The line has equation $Y = mX - 2$ where m is a constant.

The curve C has equation $Y = 2X^2 + X + 6$

The line does not cross or touch the curve C

a) Show that m satisfies the inequality $m^2 - 2m - 63 < 0$

Equations and Inequalities

3.4) Linear Inequalities

Review: Number Lines



Equations and Inequalities

Example:



Equations and Inequalities



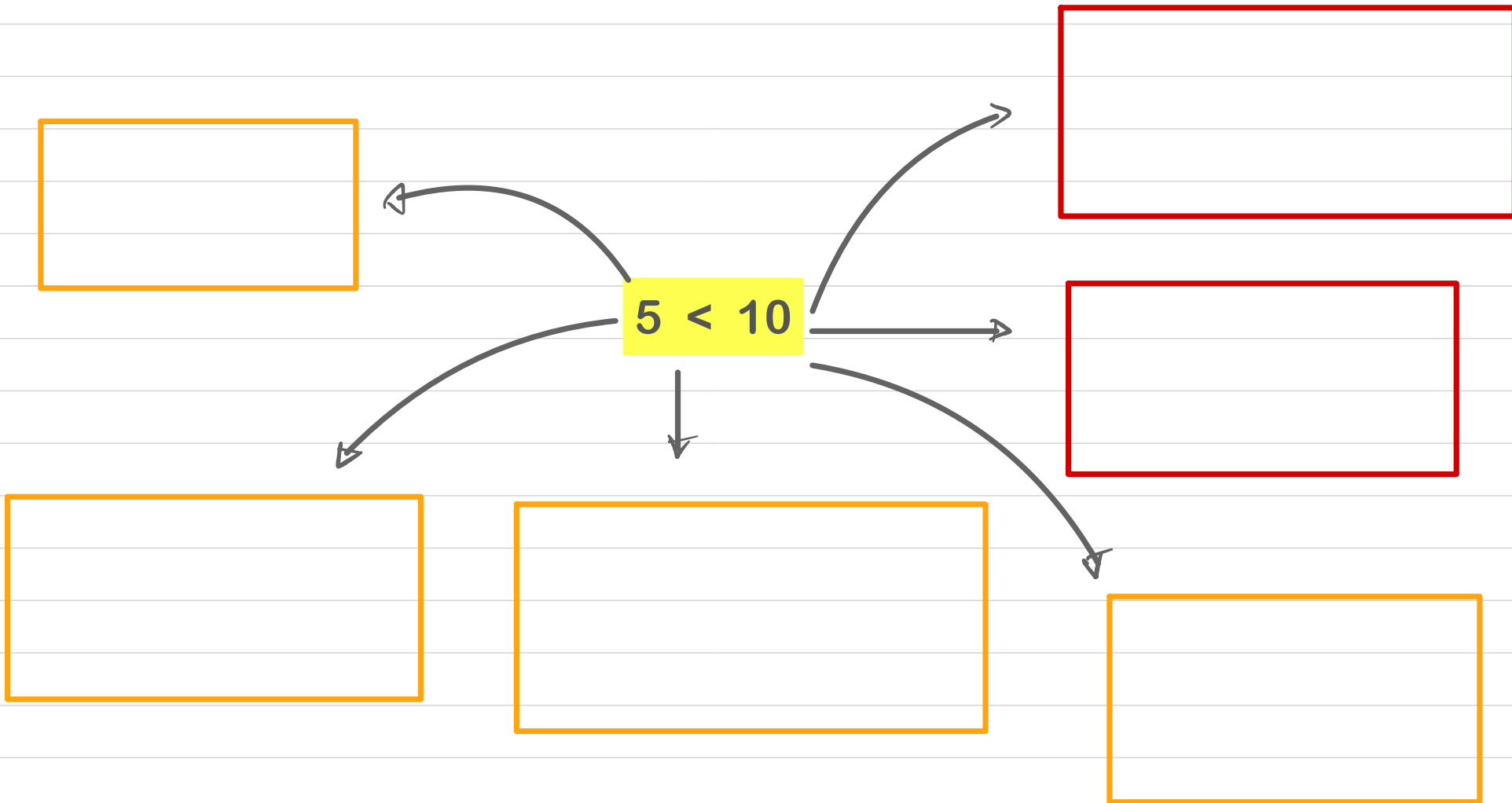
Equations and Inequalities

Example: Find the set of values of x for which.

$$1) 5x + 9 \geq x + 20$$

Equations and Inequalities

Negative values for Inequalities



Equations and Inequalities

$$2) \quad 12 - 3X < 27$$

Equations and Inequalities

$$3) \quad 3(x - 5) > 5 - 2(x - 8)$$

Equations and Inequalities

4) $3X - 5 < X + 8$ and $5X > X - 8$

Equations and Inequalities

$$5) \quad 2(x - 3) \leq x + 3 < 4(x + 5)$$

Equations and Inequalities

$$6) \quad X - 5 > 1 - X \quad \text{or} \quad 15 - 3X > 5 + 2X$$

Equations and Inequalities

3.5) Quadratic Inequalities.

Step

- 1) Rearrange so that the right - hand side of the inequality is 0.
- 2) Solve the corresponding quadratic equation to find the critical values.
- 3) Sketch the graph of the quadratic function.
- 4) Use your sketch to find the required set of values.

Equations and Inequalities

Example: Find the set of values of x for which.

1) $x^2 - 9 > 0$

Equations and Inequalities

$$2) \quad x^2 - 4x < 5$$

Equations and Inequalities

$$3) -3x^2 + 7x - 2 < 0$$

Equations and Inequalities

$$4) \quad 2x^2 + 6x - 1 > 0$$

Equations and Inequalities

5) Both $2X - 3 < 5$ and $X^2 - 4X < 12$

Equations and Inequalities

$$6) \frac{5}{x} > 2$$

Equations and Inequalities

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p .

(4)

Equations and Inequalities

The equation

$$(p - 1)x^2 + 4x + (p - 5) = 0, \text{ where } p \text{ is a constant}$$

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Equations and Inequalities

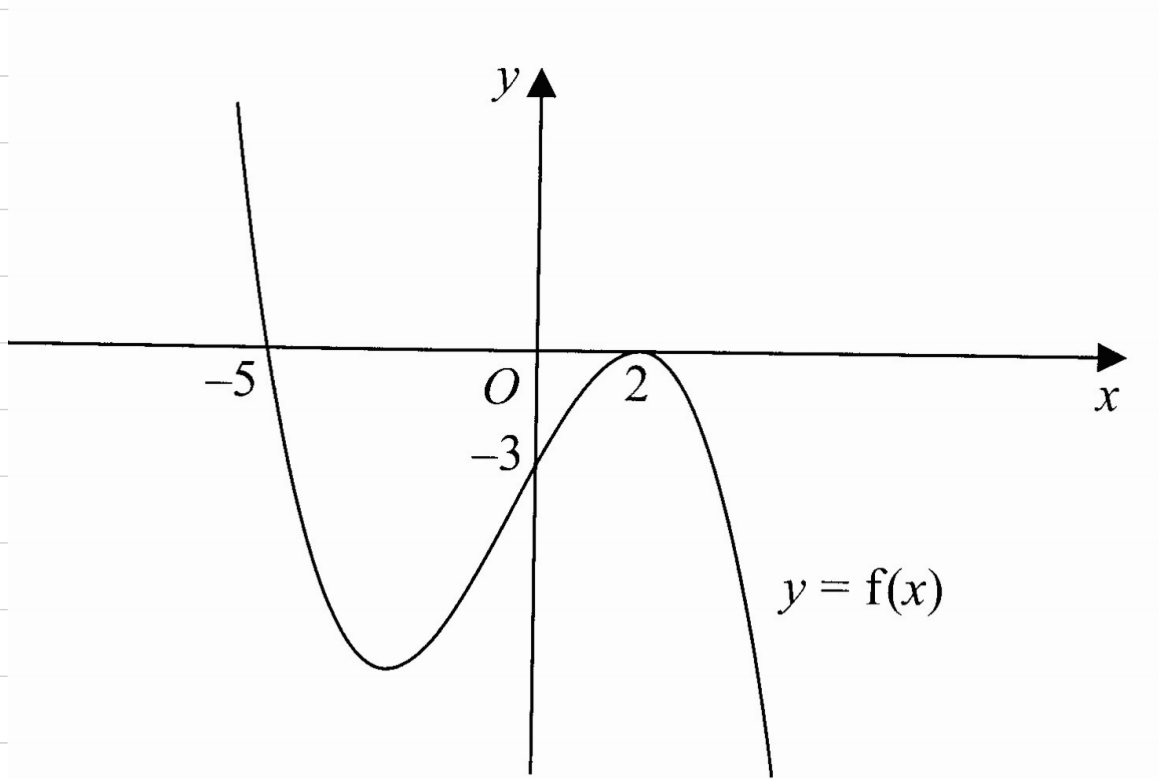
3.6) Inequalities on graphs

Example: Find the values of x .

$$f(x) = 0$$

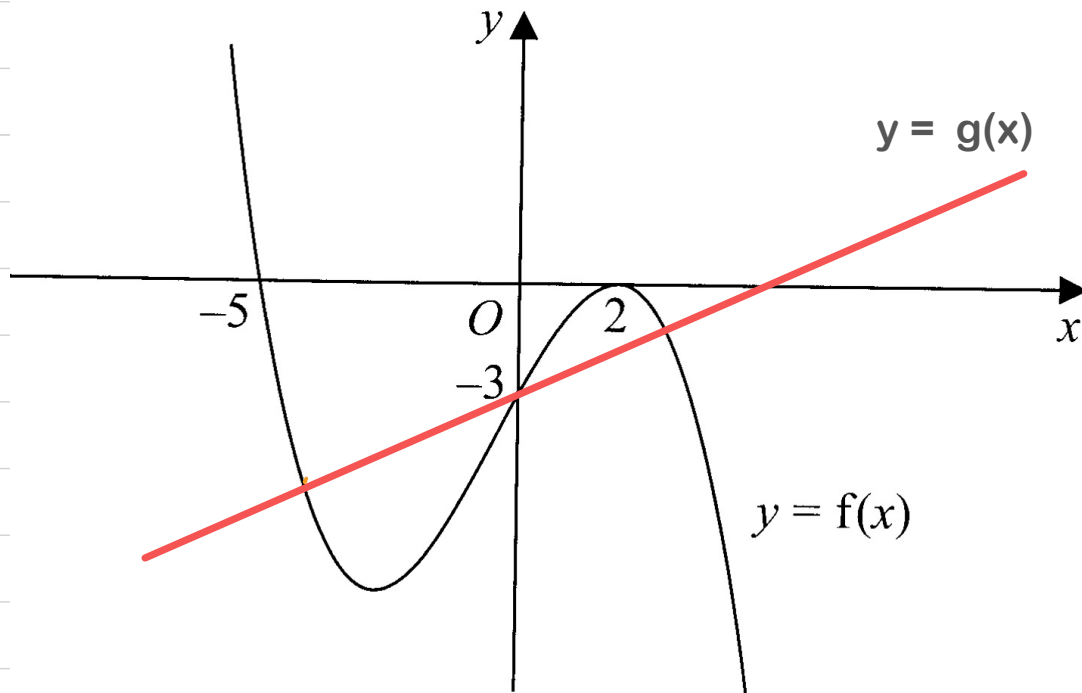
$$f(x) > 0$$

$$f(x) < 0$$



Equations and Inequalities

Example: Find the values of x .



$$f(x) = g(x)$$

$$f(x) > g(x)$$

$$f(x) < g(x)$$

Equations and Inequalities

Example: Find the values of x .

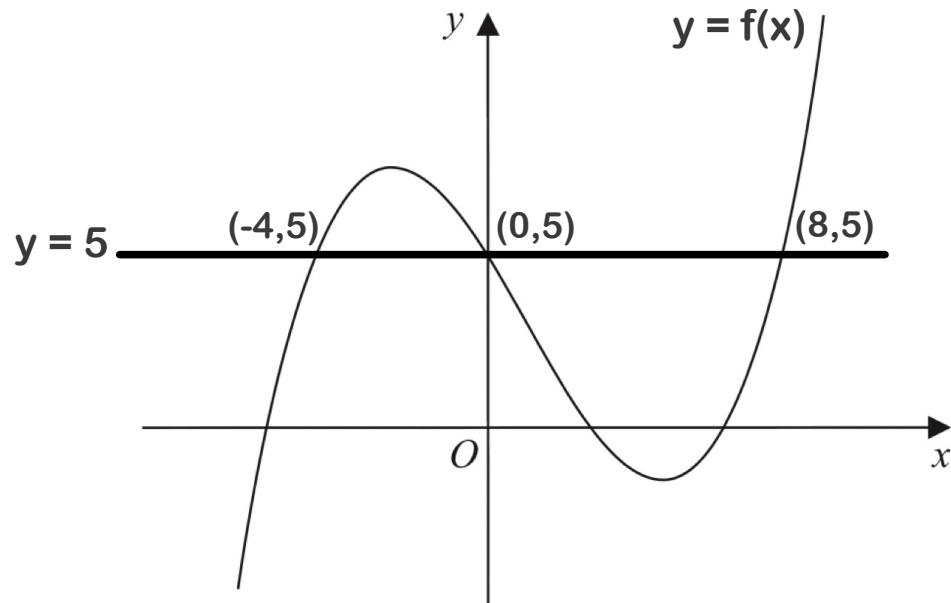


Figure 3

$$f(x) > 5$$

$$f(x) < 5$$

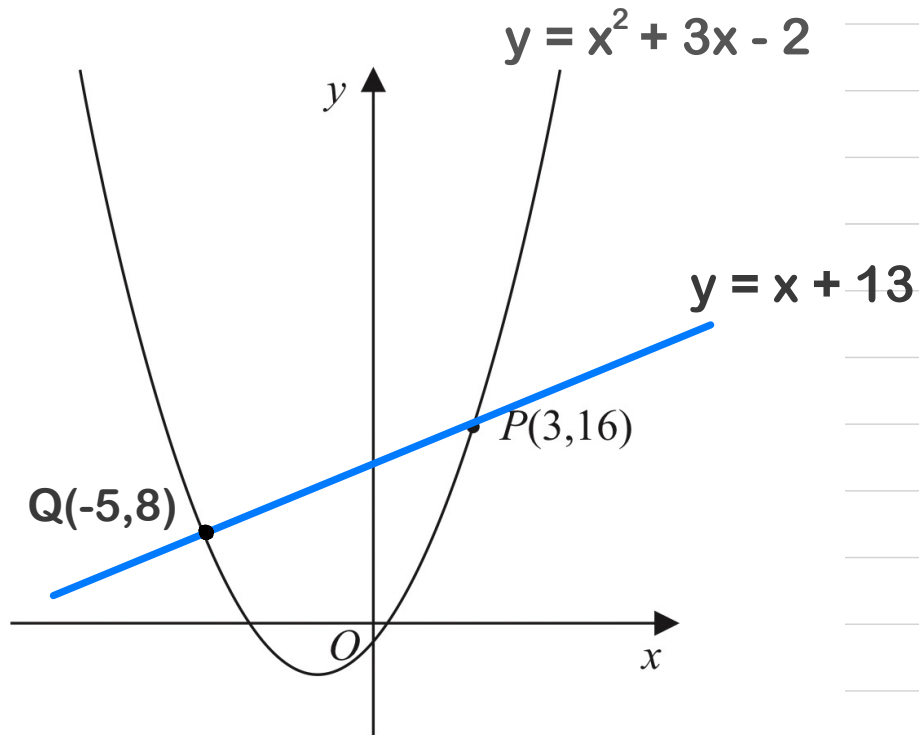
Equations and Inequalities

Example: Find the values of x .

$$x^2 + 3x - 2 = x + 13$$

$$x^2 + 3x - 2 > x + 13$$

$$x^2 + 3x - 2 < x + 13$$



Equations and Inequalities

Exercise:

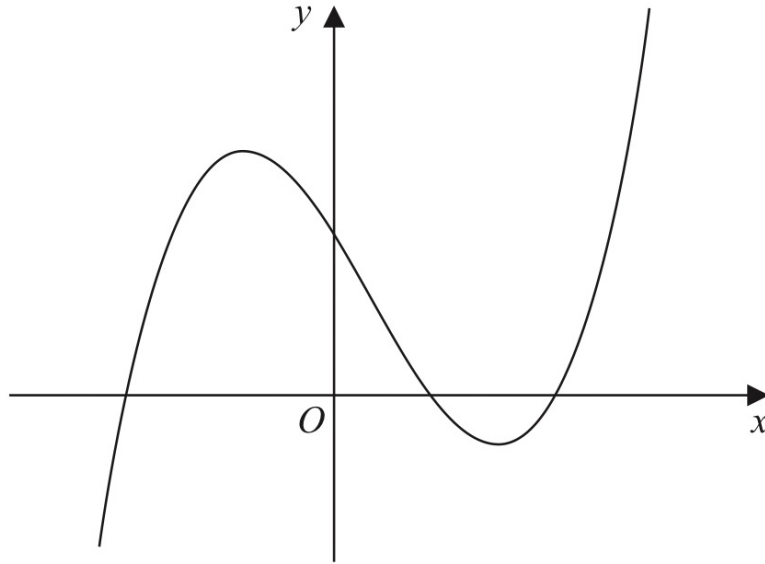


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

(a) Use the given information to state the values of x for which

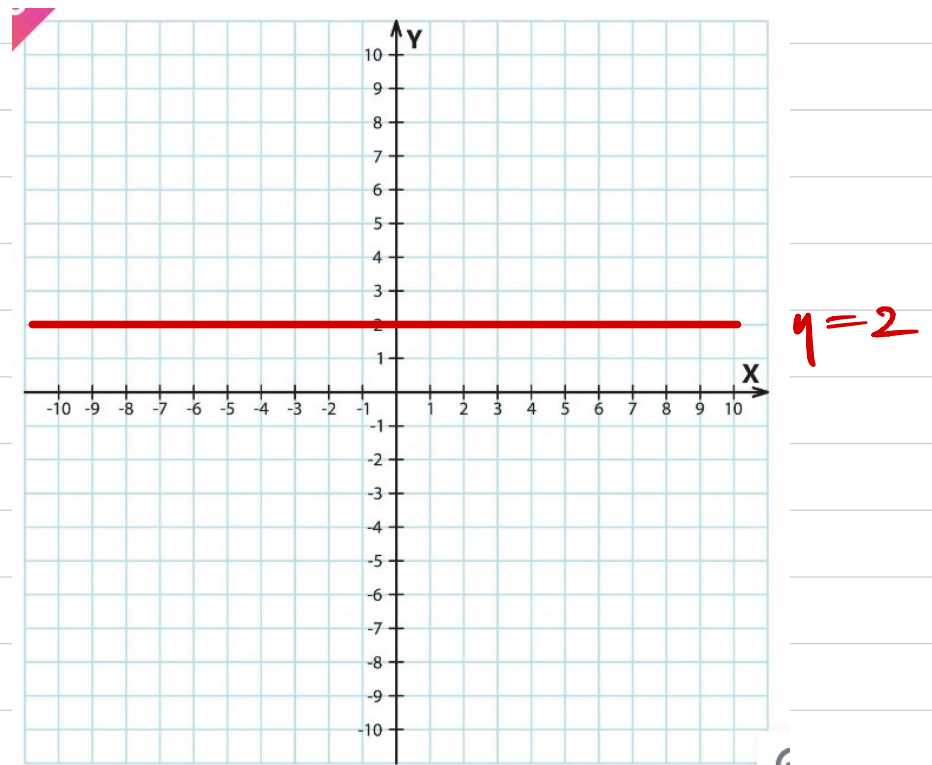
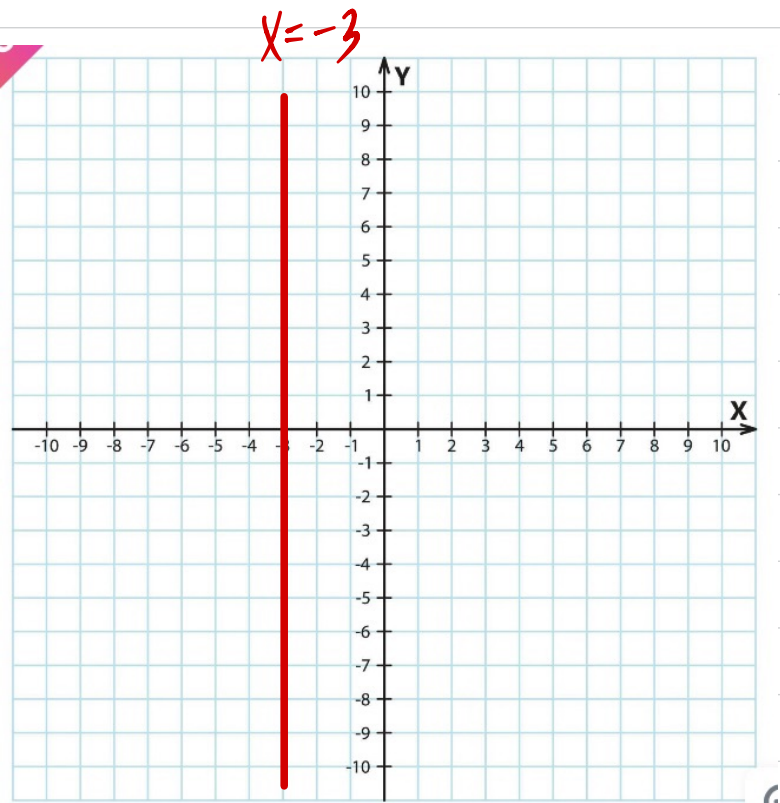
$$f(x) > 0$$

(2)

Equations and Inequalities

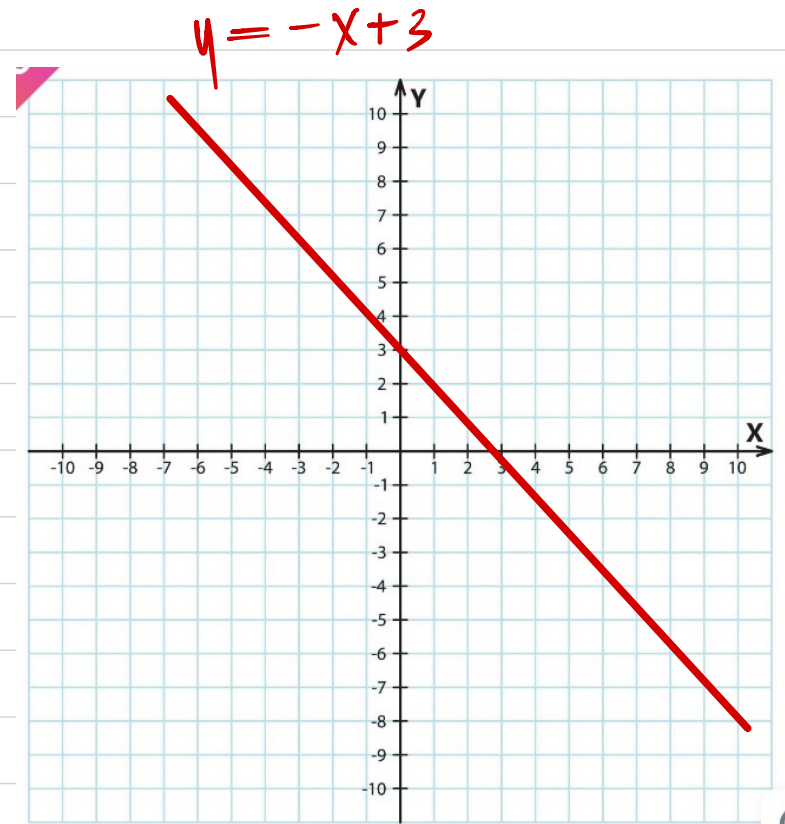
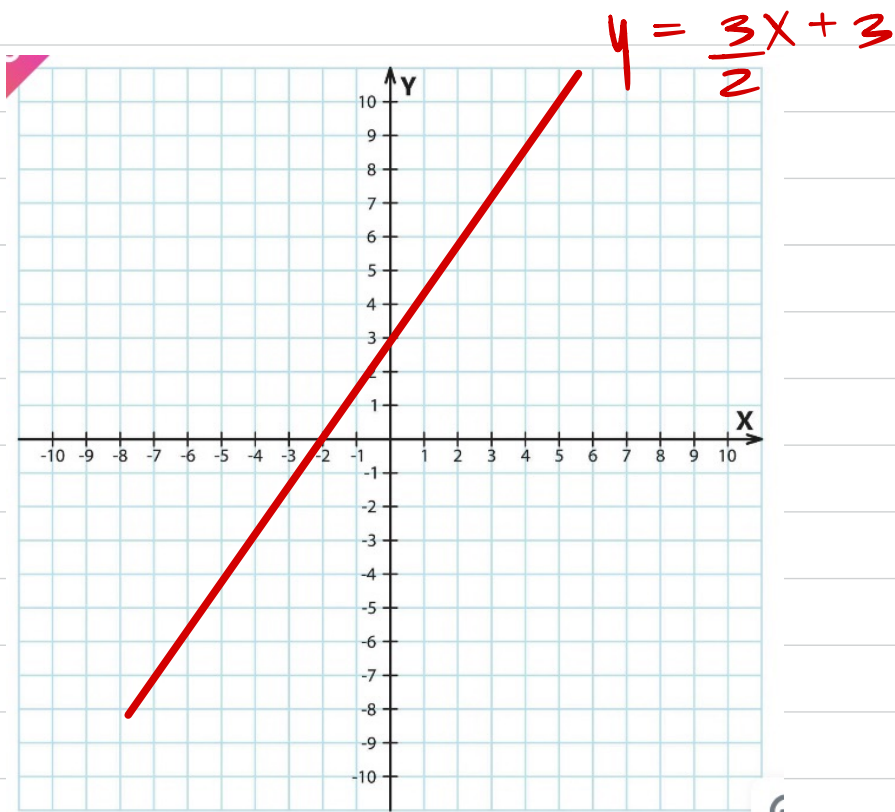
3.7) Regions

Example: Find shaded region.



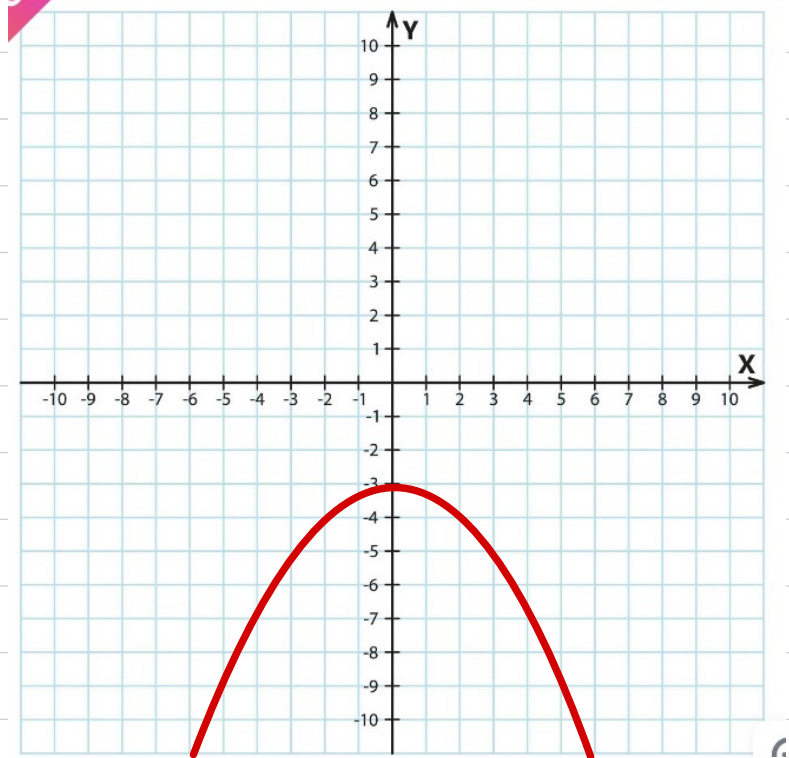
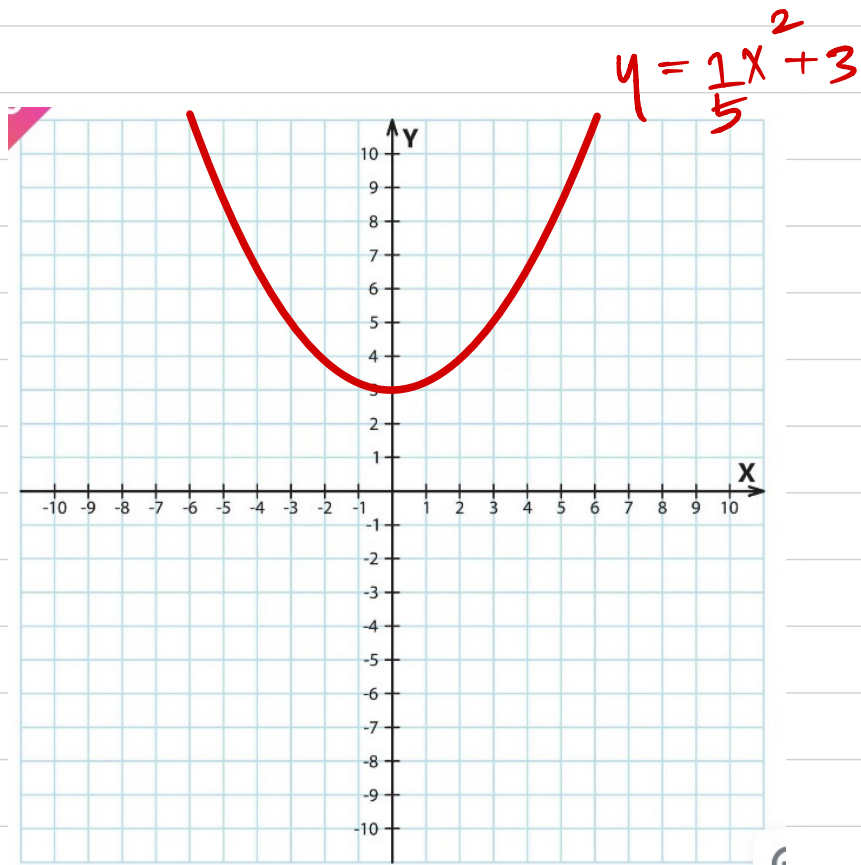
Equations and Inequalities

Example: Find shaded region.



Equations and Inequalities

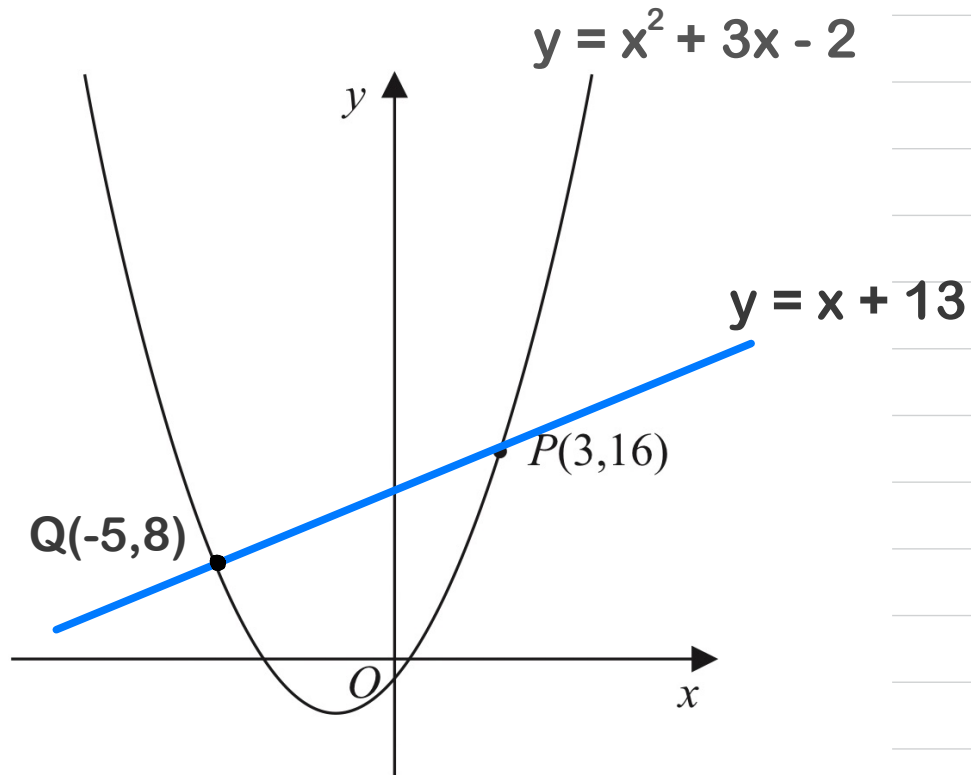
Example: Find shaded region.



$$y = -\frac{1}{5}x^2 - 3$$

Equations and Inequalities

Example: Find shaded region.

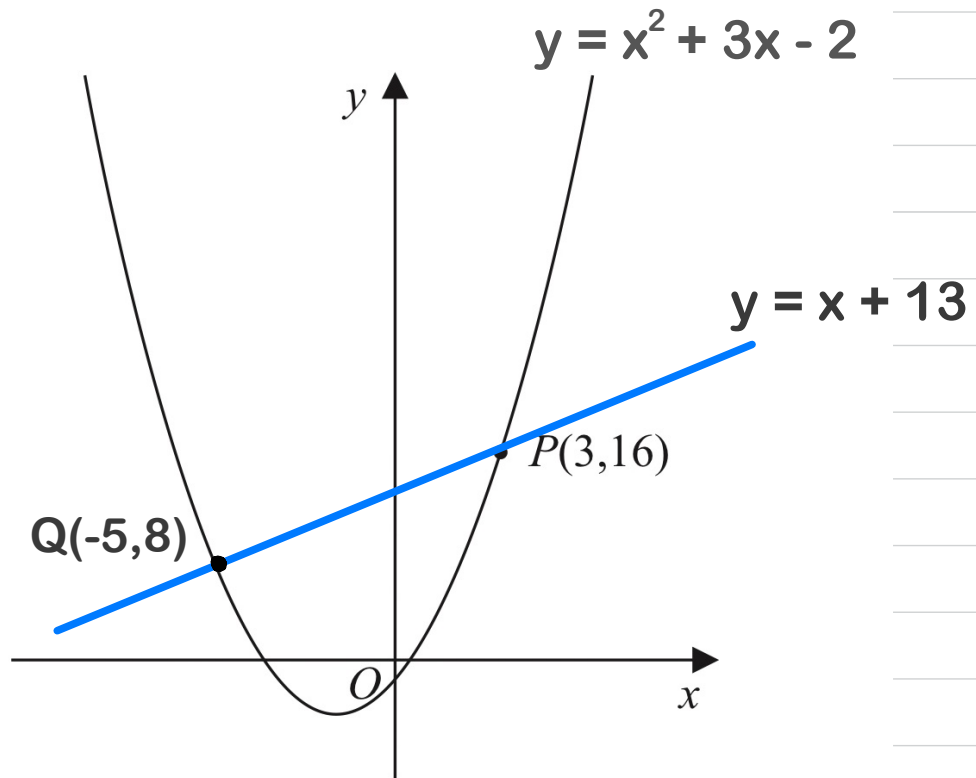


$$y \geq x^2 + 3x - 2$$

$$y \leq x + 13$$

Equations and Inequalities

Example: Find shaded region.

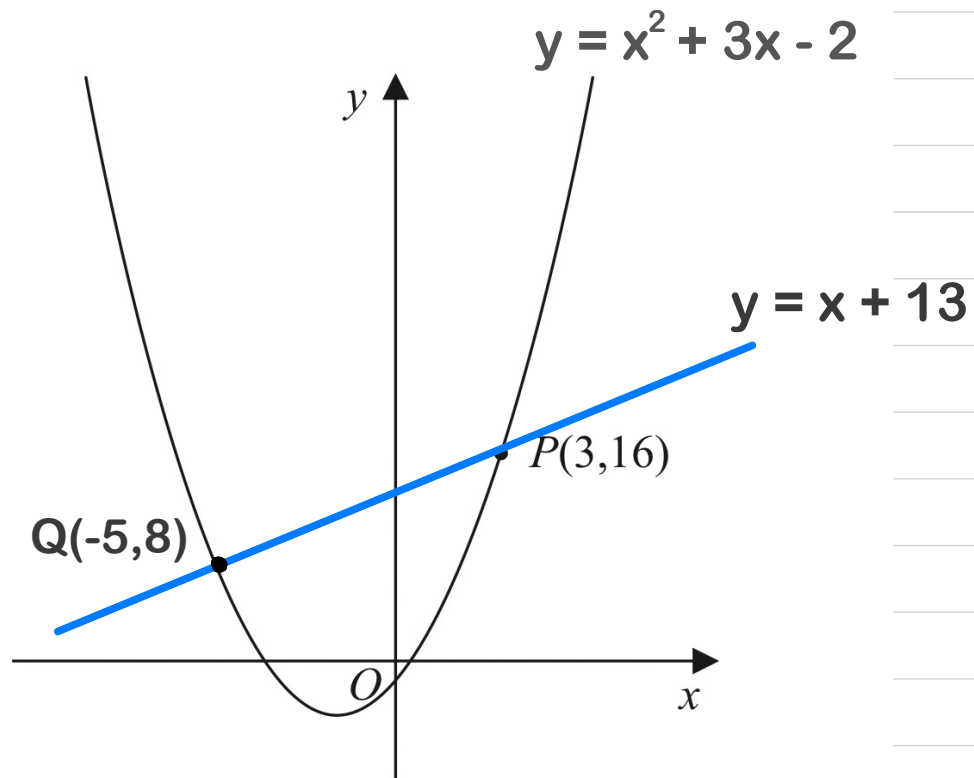


$$y \leq x^2 + 3x - 2$$

$$y > x + 13$$

Equations and Inequalities

Example: Find shaded region.



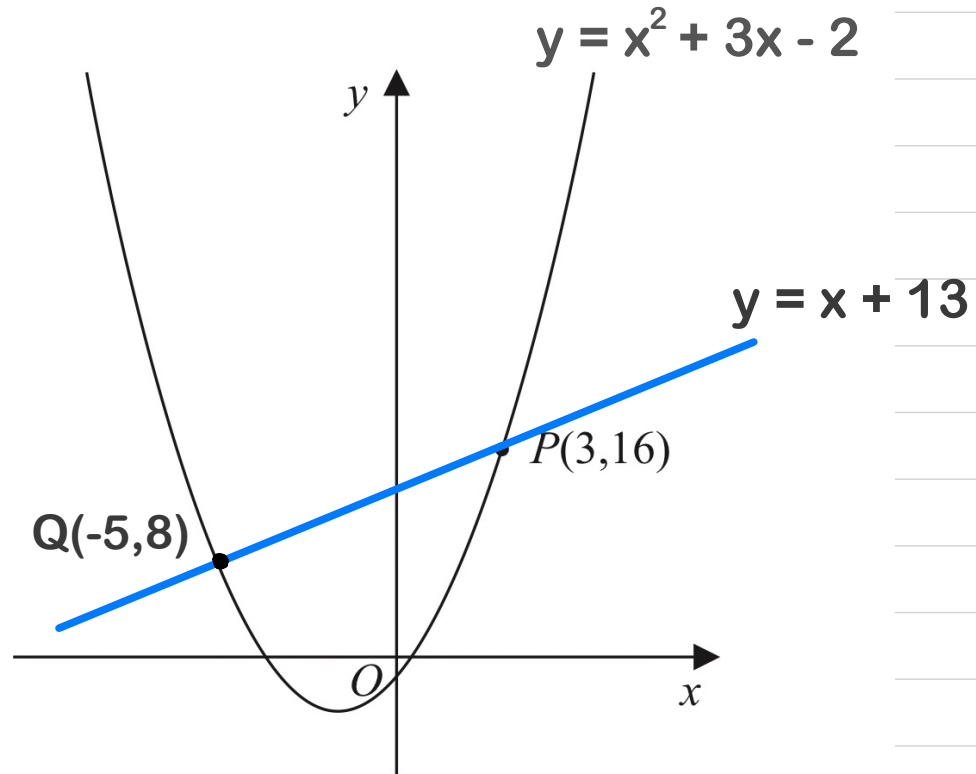
$$y \leq x^2 + 3x - 2$$

$$y \geq x + 13$$

$$x \geq 3$$

Equations and Inequalities

Example: Find shaded region.



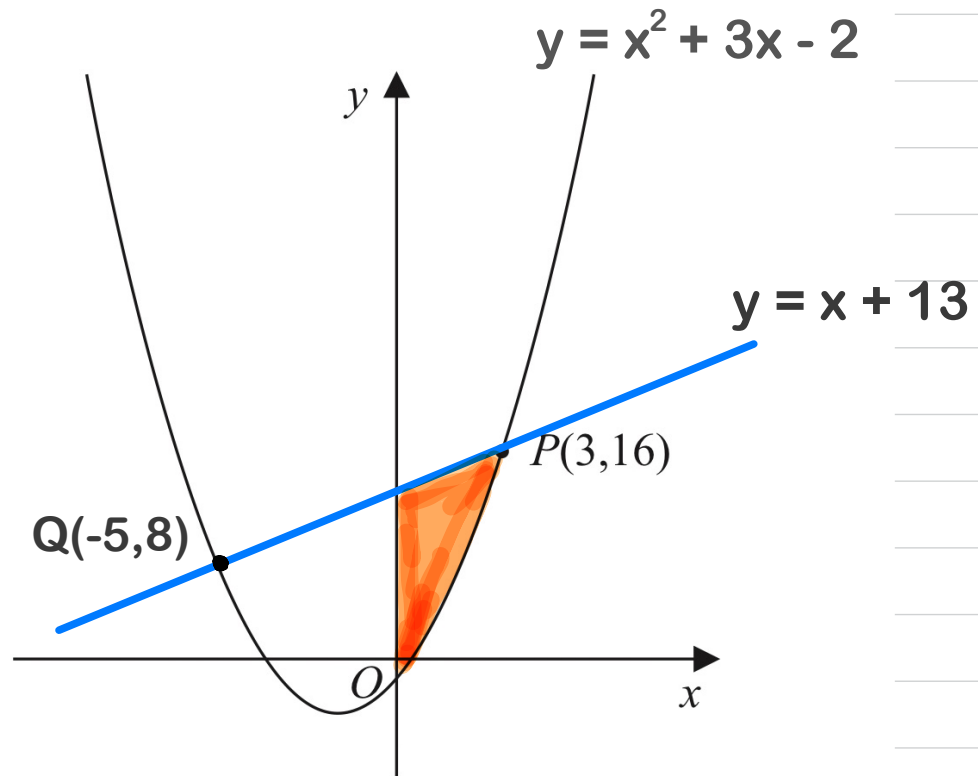
$$y \leq x^2 + 3x - 2$$

$$y \geq x + 13$$

$$x \leq -5$$

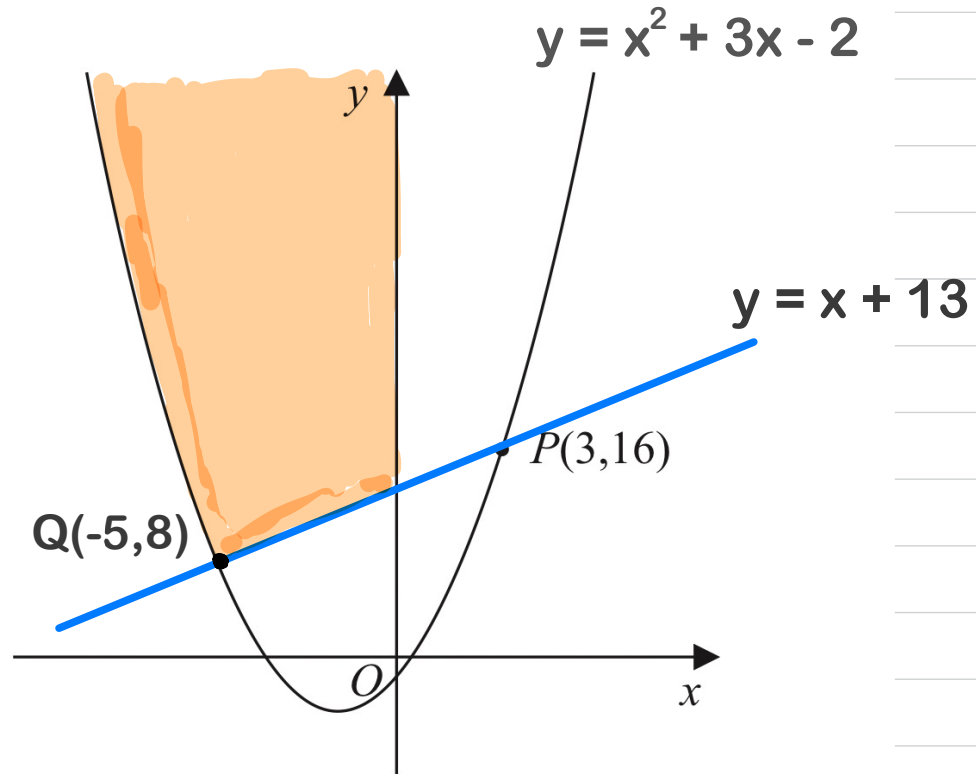
Equations and Inequalities

Example: Use inequalities to define the region R.



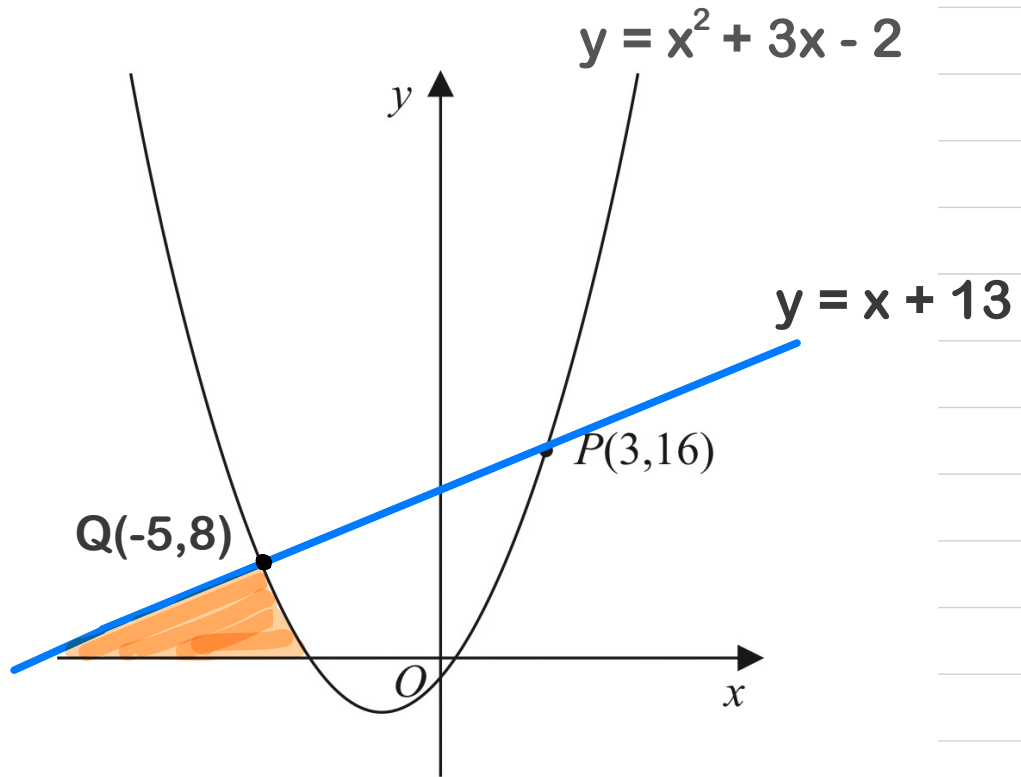
Equations and Inequalities

Example: Use inequalities to define the region R.



Equations and Inequalities

Example: Use inequalities to define the region R.



Equations and Inequalities

Exercise:

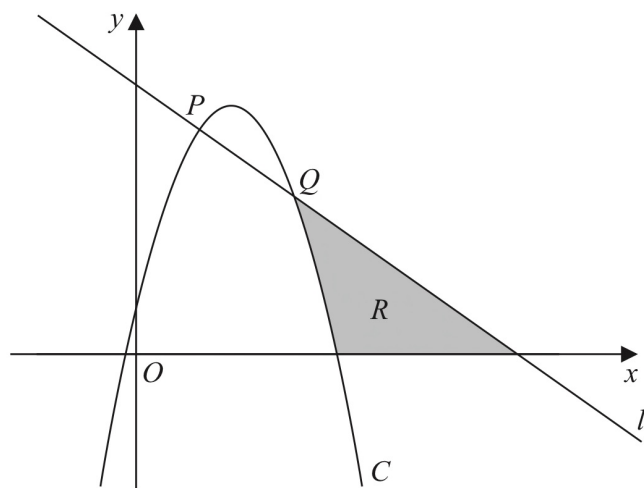


Figure 1

Figure 1 shows a line l with equation $x + y = 6$ and a curve C with equation $y = 6x - 2x^2 + 1$

The line l intersects the curve C at the points P and Q as shown in Figure 1.

(a) Find, using algebra, the coordinates of P and the coordinates of Q .

Equations and Inequalities

Exercise:

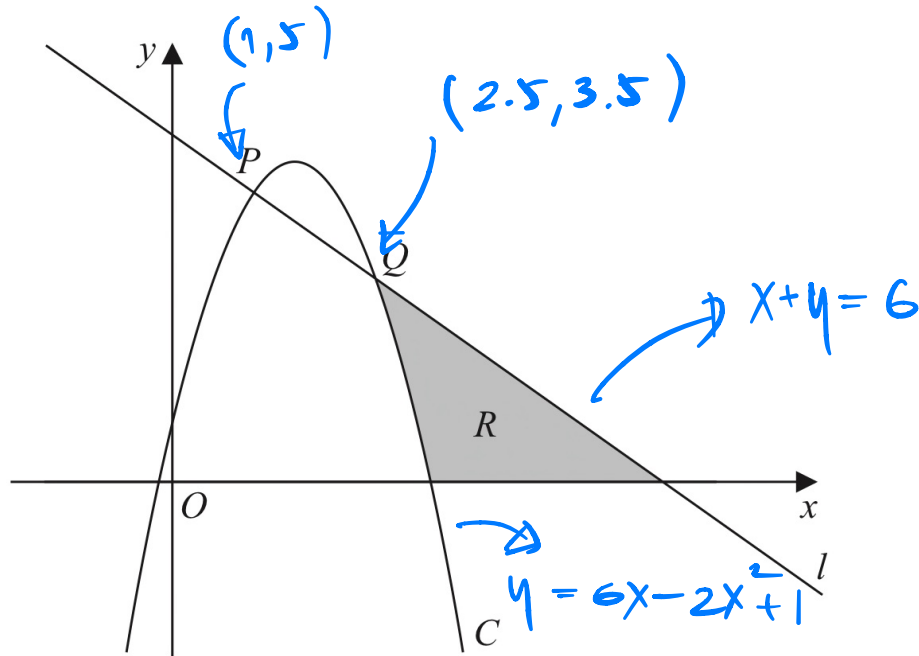


Figure 1

(b) Use inequalities to define the region R .

Equations and Inequalities

Exercise:

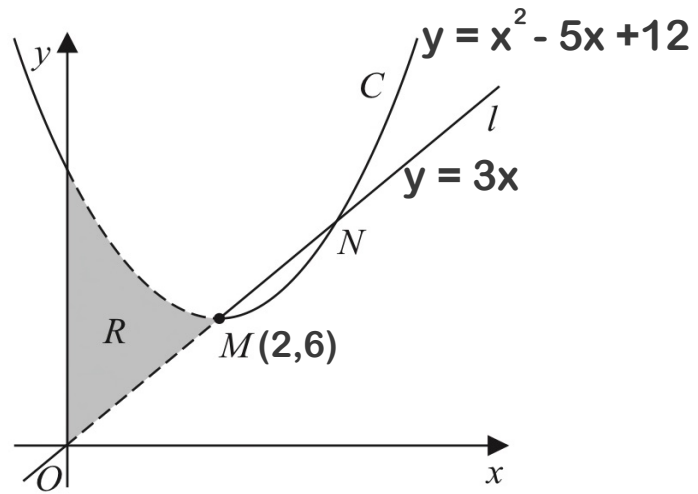


Figure 3

Figure 3 shows the curve C and the line l . The finite region R , shown shaded in Figure 3, is bounded by C , l and the y -axis.

(c) Use inequalities to define the region R .