

PURE 1

GRAPHS &

TRANSFORMATIONS

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## Type of graphs

- ★ Quadratic graphs
- ★ Cubic graphs
- ★ Reciprocal graphs
- ★ Points of intersection

Form of quadratic graph

$$y = ax^2 + bx + c$$

or

$$y = a(x + m)^2 + n$$



# Graphs : Quadratic graphs

TOP Maths A\*level



# Graphs : Quadratic graphs

TOP Maths A\*level

Sketch each curve showing the coordinates of any points of intersection with the coordinate axes.

$$y = x^2 - 3x + 2$$

$$y = -x^2 + 5x - 4$$

Sketch each curve showing the exact coordinates of its turning point and the point where it crosses the  $y$ -axis.

$$y = x^2 - 4x + 3$$

$$y = -2x^2 + 8x - 15$$

$$y = ax^3 + bx^2 + cx + d$$

## Shapes of cubic graphs

## Sketching cubic graphs

Sketch each graph showing the coordinates of any points of intersection with the coordinate axes.

$$y = (x + 1)(x - 1)(x - 3)$$

$$y = 2x(x - 1)(x - 5)$$

$$y = -(x + 2)(x + 1)(x - 2)$$

$$y = x^2(x - 4)$$

$$y = (x + 2)(x - 1)^2$$



# Graphs : Cubic graphs

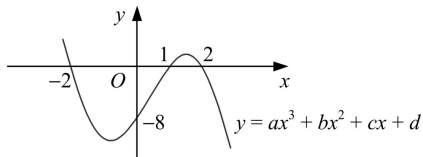
TOP Maths A\*level

Factorise fully  $x^3 + 6x^2 + 9x$ .

Hence, sketch the curve  $y = x^3 + 6x^2 + 9x$ , showing the coordinates of any points where the curve meets the coordinate axes.

# Graphs : Cubic graphs

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The diagram shows the curve with equation  $y = ax^3 + bx^2 + cx + d$ .

Given that the curve crosses the  $y$ -axis at the point  $(0, -8)$  and crosses the  $x$ -axis at the points  $(-2, 0)$ ,  $(1, 0)$  and  $(2, 0)$ , find the values of the constants  $a$ ,  $b$ ,  $c$  and  $d$ .



# Graphs : Reciprocal graphs

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$$y = \frac{a}{x}$$



# Graphs : Reciprocal graphs

TOP Maths A\*level

$$y = \frac{a}{x^2}$$

# Graphs : Reciprocal graphs

TOP Maths A\*level

Sketch and label each pair of graphs on the same set of axes showing the coordinates of any points where the graphs intersect. Write down the equations of any asymptotes.

$$y = \frac{1}{x} \quad \text{and} \quad y = \frac{2}{x}$$

# Graphs : Reciprocal graphs

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$$y = \frac{1}{x} \quad \text{and} \quad y = \frac{1}{x^2}$$

# Graphs : Reciprocal graphs

TOP Maths A\*level

$$y = \frac{2}{x} \quad \text{and} \quad y = \frac{-2}{x^2}$$



# Graphs : Exponential graphs

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You can sketch curve of functions to show points of intersection and solutions.

The x coordinates at points of intersection of the curve with equations

$y = f(x)$  and  $y = g(x)$  are the solutions to the equation  $f(x) = g(x)$



- a** Sketch on the same diagram the straight line  $y = 2x - 5$  and the curve  $y = x^3 - 3x^2$ , showing the coordinates of any points where each graph meets the coordinate axes.
- b** Hence, state the number of real roots that exist for the equation

$$x^3 - 3x^2 - 2x + 5 = 0,$$



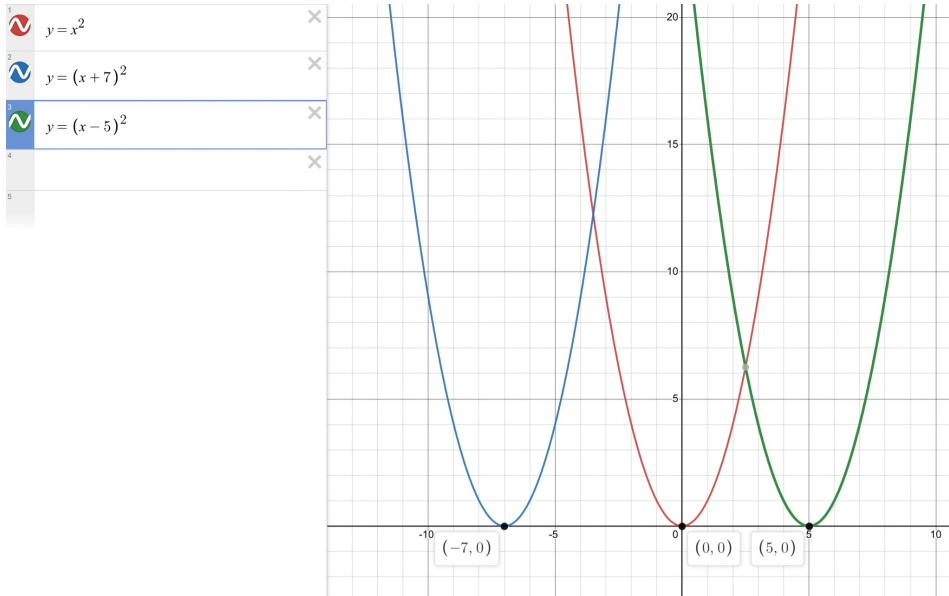
## Type of Transformations

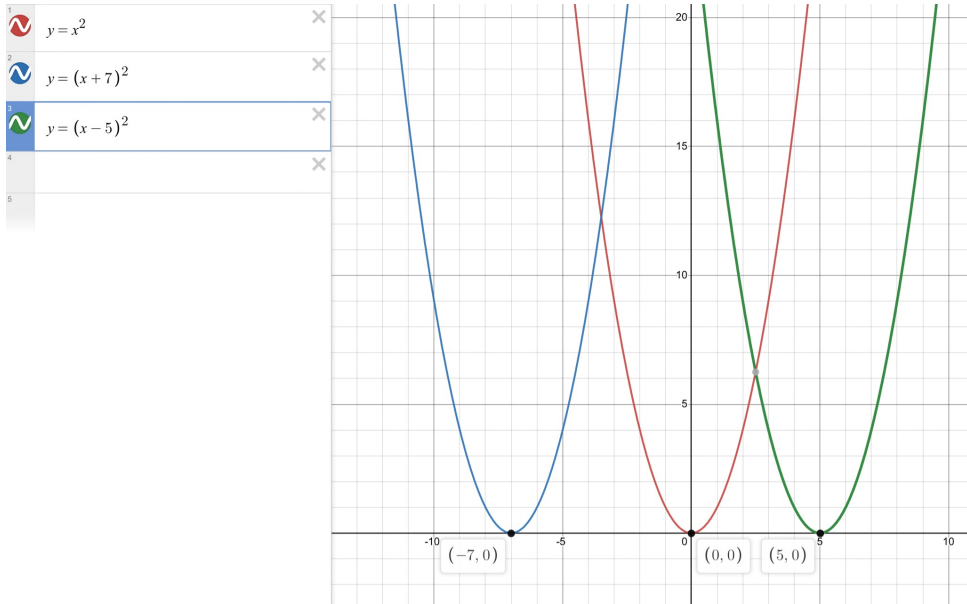
★ Translation

★ Reflection

★ Stretching

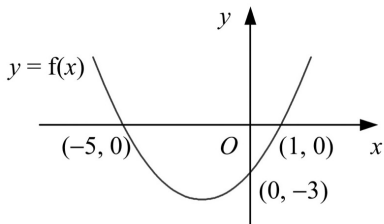
# Translations





# Note

## Question

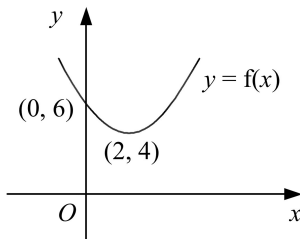


Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

$$y = f(x - 5)$$

$$y = f(x + 1)$$

## Question






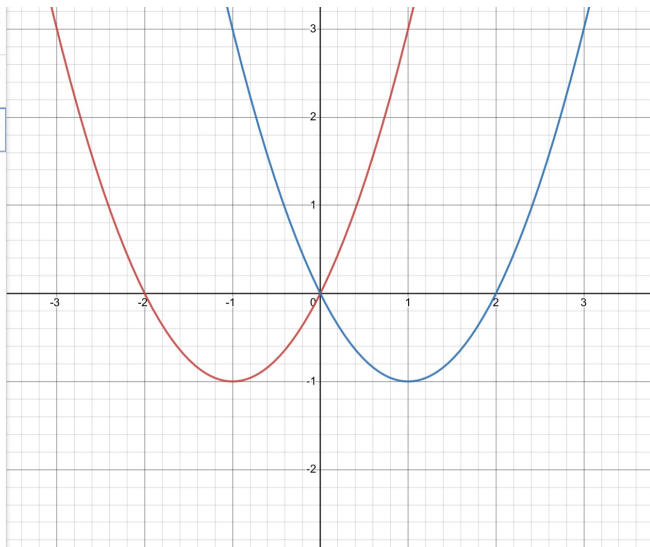
Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

$$y = 4 + f(x)$$

$$y = f(x) - 3$$

# Reflections

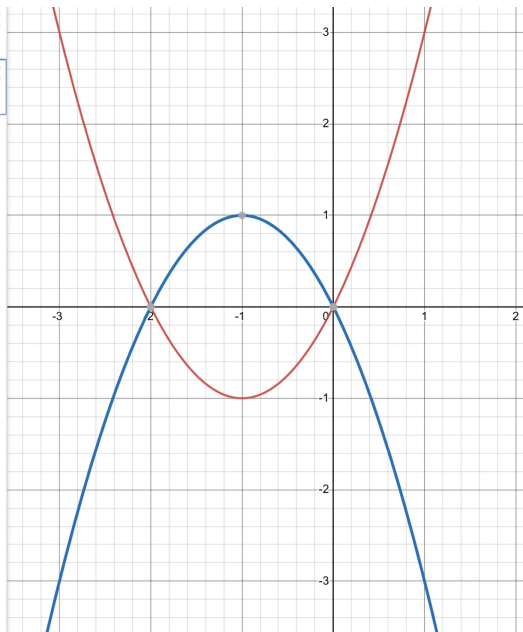
- 1   $y = x^2 + 2x$  ×
- 2   $y = (-x)^2 + 2(-x)$  ×
- 3  | ×
- 4



1   $y = x^2 + 2x$  ×

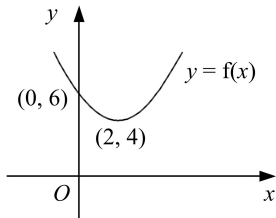
2   $y = -(x^2 + 2x)$  ×

3



# Note

## Question



Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

$$y = f(-x)$$

$$y = -f(x)$$

## Question

Find and simplify an equation of the graph obtained when the graph of  $y = 4x - 7$  is reflected in the  $x$ -axis.

**Question**

Describe a single transformation that would map the graph of  $y = x^3$  onto the graph of

$$y = -x^3$$

# Stretching





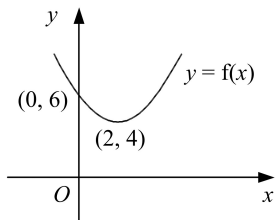






# Note

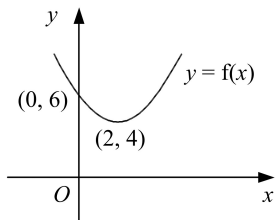
## Question



Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

$$y = f(2x)$$

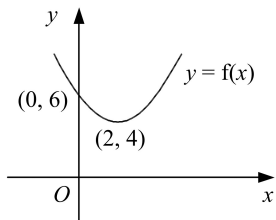
## Question



Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

$$y = f\left(\frac{1}{2}x\right)$$

## Question

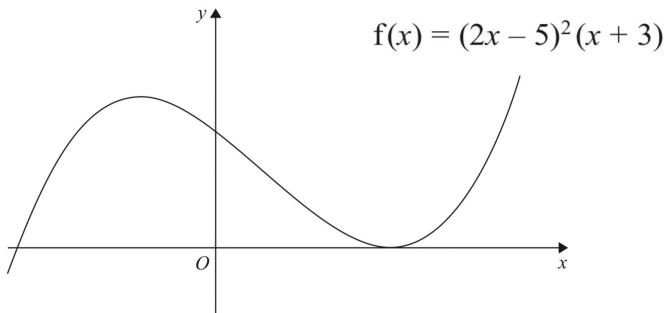


Showing the coordinates of the turning point and of any points of intersection with the axes, sketch on separate diagrams the graphs of

$$y = 3f(x)$$



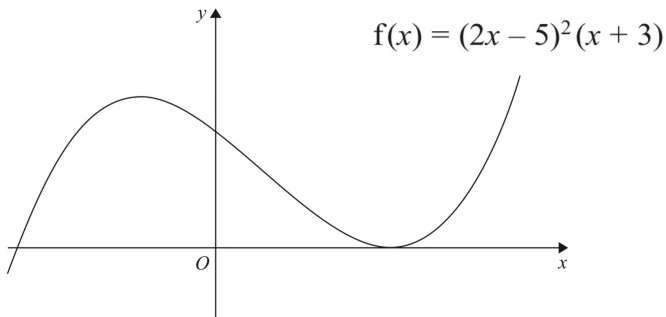
## Question



Given that

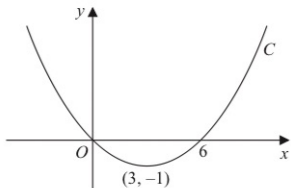
- (i) the curve with equation  $y = f(x) - k$ ,  $x \in \mathbb{R}$ , passes through the origin, find the value of the constant  $k$ ,

## Question



- (ii) the curve with equation  $y = f(x + c)$ ,  $x \in \mathbb{R}$ , has a minimum point at the origin, find the value of the constant  $c$ .

## Question



The curve  $C$  passes through the origin and through  $(6, 0)$

The curve  $C$  has a minimum at the point  $(3, -1)$

On separate diagrams, sketch the curve with equation

(a)  $y = f(2x)$

(b)  $y = f(x + p)$ , where  $p$  is a constant and  $0 < p < 3$

On each diagram show the coordinates of any points where the curve intersects the  $x$ -axis and of any minimum or maximum points.