MATHS ONLINE

By: Kru Tar

TOPMaths
A* Level

Trigonometric ratios

P1

Objectives

1) Sine rule

2) Cosine rule

3) Bearing

4) Area of triangle

5) Graph of sin(x), cos(x), tan(x)

6) Transformation of trigonometric functions

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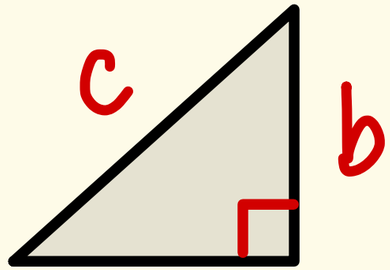
Recap

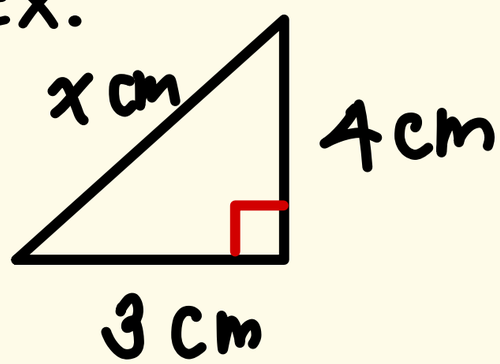
For right-angled triangles

1) Pythagoras' theorem (Sides)

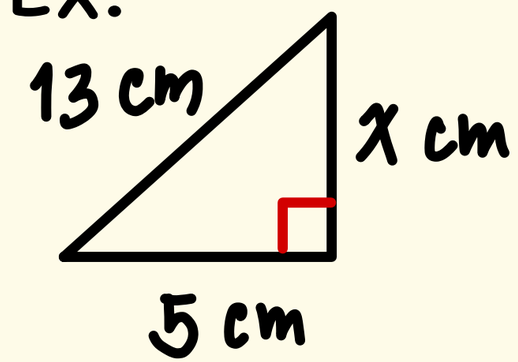


$$c^2 = a^2 + b^2$$

Ex.



Ex.



Recap

For right-angled triangles

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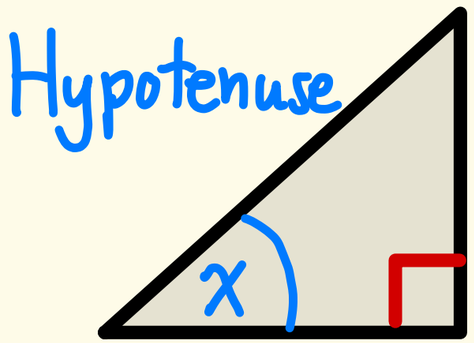
2) Trigonometry (Sides + Angles)



Sides

Angles

Sine

Cosine

Tangent

SOH CAH TOA

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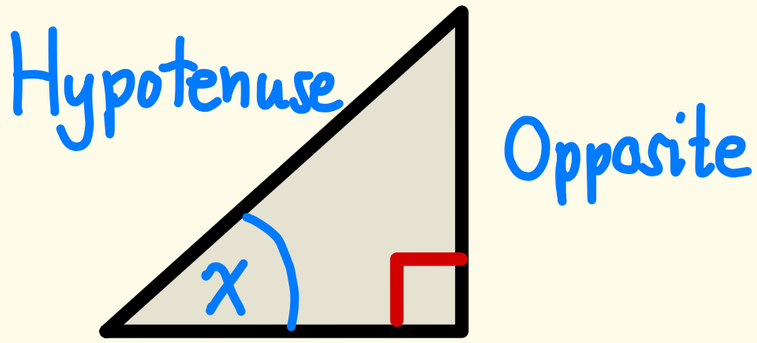
Recap

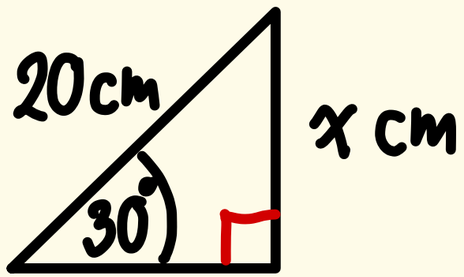
For right-angled triangles

2) Trigonometry

SOH CAH TOA



Ex.



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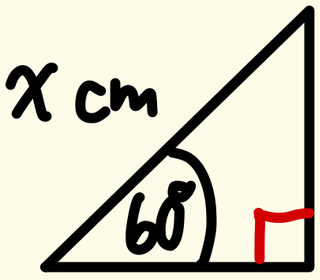
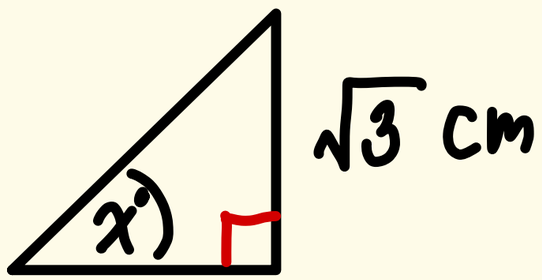


Ex.



For NOT right-angled triangles

1) Sine rule (2 sides + 2 angles)

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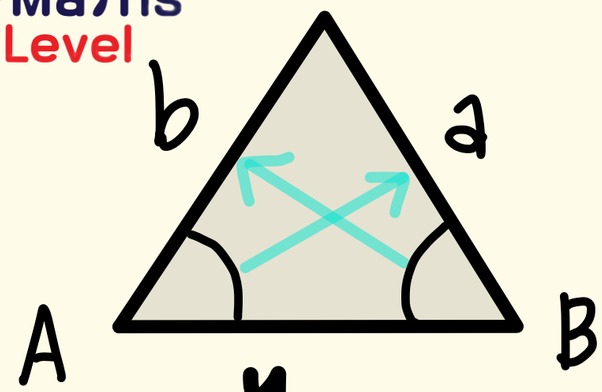


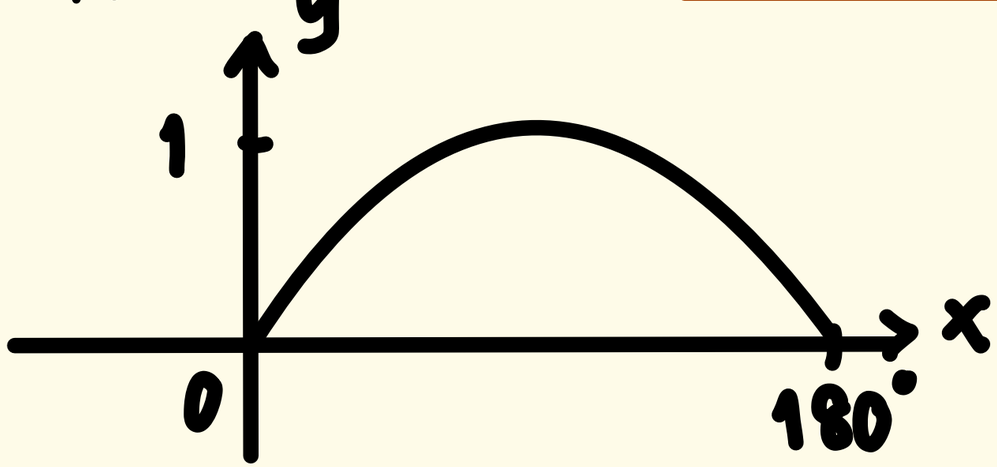
Sides

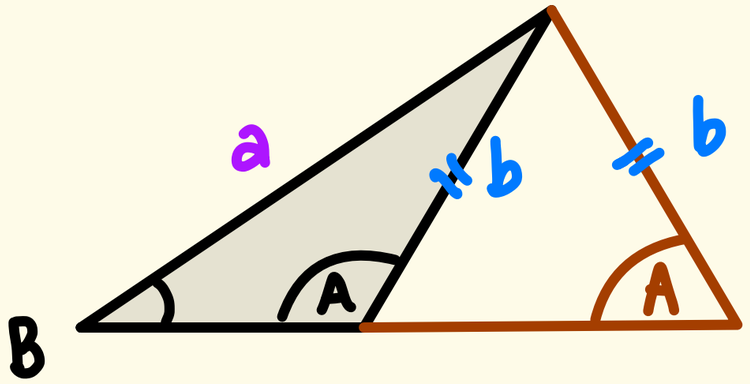
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

or

Angles

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$





Two possible angles

1) Acute angle =

2) Obtuse angle =

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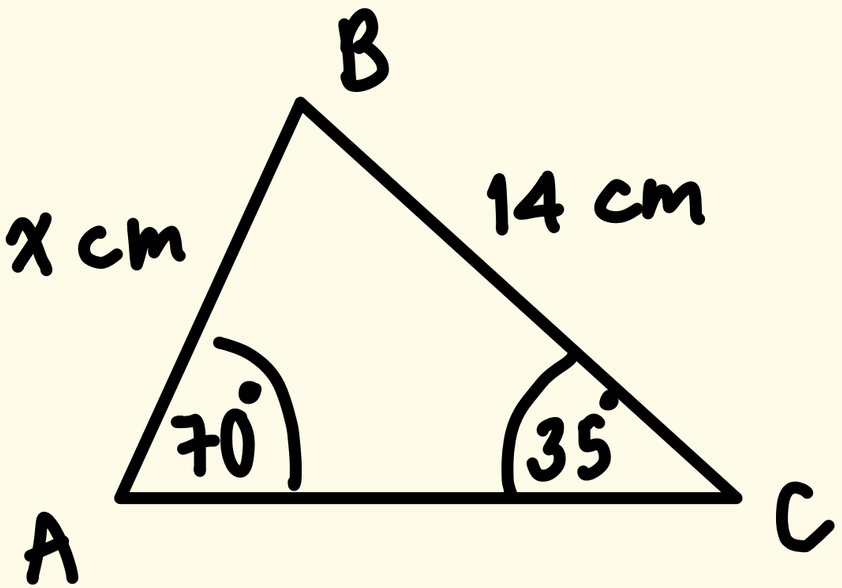
For NOT right-angled triangles

1) Sine rule

Ex. Find a side AB.



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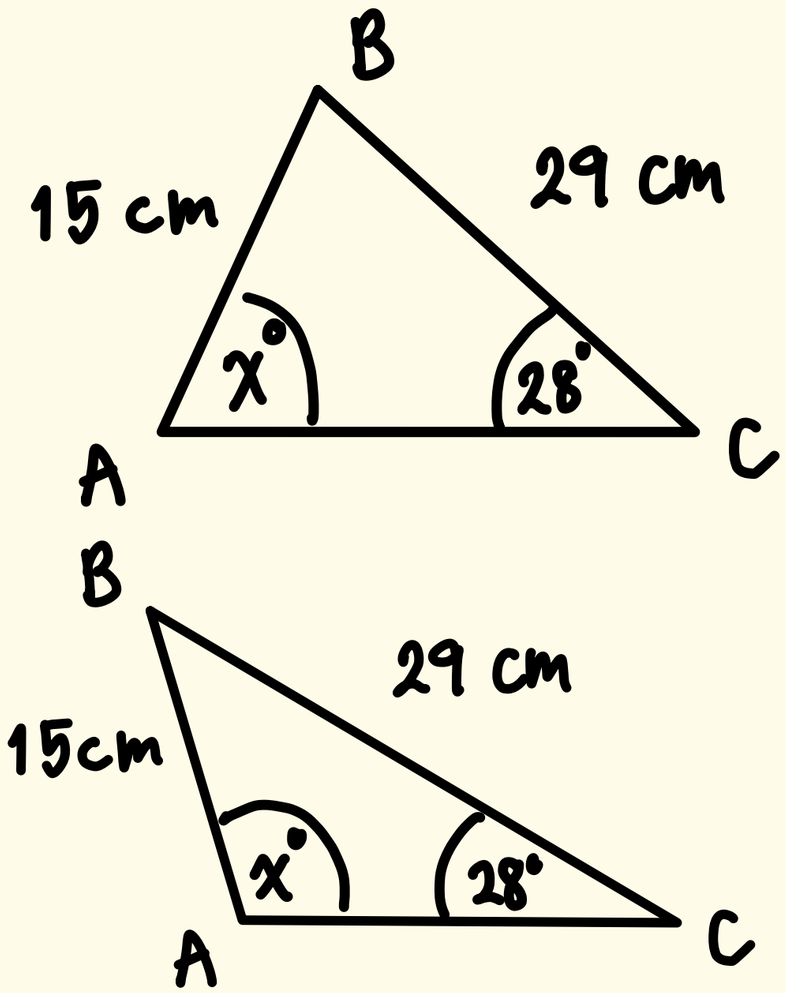
For NOT right-angled triangles

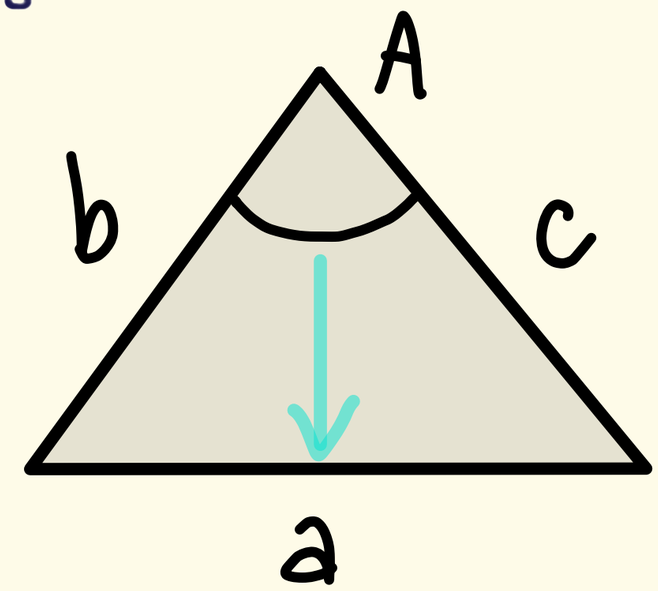
1) Sine rule

Ex. Find two possible angles BAC.



For NOT right-angled triangles

2) Cosine rule (1 angle + 3 sides)



Side

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

Angle

$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

Only one possible angle.

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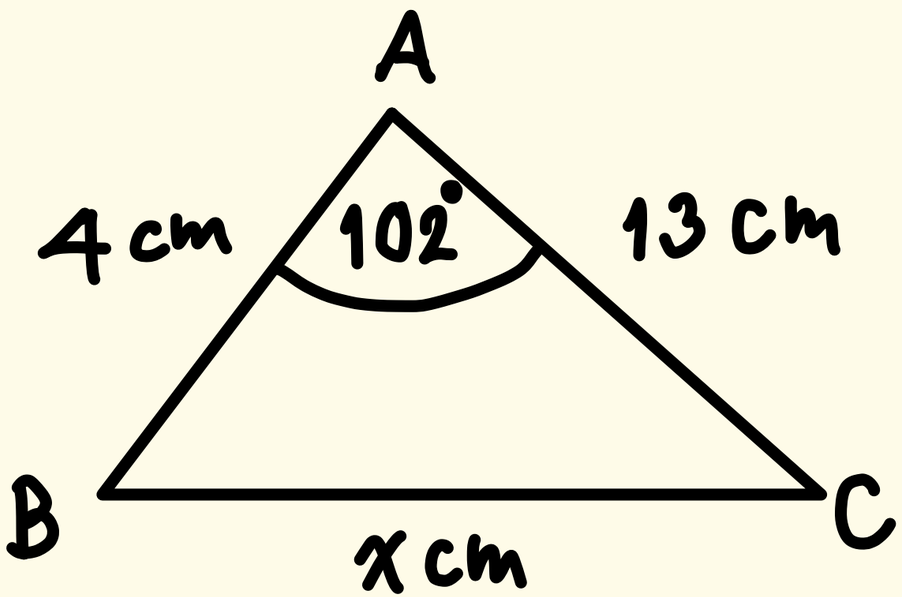
For NOT right-angled triangles

2) Cosine rule

Ex. Find a side BC.



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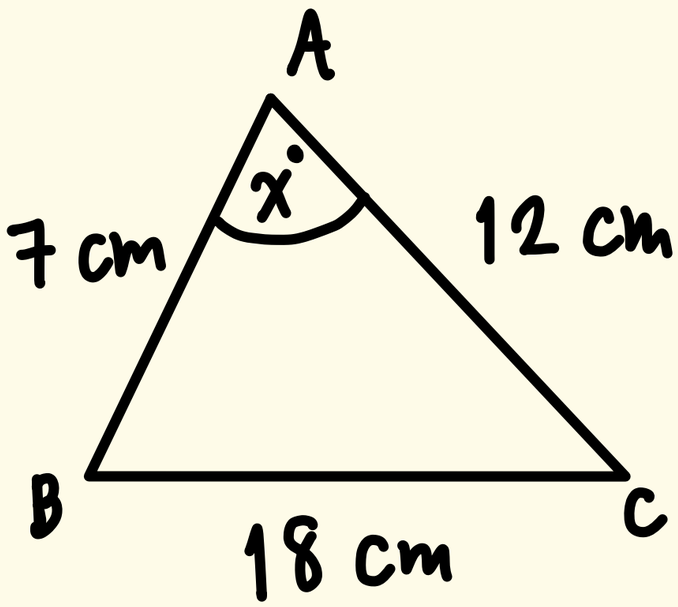
For NOT right-angled triangles

2) Cosine rule

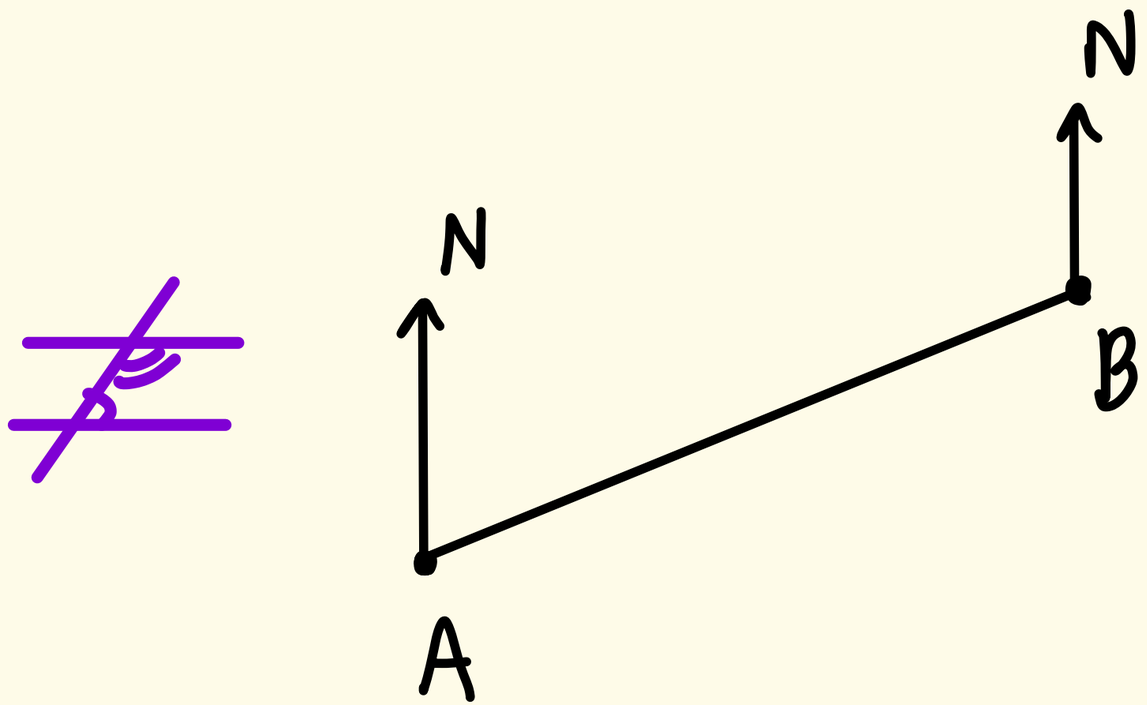
Ex. Find an angle BAC.



Bearing

Clockwise

Bearing of B from A is 030°.



Bearing of A from B is

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Ex. The school B is 10 km, on a bearing of 070°, from the bus station A. John is 5.2 km, on a bearing of 020°, away from A. How far John is from B.

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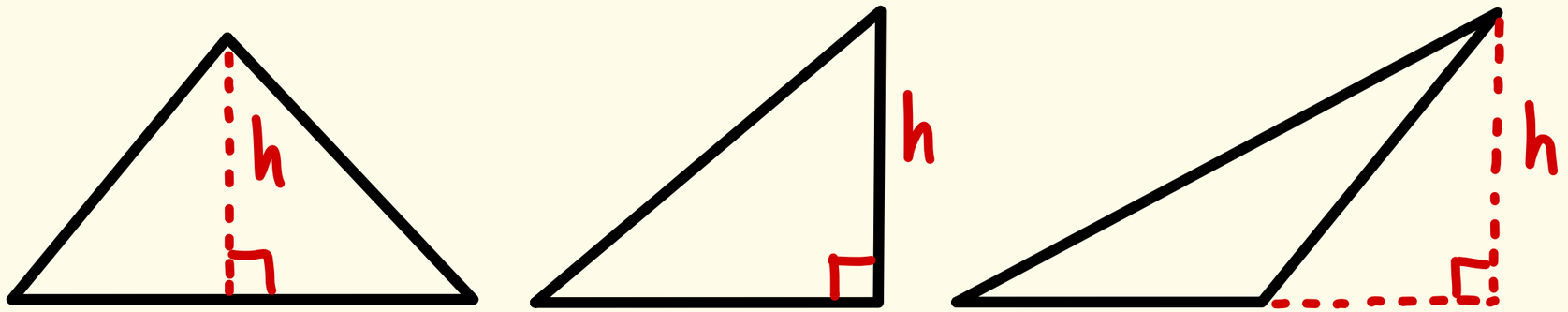
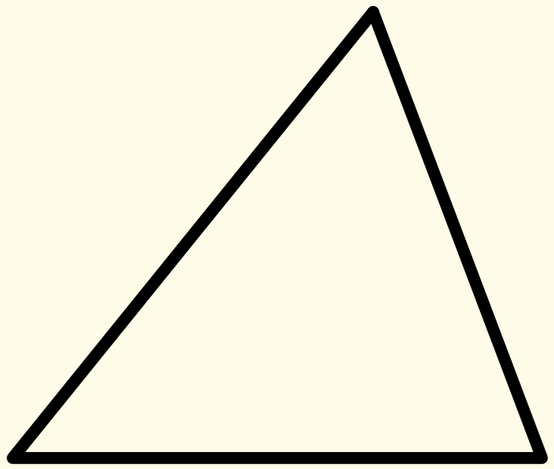
Area of triangles



$$\text{Area} = \frac{1}{2} \times b \times h$$

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin(C)$$



Two possible angles

1) Acute angle =

2) Obtuse angle =

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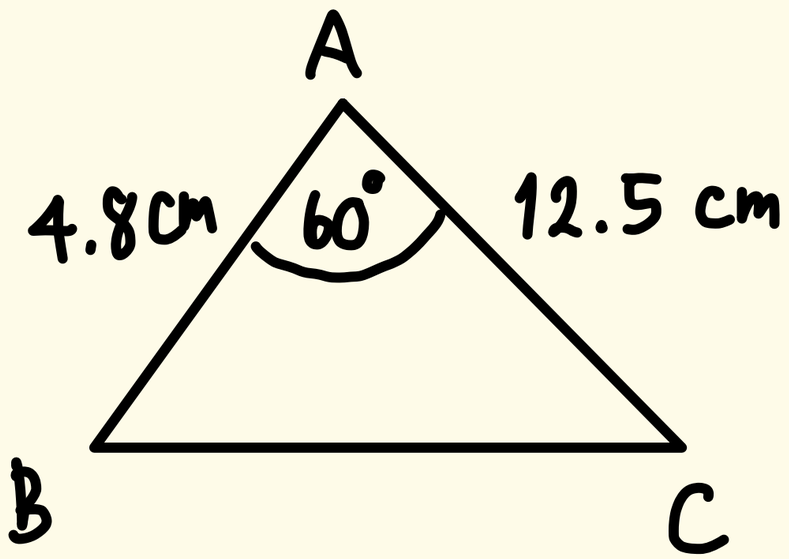
Ex. Find the exact area of triangle ABC.



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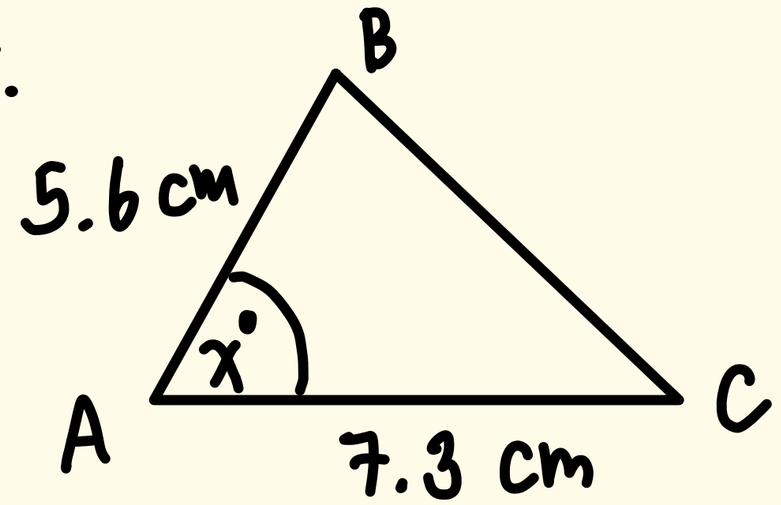
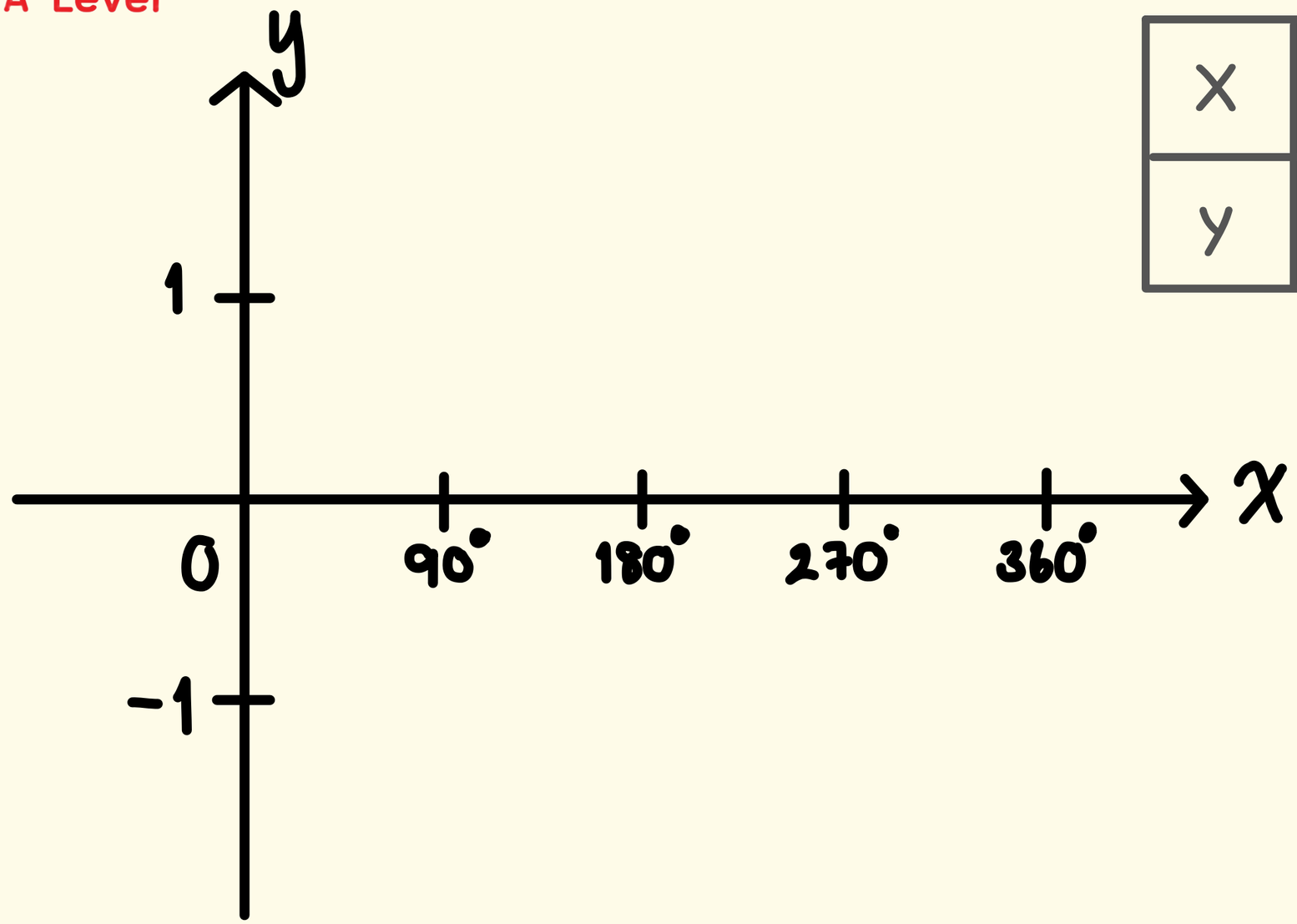
Ex. Given that the area of triangle ABC is 20 cm^2.
Find two possible angles BAC.



Graph of Sine, Cosine and Tangent

$$y = \sin(x)$$

X	0°	90°	180°	270°	360°
y					



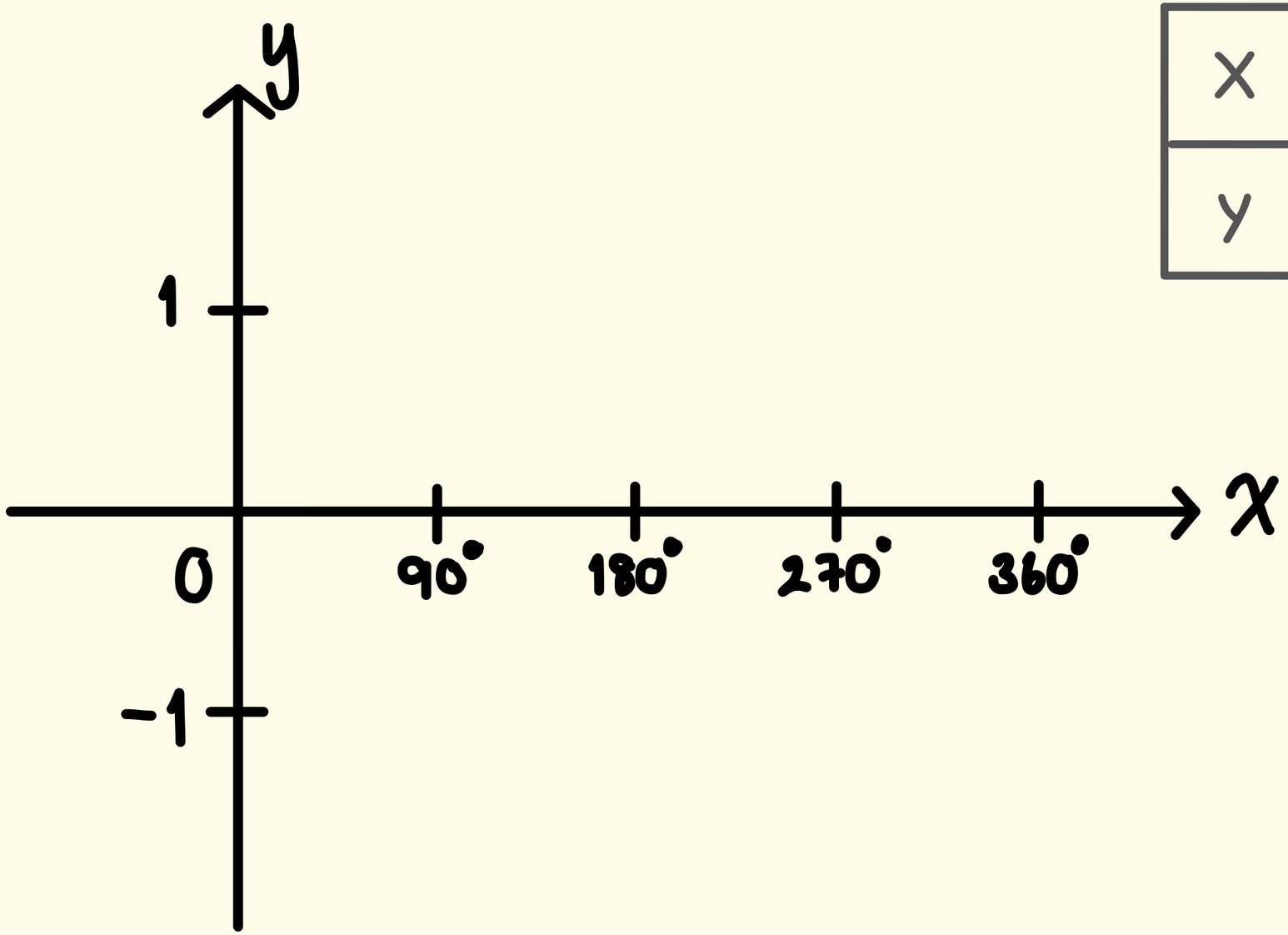
Max =

Min =

Graph of Sine, Cosine and Tangent

$$y = \cos(x)$$

X	0°	90°	180°	270°	360°
y					



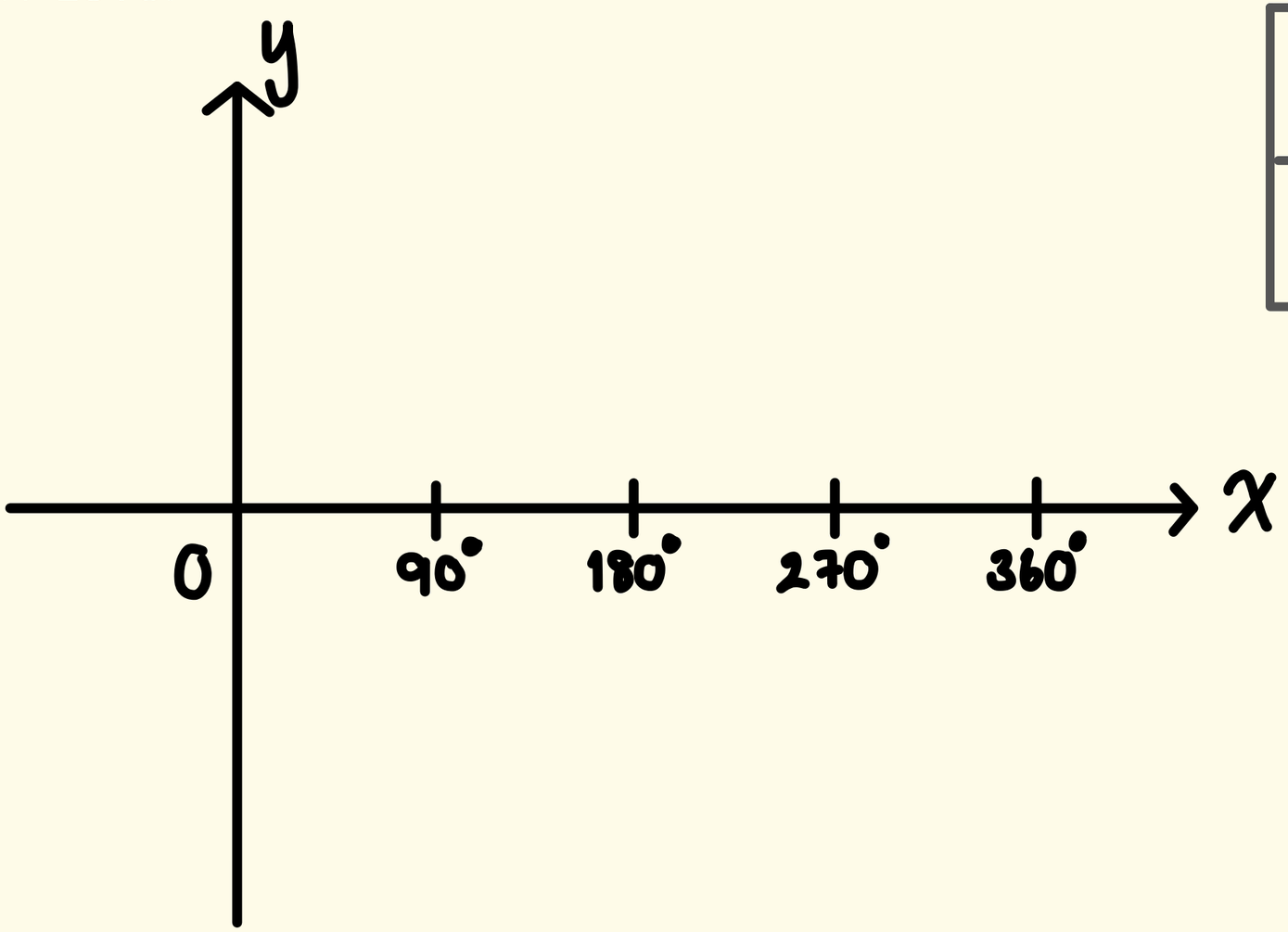
Max =

Min =

Graph of Sine, Cosine and Tangent

$$y = \tan(x)$$

X	0°	90°	180°	270°	360°
y					



No Max/Min

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Recap

Transformations of y=f(x)

1) $y=f(x+a)$ Translation by $\begin{pmatrix}-a\\0\end{pmatrix}$.

2) $y=f(x)+a$ Translation by $\begin{pmatrix}0\\a\end{pmatrix}$.

3) $y=f(-x)$ Reflection in the y-axis.

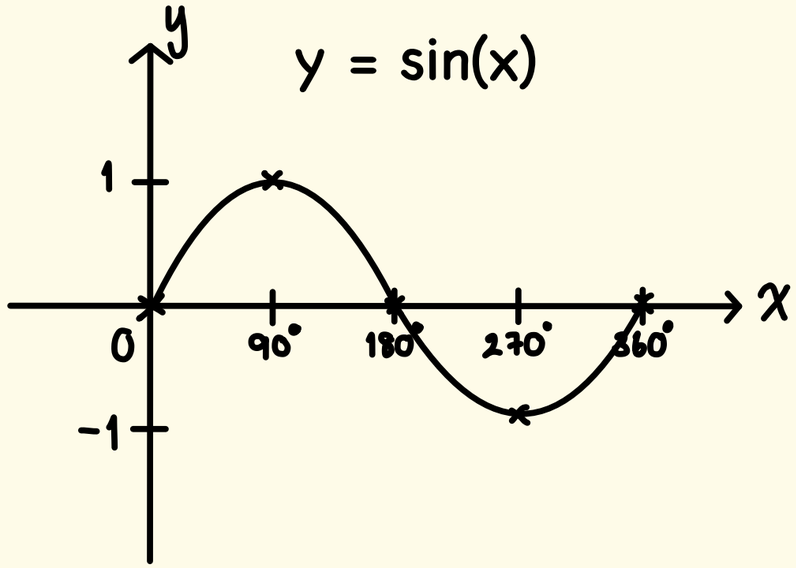
4) $y=-f(x)$ Reflection in the x-axis.

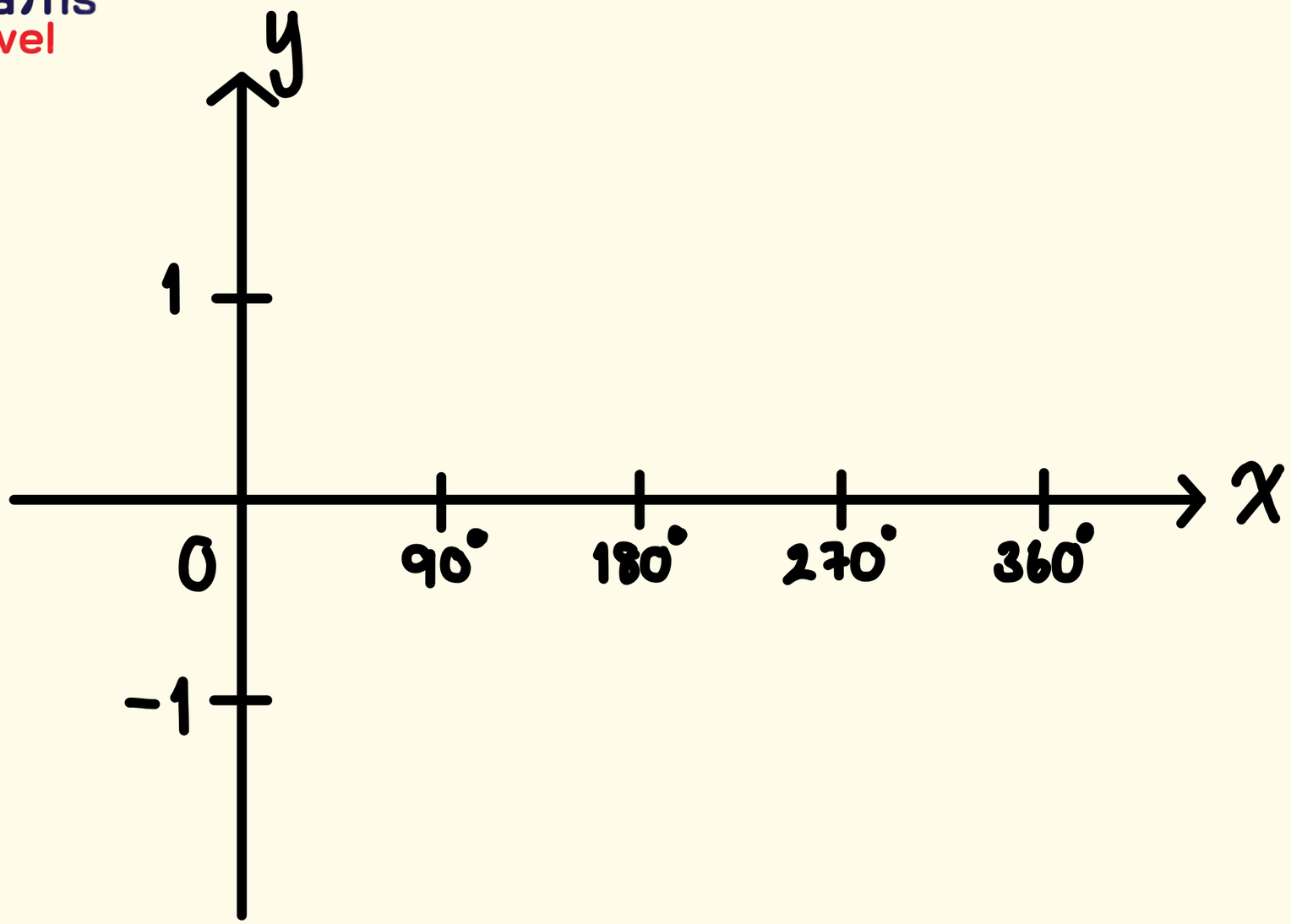
5) $y=f(ax)$ Stretch sf. $\frac{1}{a}$ in the horizontal direction.

6) $y=af(x)$ Stretch sf. a in the vertical direction.

Transformation graphs of y = sin(x)

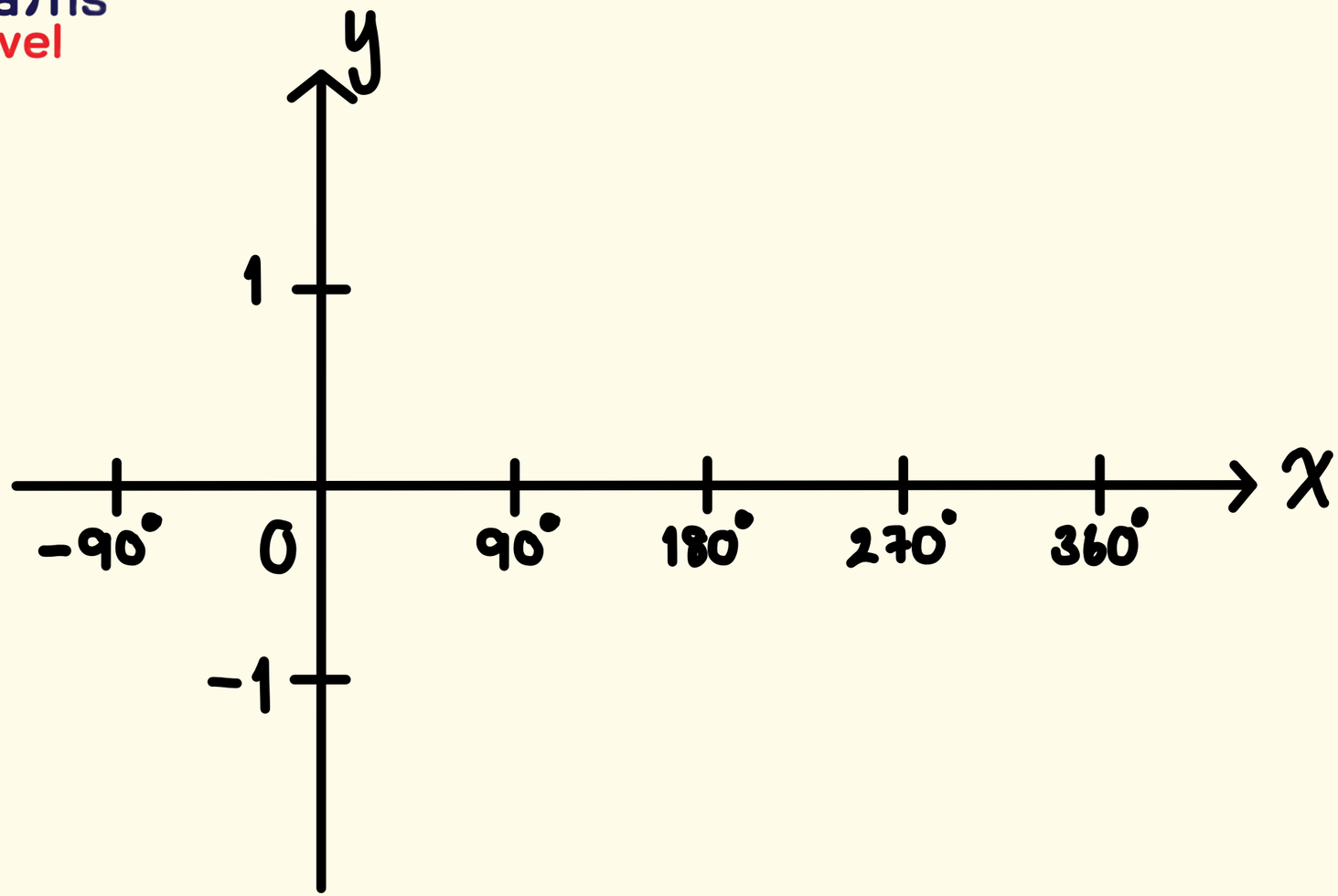
1) $y = \sin(x - 30^{\circ})$

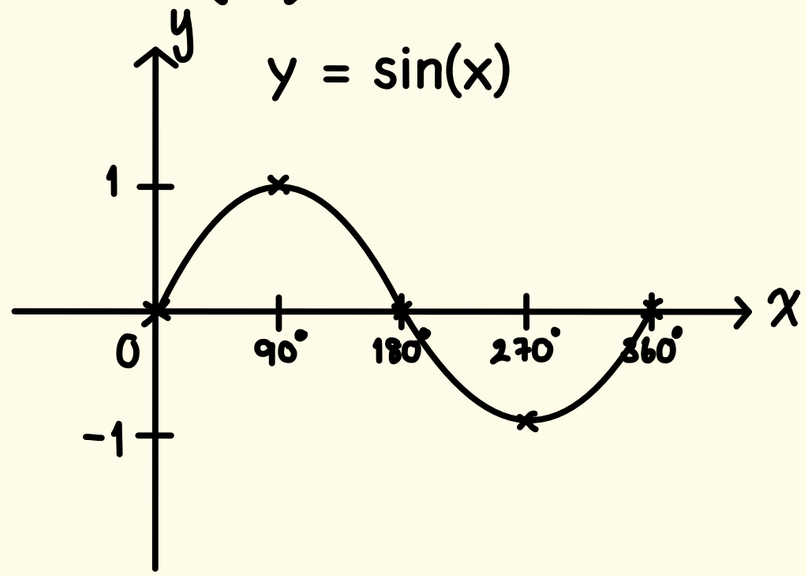




Transformation graphs of y = sin(x)

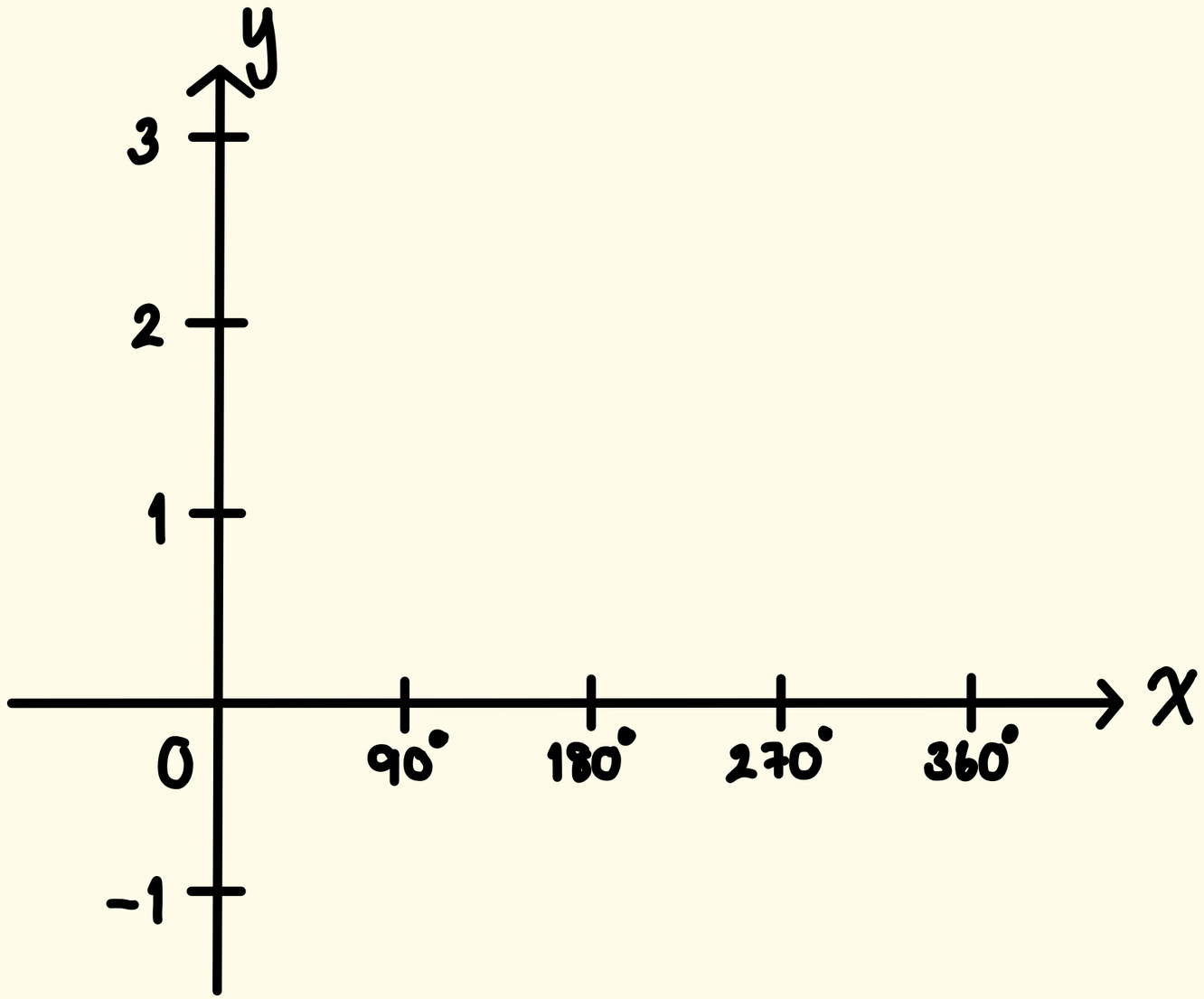
2) $y = \sin(x+30^\circ)$

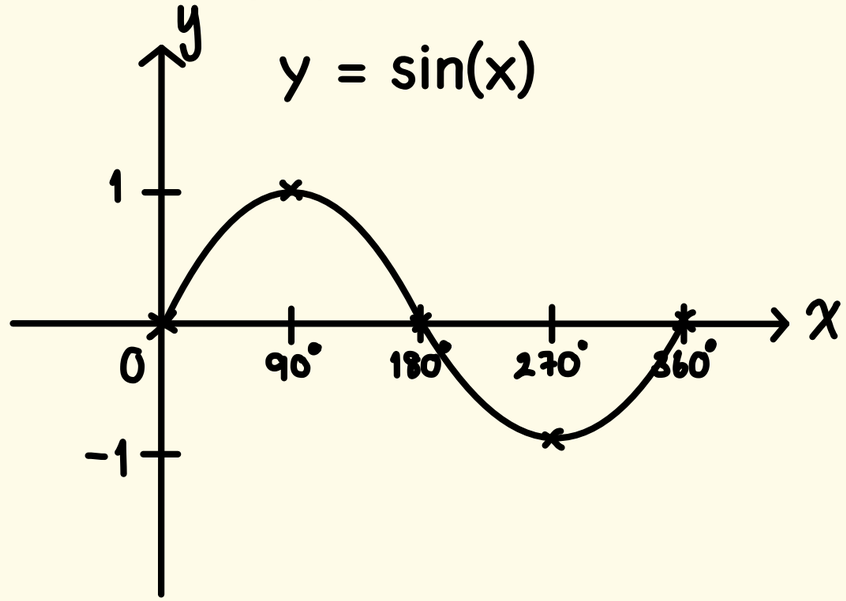




Transformation graphs of y = sin(x)

3) $y = \sin(x) + 2$





Transformation graphs of y = sin(x)

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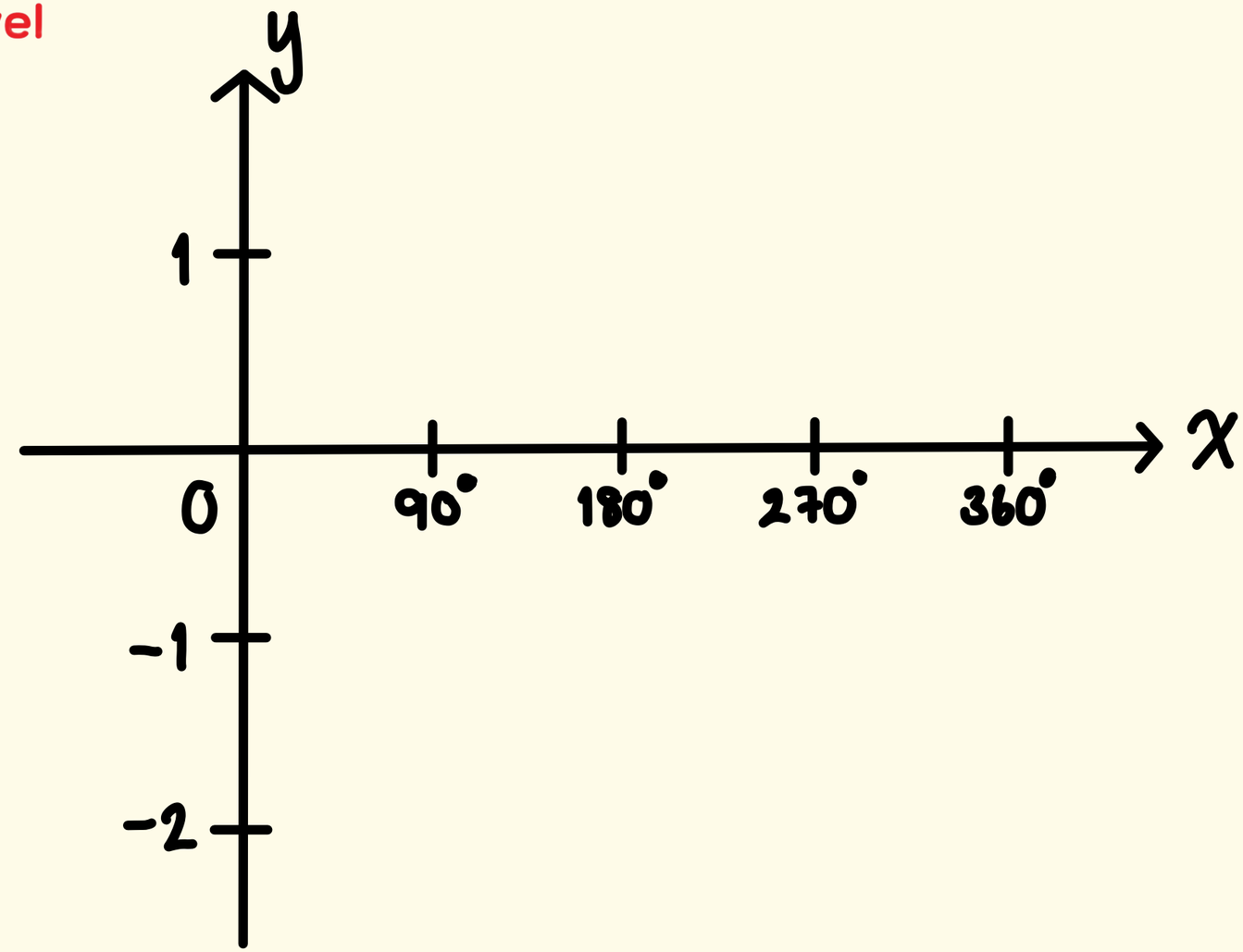
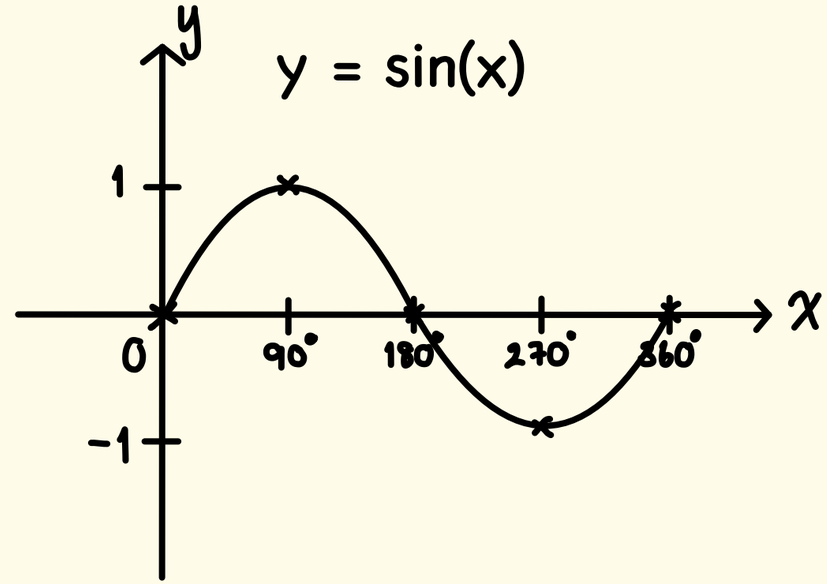
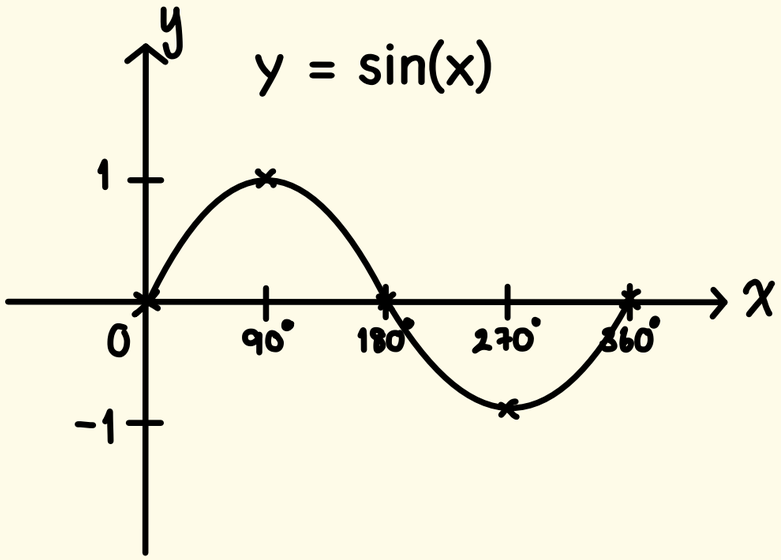
4) $y = \sin(x) - 1$

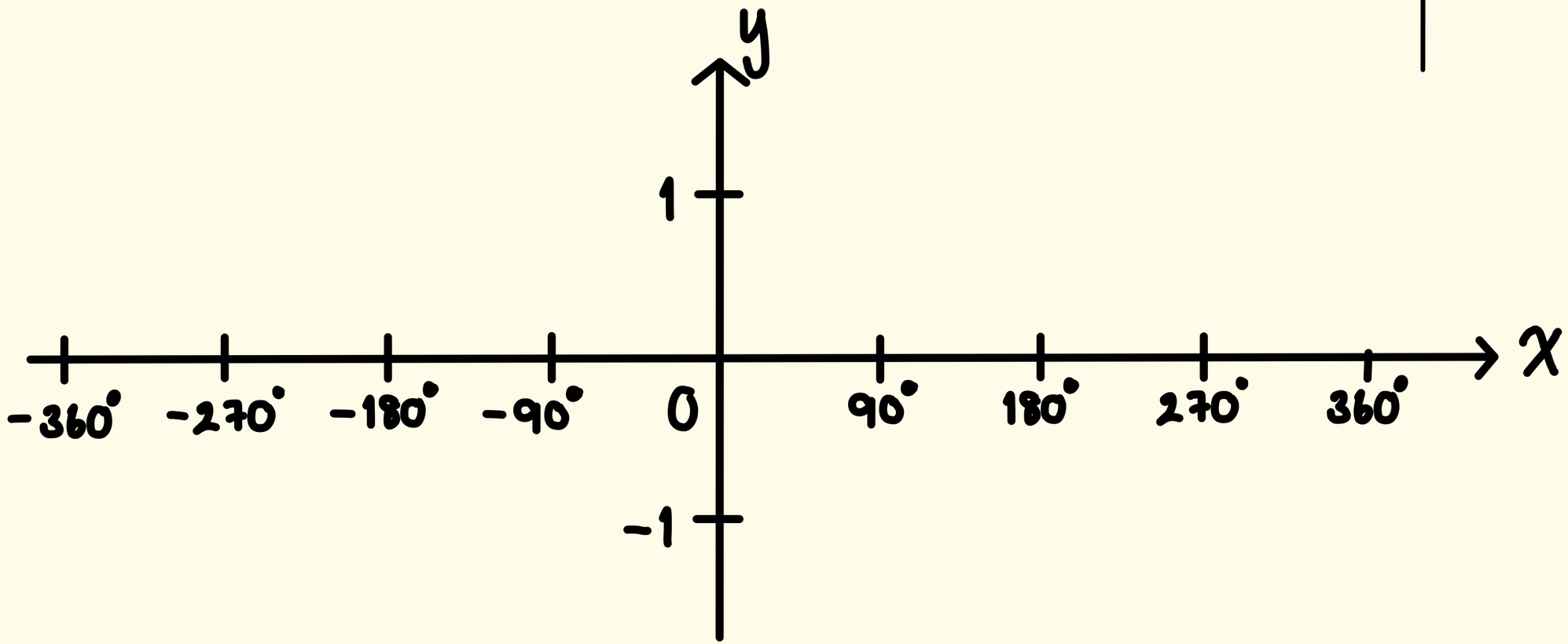




Transformation graphs of y = sin(x)

5) $y = \sin(-x)$





Transformation graphs of y = sin(x)

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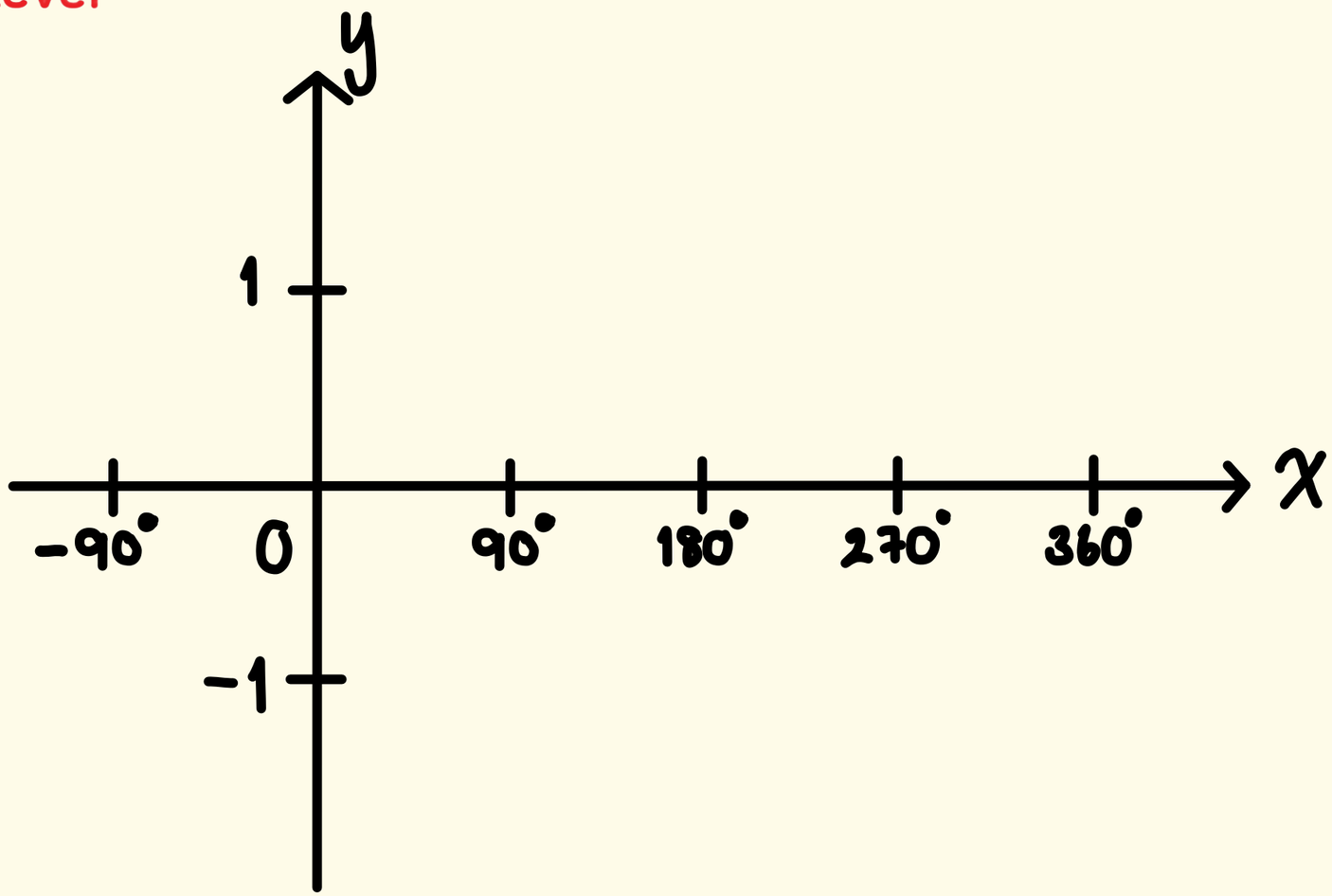
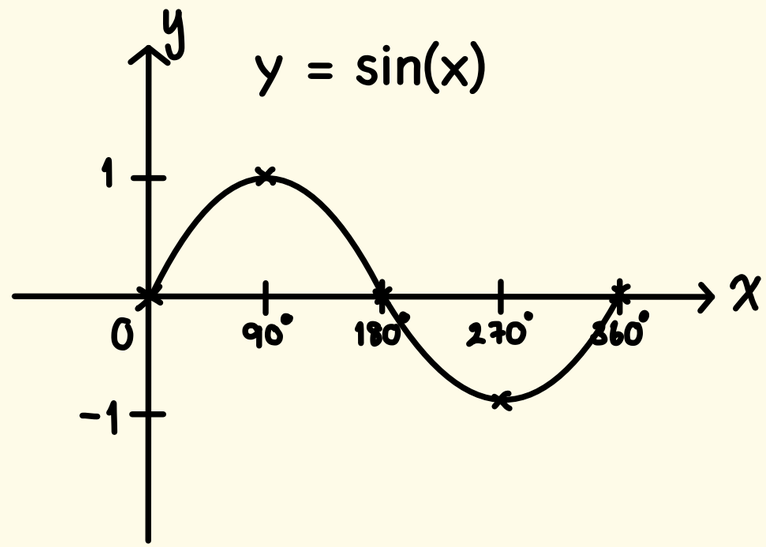
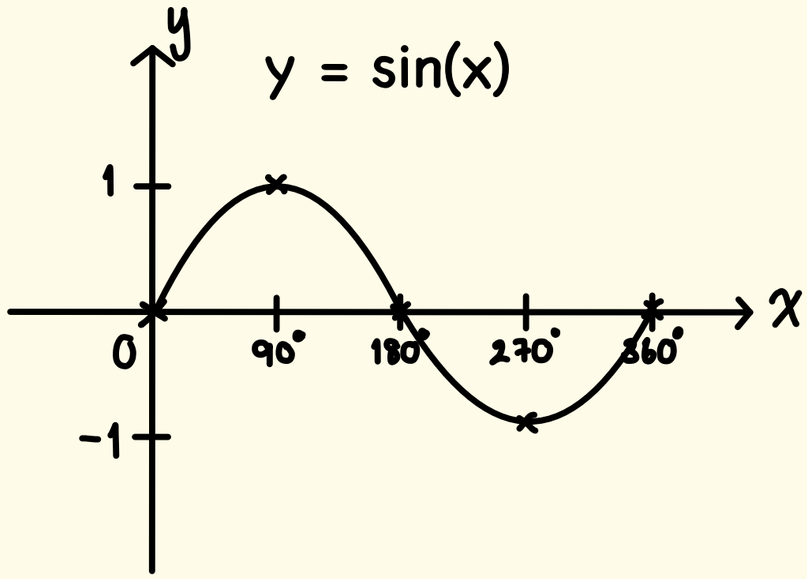
6) $y = -\sin(x)$

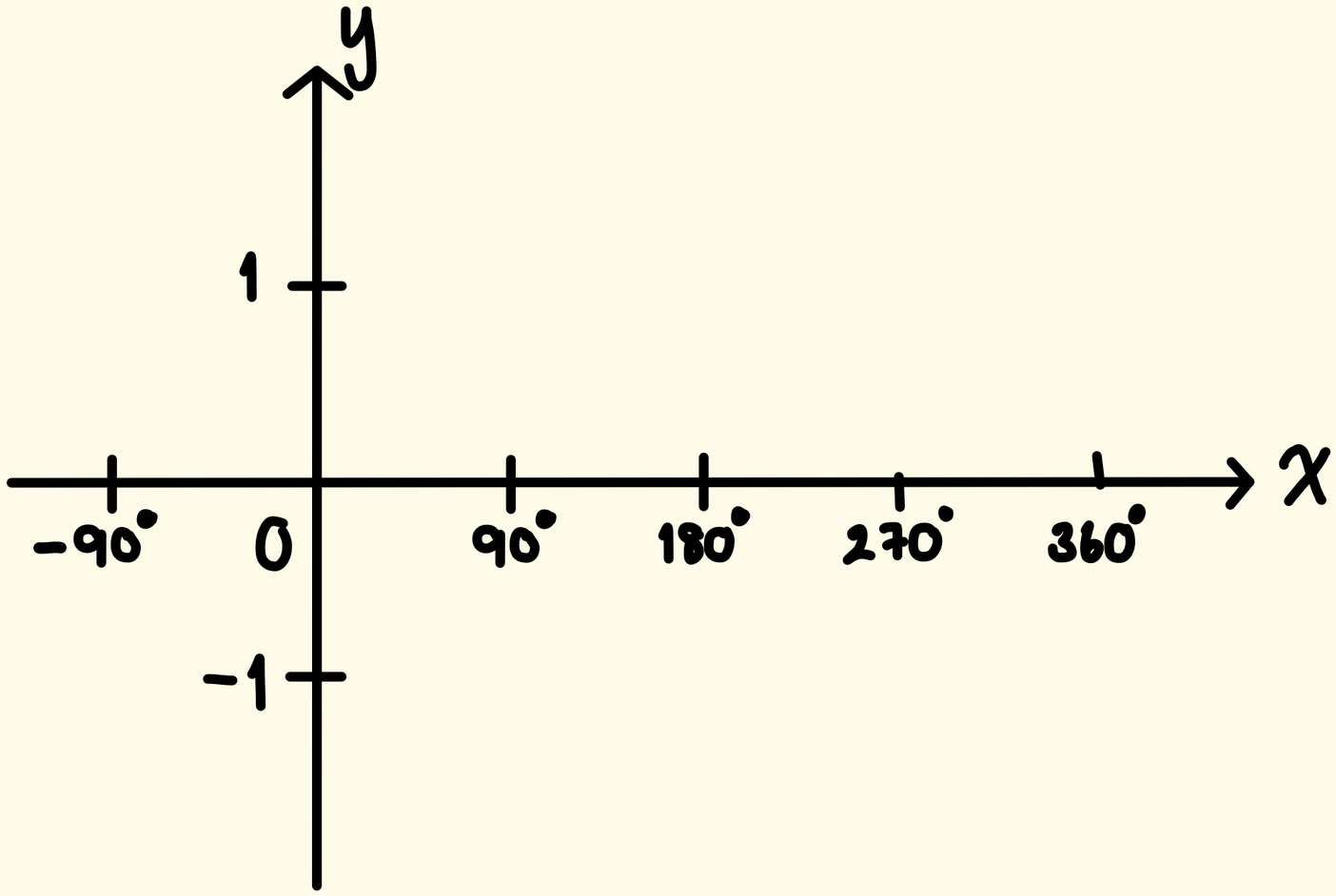




Transformation graphs of y = sin(x)

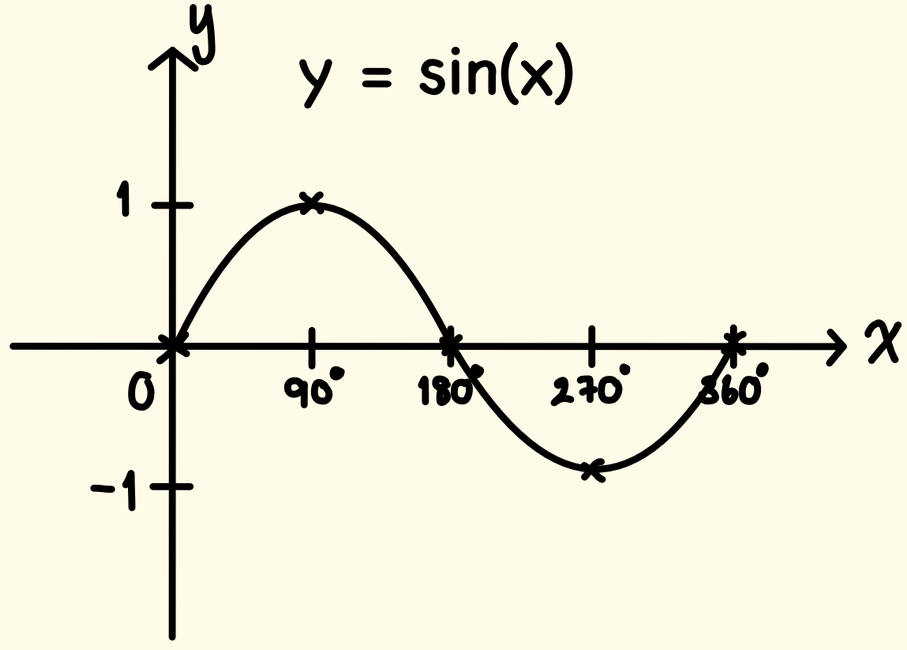
7) $y = \sin(2x)$

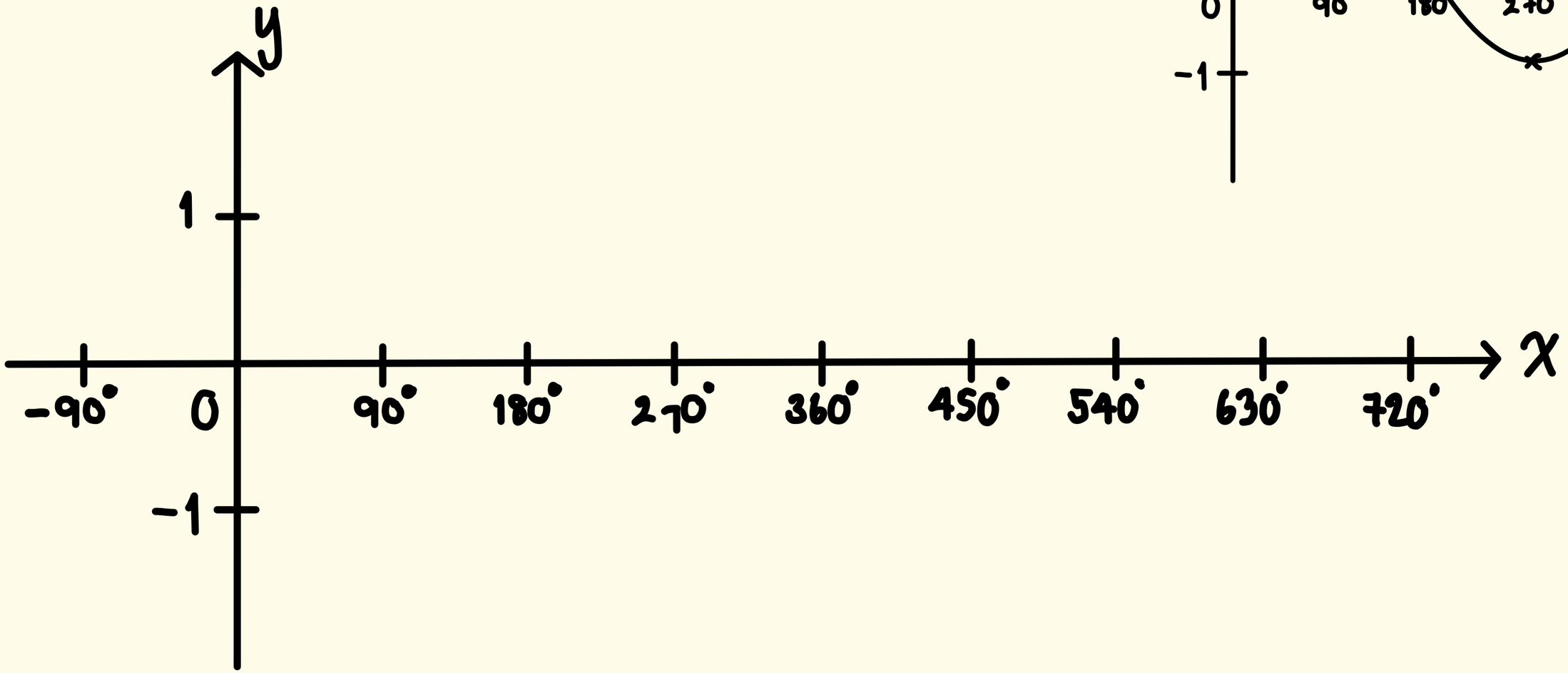




Transformation graphs of y = sin(x)

8) $y = \sin(0.5x)$





Transformation graphs of y = sin(x)

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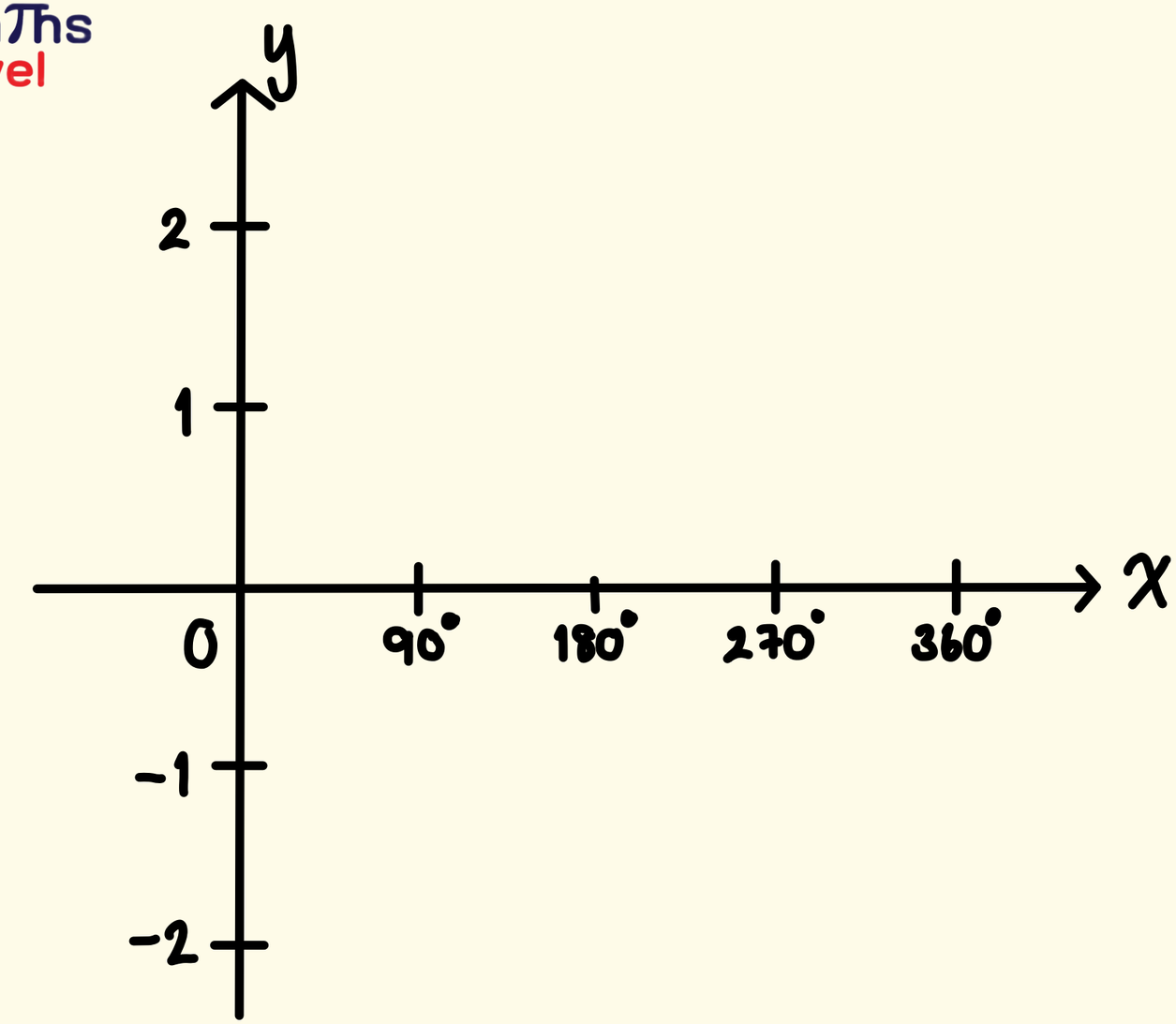
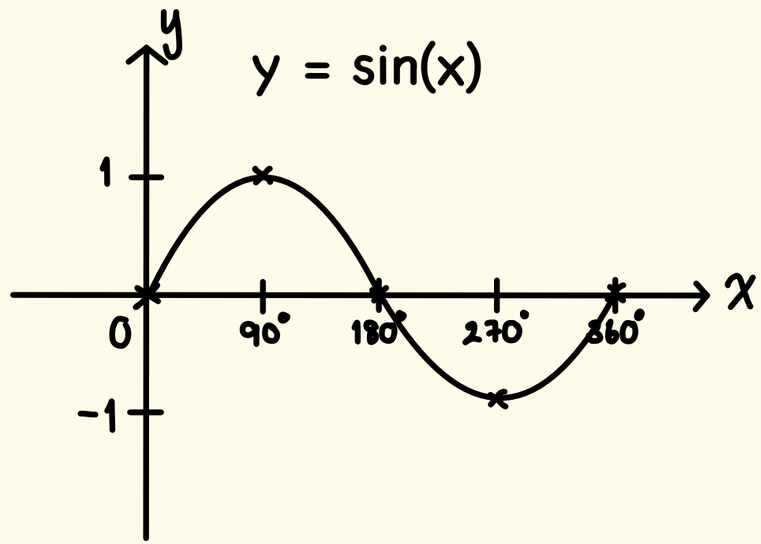
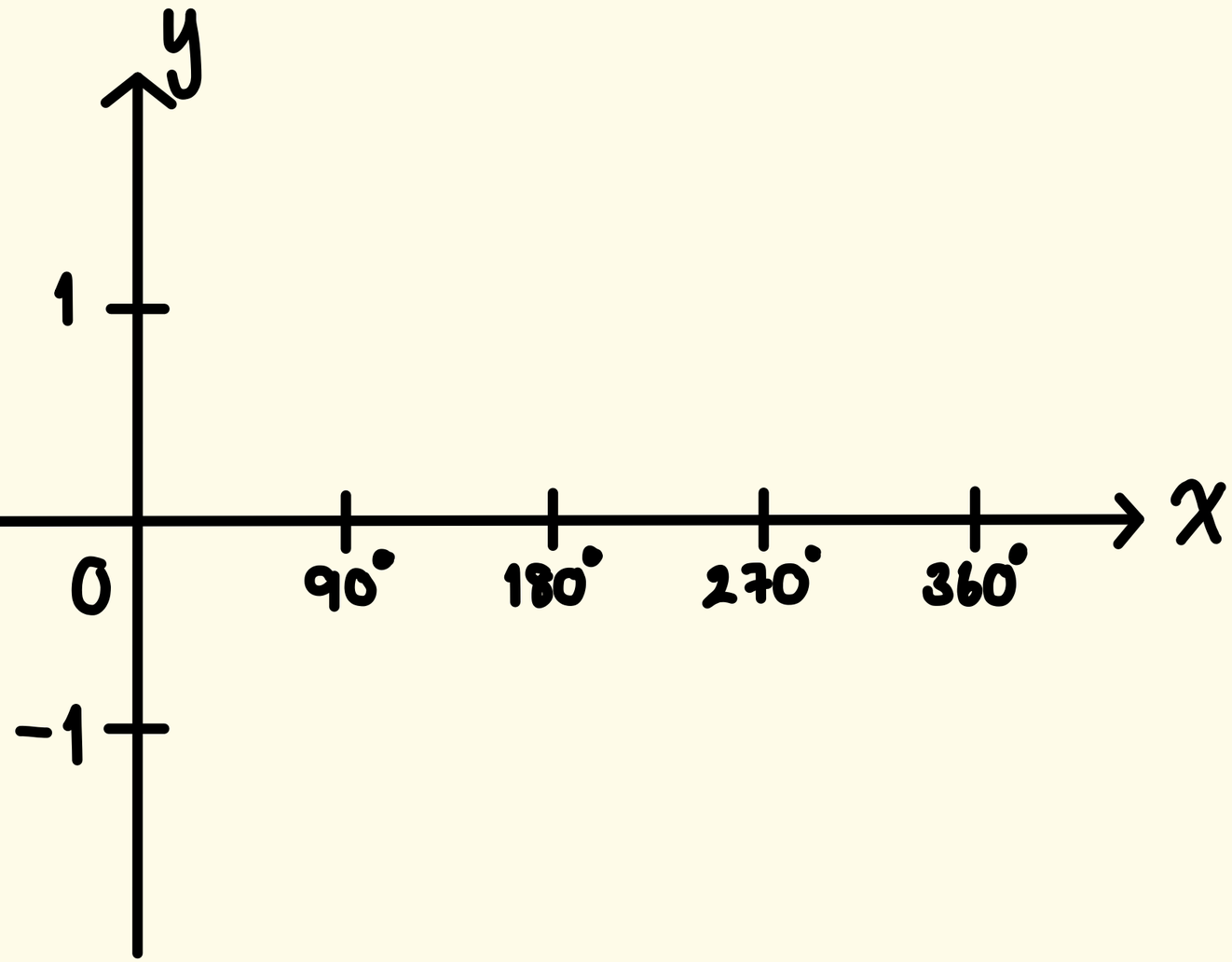
9) $y = 2\sin(x)$

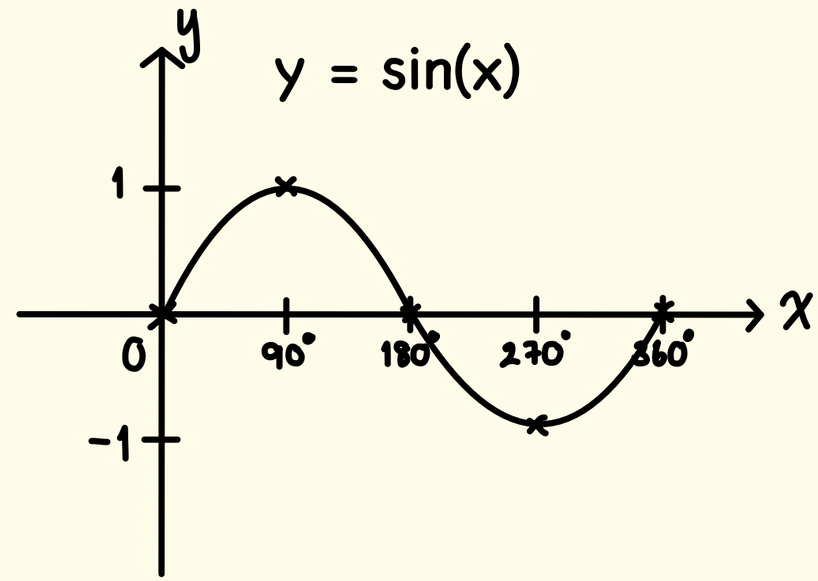




Transformation graphs of y = sin(x)

10) $y = 0.5\sin(x)$





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$$y = -a\sin(-bx+c)+d$$

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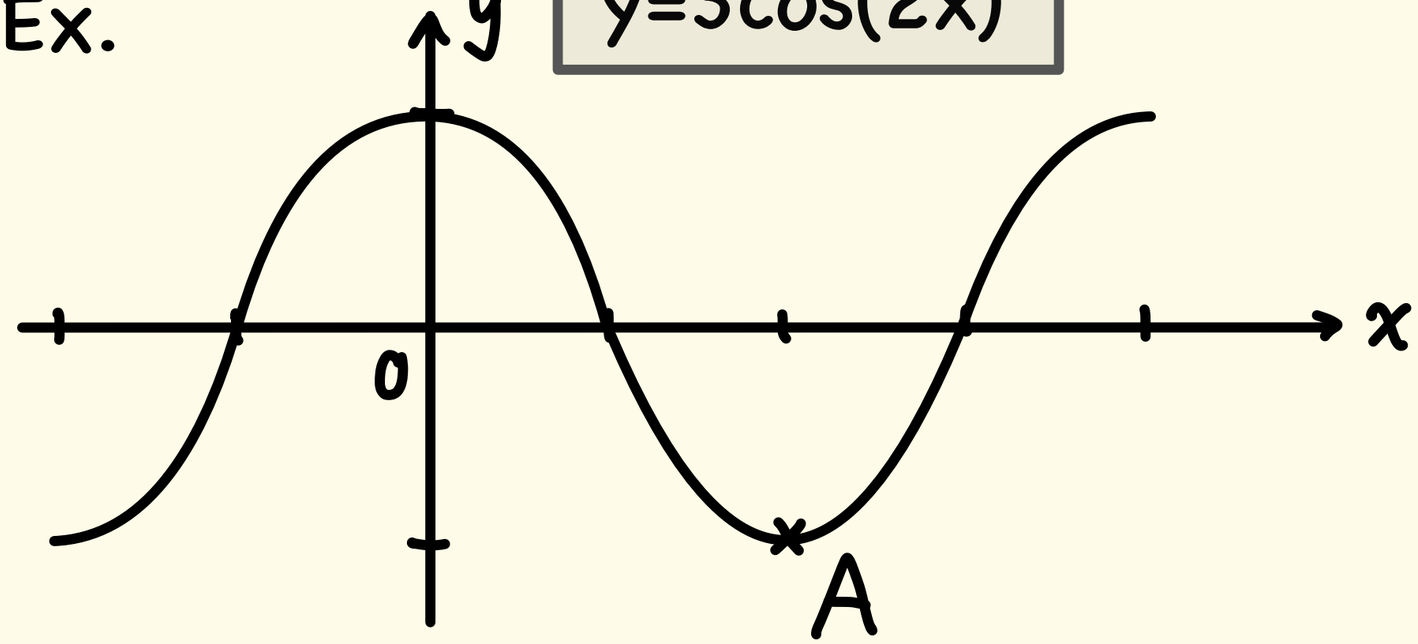
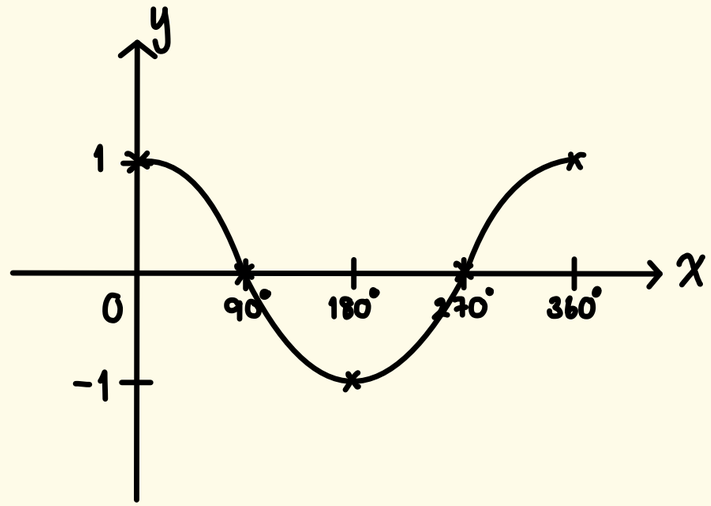
Ex.

$y=3\cos(2x)^{\circ}$





a) Find the coordinate of A.

b) Write down the period.

c) State the number of solutions to the equation $3\cos(2x) = 1$

i) for $-90^{\circ}<x<180^{\circ}$

ii) for $-90^{\circ}<x<1800^{\circ}$

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Conclusion

1) Pythagoras' theorem $c^2 = a^2 + b^2$

2) Trigonometry SOH CAH TOA

3) Sine rule $\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$ or $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$

Two possible angles
1) Acute angle = $\sin^{-1}(\ldots)$ = Ans
2) Obtuse angle = 180° − Ans

4) Cosine rule $a^2 = b^2 + c^2 - 2bc\cos(A)$ and $A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$

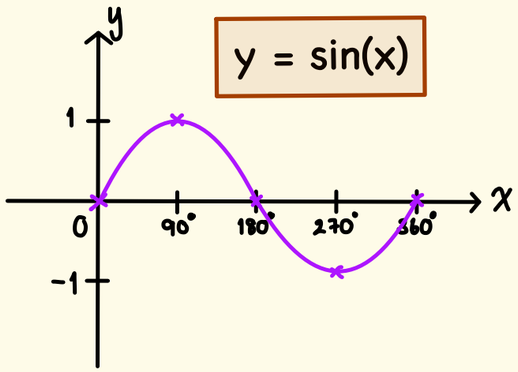
5) Area of triangles

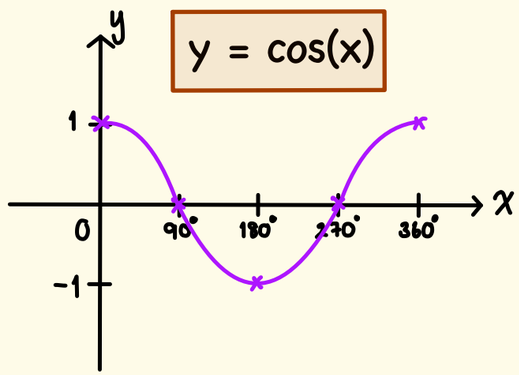
$\text{Area} = \frac{1}{2} \times b \times h$

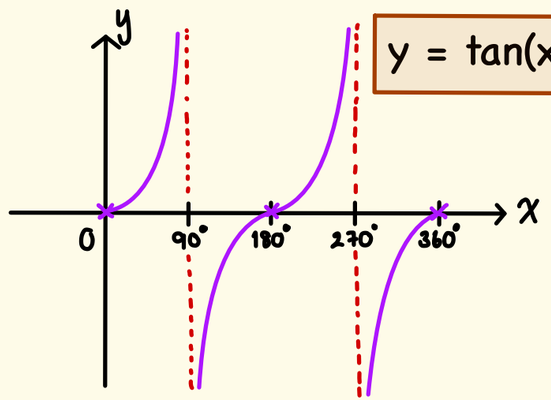
$\text{Area} = \frac{1}{2} \times a \times b \times \sin(C)$

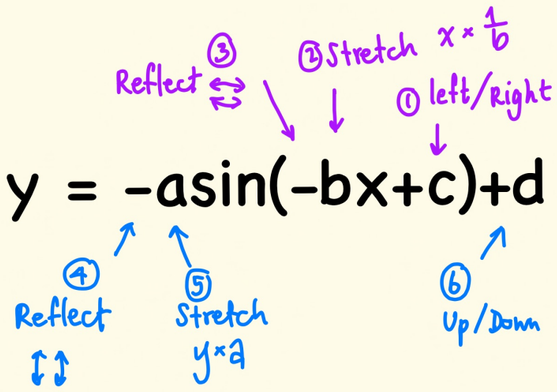
Two possible angles
1) Acute angle = $\sin^{-1}(\ldots)$ = Ans
2) Obtuse angle = 180° − Ans

6) Graph of Sine, Cosine, Tangent and Transformations









TOPMaths
A* Level

Ex.1 A triangle has vertices at A(1,2), B(5,3) and C(-1,-2).

a) Find the value of cosine of angle BAC.

b) Calculate the area of triangle ABC.

TOPMaths
A* Level

Ex.2 John walks 800 m due North from a coffee shop,A to a school,B. He then walks 950 m on a bearing of 100° from B to his home,C.

a) Find how far he is from his home,C when he is at a coffee shop,A.

b) Calculate the bearing of his home,C from a coffee shop,A.

c) Find the area enclosed by his walk.

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A* Level

Ex.3 A triangle ABC, with AB=4 cm, BC=(2x+1) cm, CA=(3x-1) cm and angle ABC= 60°.

a) Find the value of x.

b) Find the area of the triangle ABC.

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TOPMaths
A* Level

Ex.4 A triangle ABC with AB=21 cm, BC=13 cm and angle BAC=25°.

a) Find the value of sine of angle ACB, giving your answer to 4 decimal places.

b) Given that AB is the longest side, find the size of angle ACB, giving your answer to 2 decimal places.

TOPMaths
A* Level

Ex.5 ABCD is a parallelogram which has area 80 cm^2.

BC=20 cm, CD=12 cm and angle BCD is obtuse.

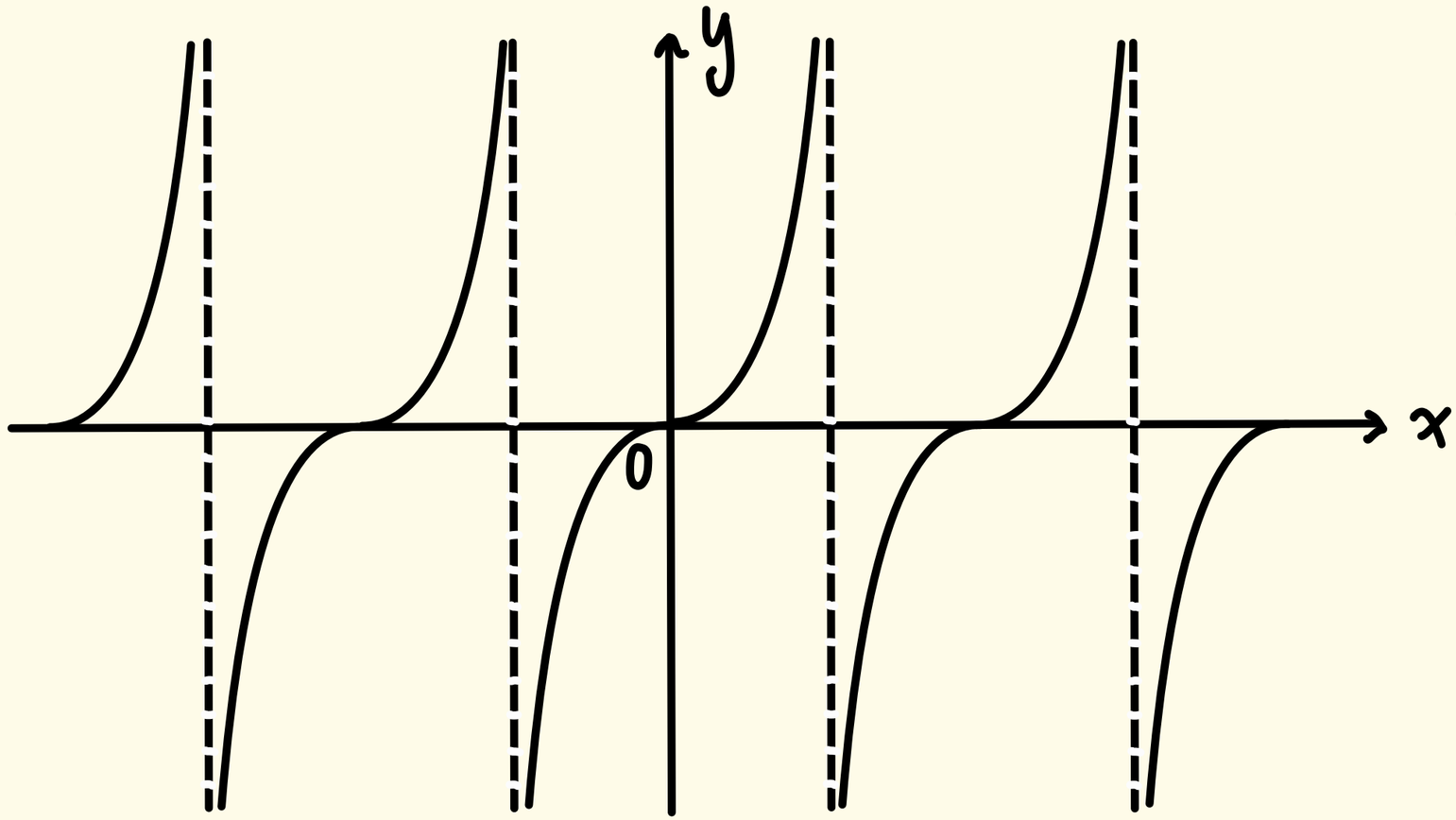
a) Find the size of angle BCD, in degrees, to 1 decimal place.

b) Find the length of diagonal BD, to 2 decimal places.

TOPMaths
A* Level

Ex.6 Given that tan(a) = k, where $0<a<90^{\circ}$

y=tan(x)



State, in term of k, the value of

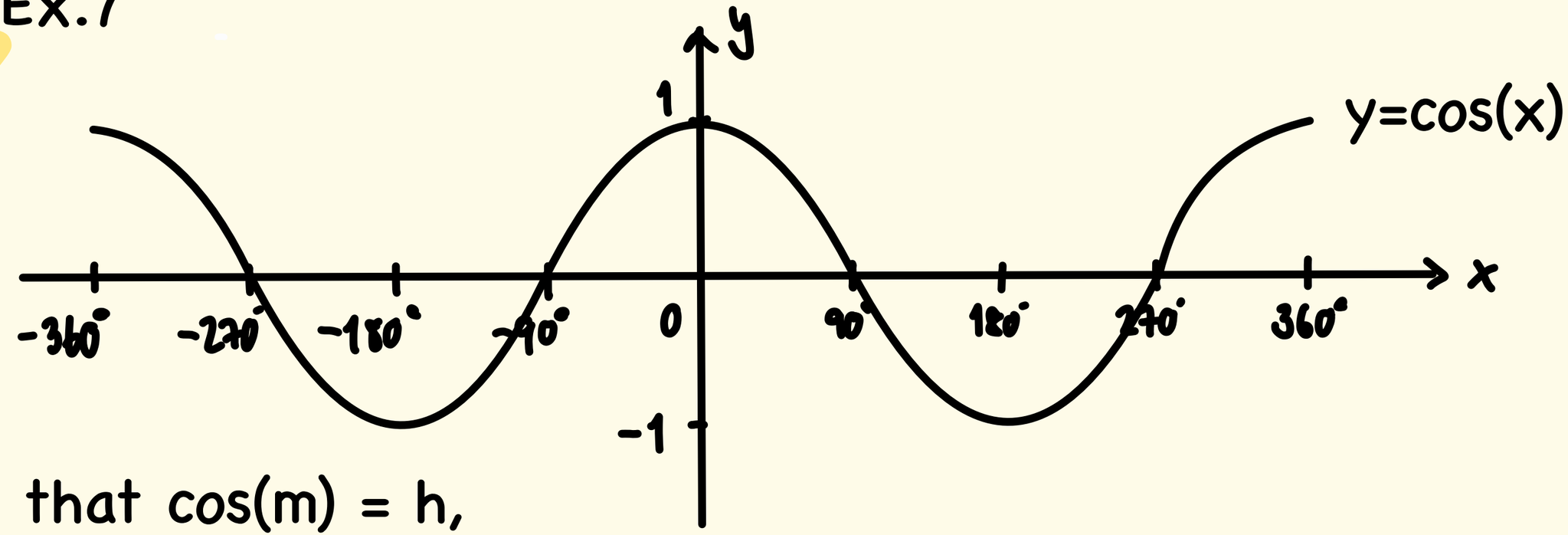
i) $3\tan(a+180^{\circ})$

ii) $\tan(a-180^{\circ}) + 2$

iii) $2\tan(-180^{\circ}-a)$

TOPMaTh
A* Level

Ex.7



Given that cos(m) = h,
where $0<m<90^{\circ}$

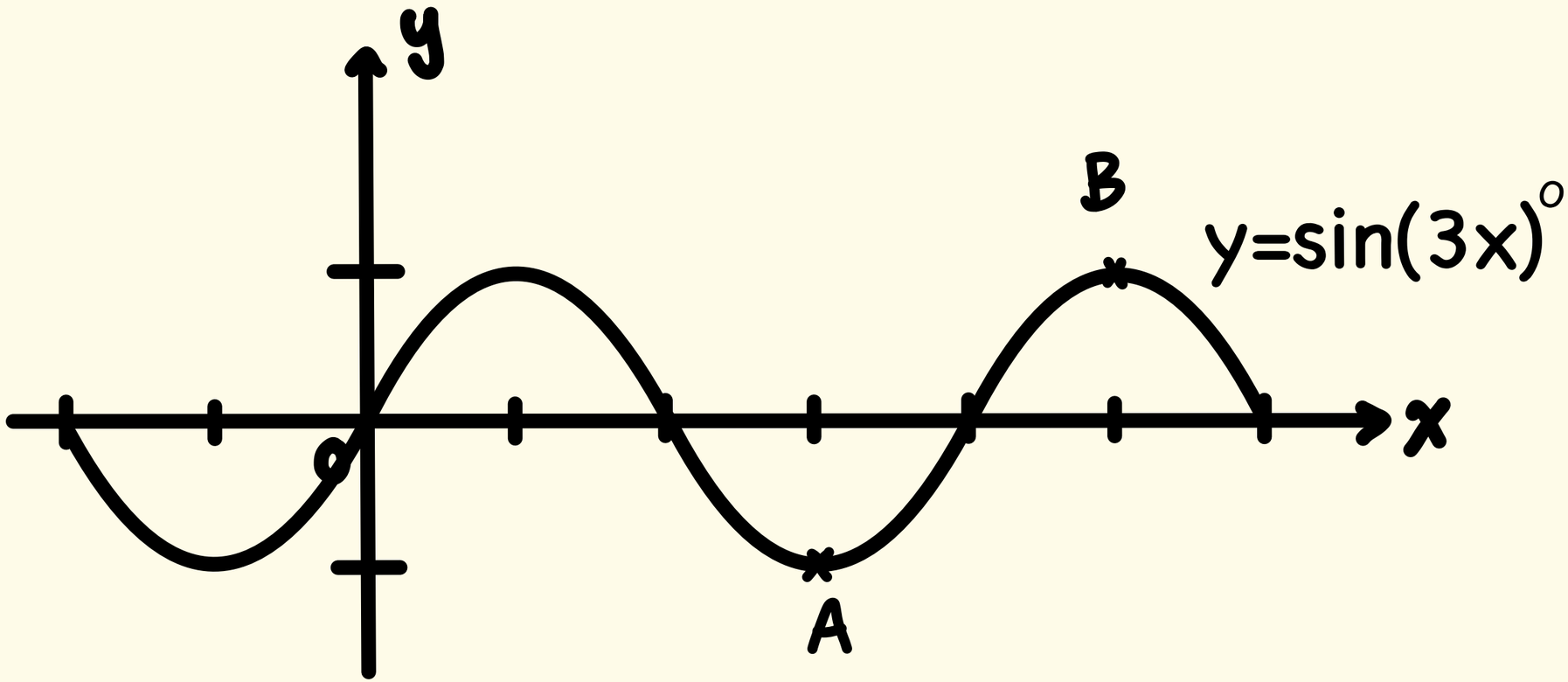
On the diagram

i) Sketch the graph of y=cos(2x)

ii) State the values of x for $0<x<180^{\circ}$ where cos(2x) = h.

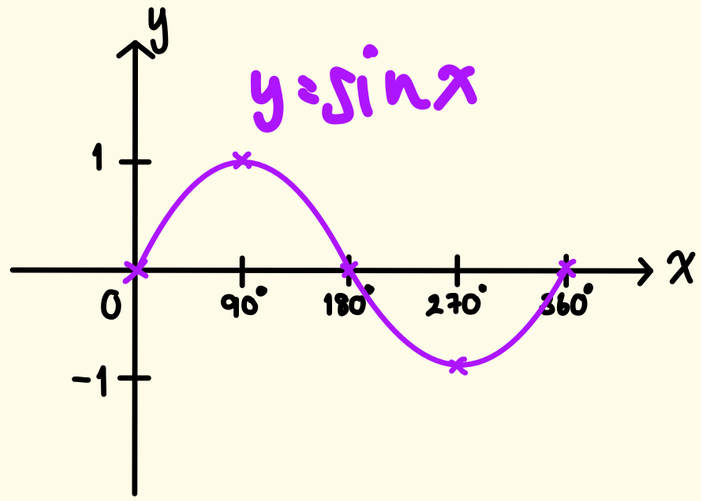
TOPMaths
A* Level

Ex.8 Curve C1

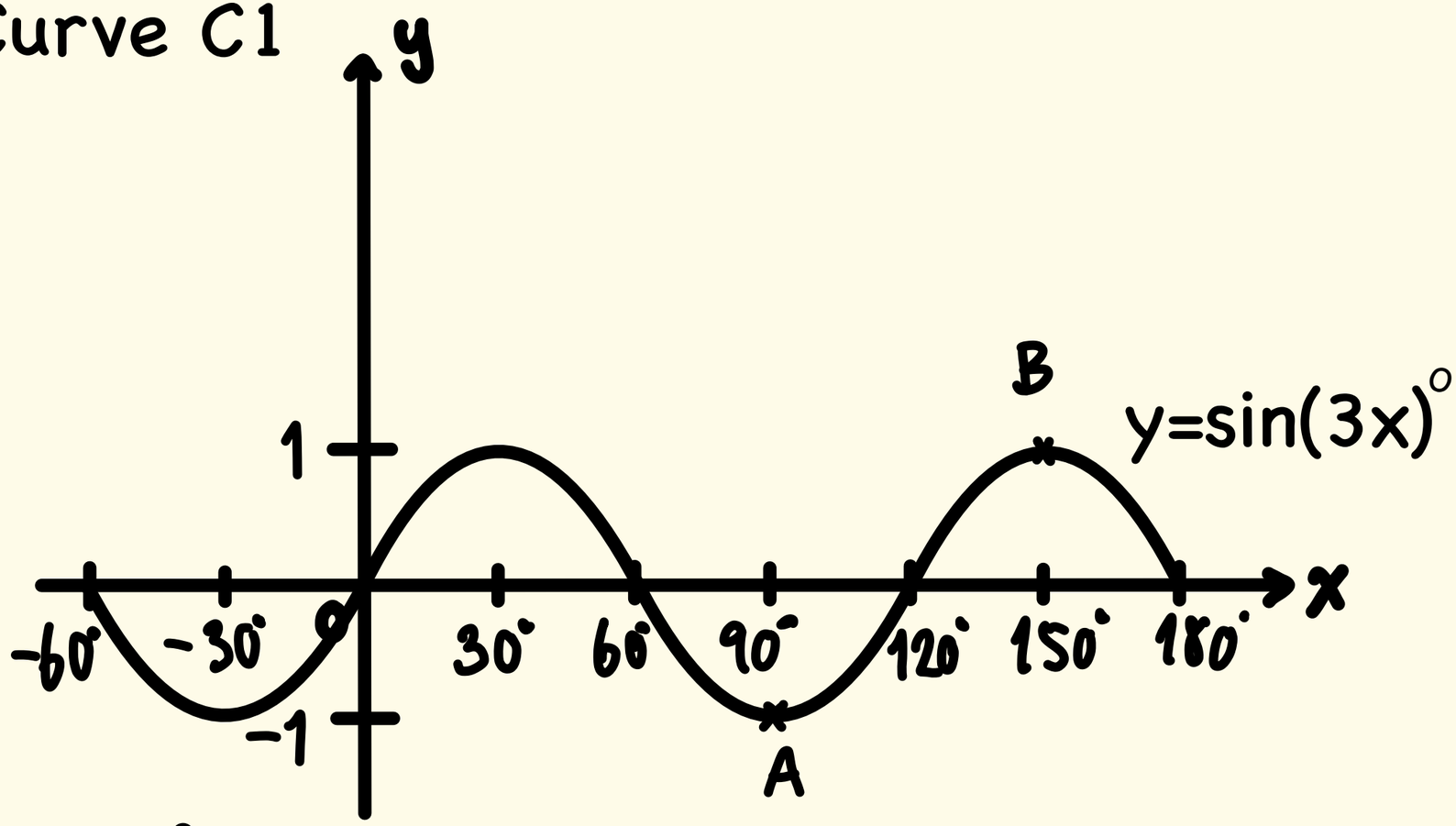


a) Find the coordinates of A.

b) Find the coordinates of B.



Ex.9 Curve C1



Curve C2, $y=\sin(3x)^{\circ}+c$,where c is a constant, has a minimum y value of 5. The point D is the maximum point on C2 with the smallest positive x coordinate.

State the coordinates of D.