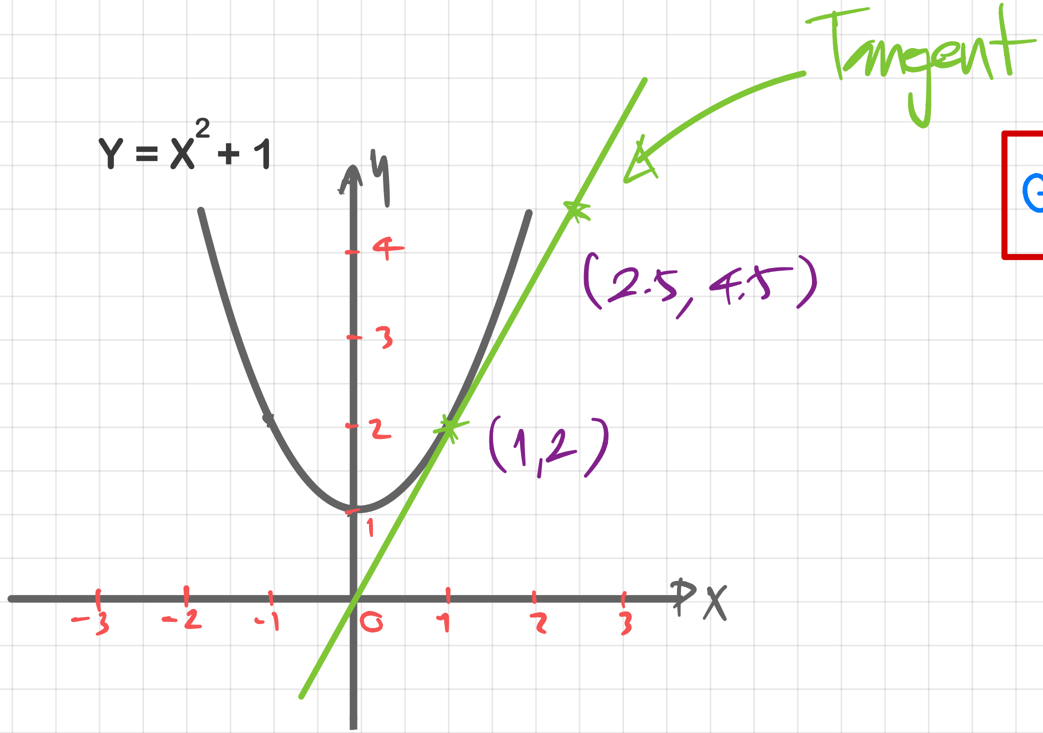
TOPMaThs A* Level

Differentiation

Differentiation

8.1) Gradients of curves

Find the gradient of curve at point (1,2)

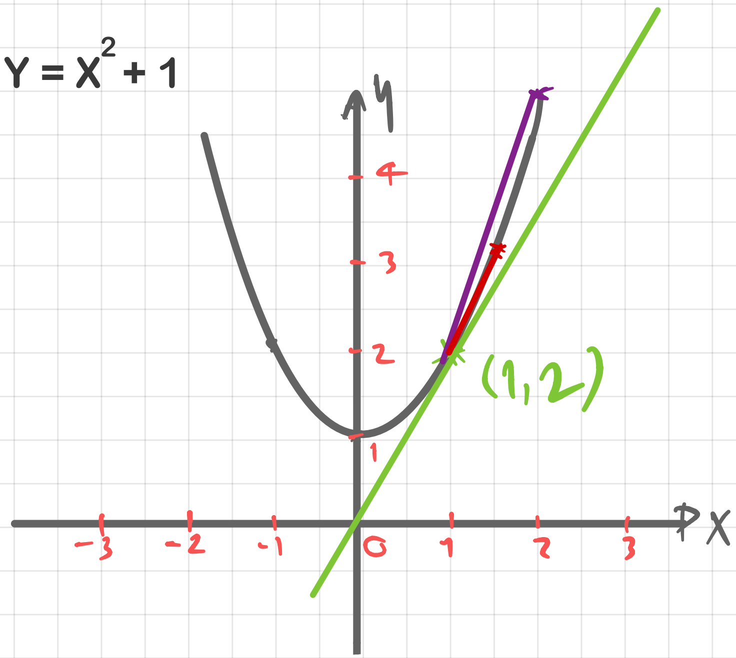


Gradient of curve = Gradient of tangent

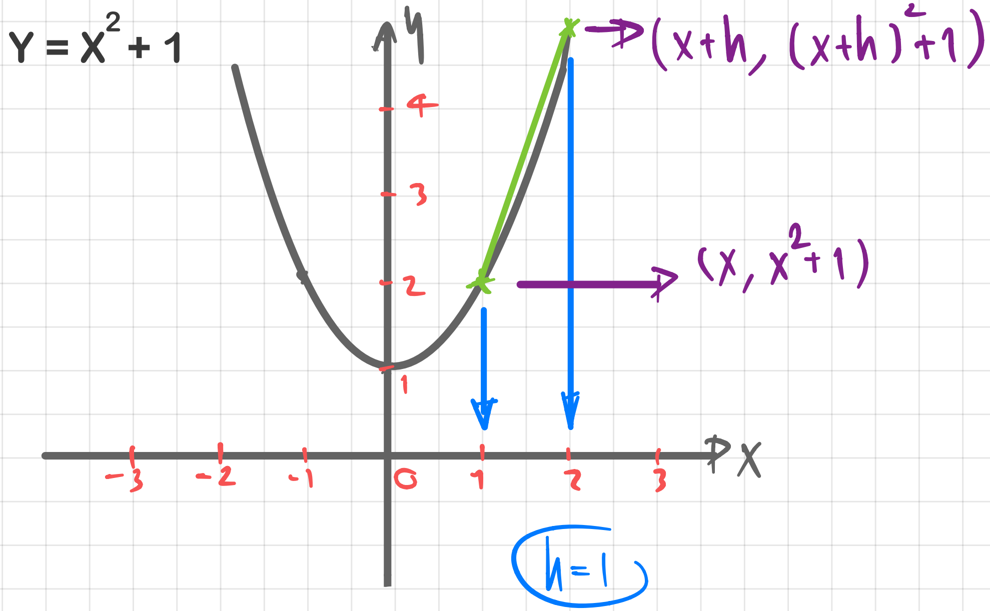
Differentiation

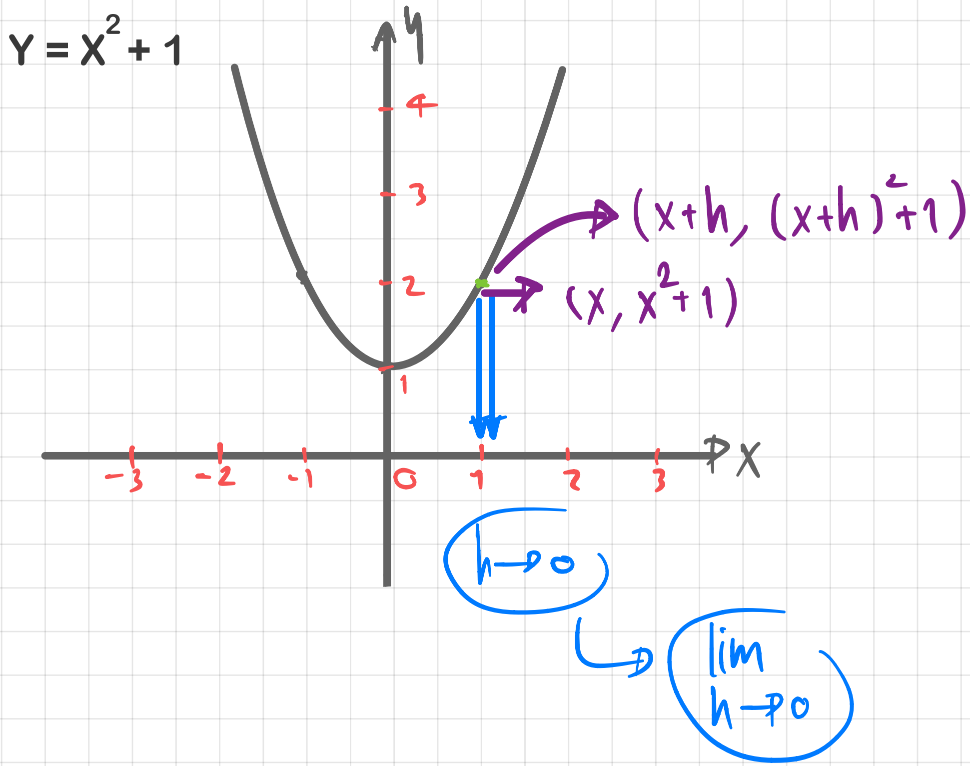
8.2) Finding the derivative (Prove)

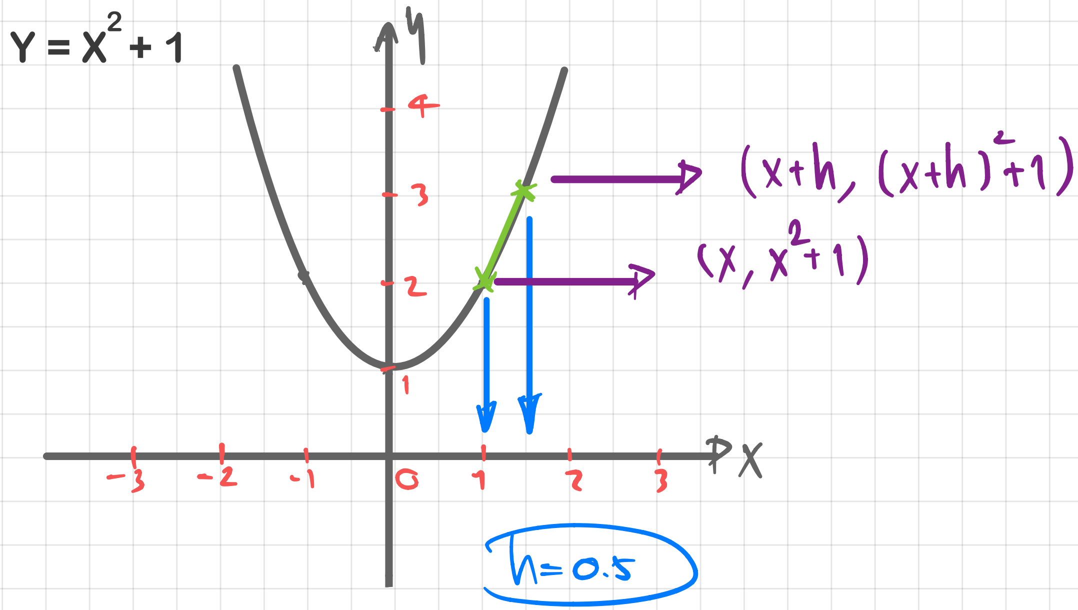
$Y = X^2 + 1$



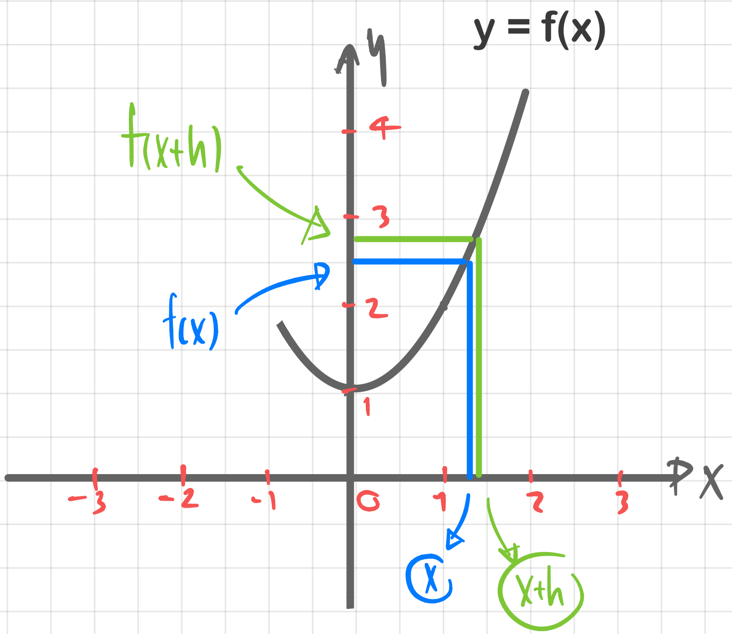
Differentiation







Differentiation



Differentiating from first principles.

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The gradient function can be used to find the gradient of the curve for any value of x.

Differentiation

$$\lim_{h \to 0} (7+h) =$$

$$\lim_{h \to 0} (h) =$$

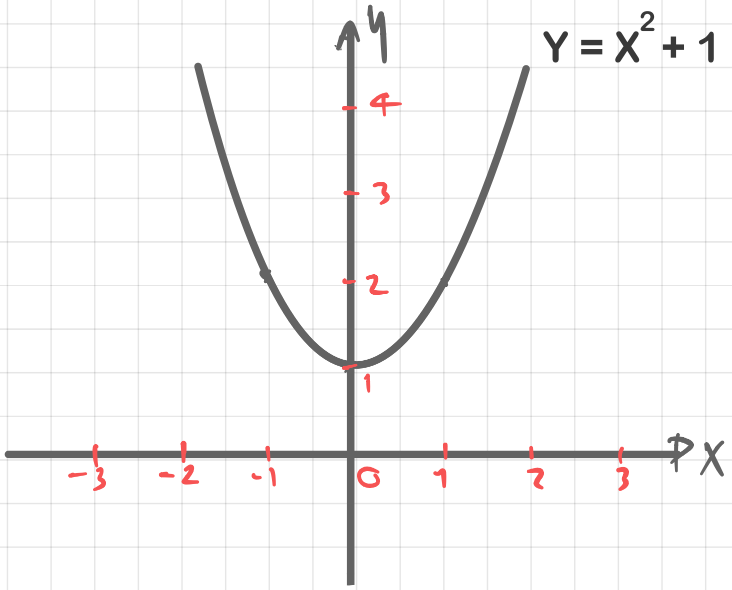
$$\lim_{h \to 0} (3x+h) =$$

$$\lim_{h \to 0} (3xh) =$$

$$\lim_{h \to 0} (h^2) =$$

Differentiation

Example: Prove, from first principles, that the derivative of $Y = X^2 + 1$



Differentiation

Example: Prove, from first principles, that the derivative of $y = 3x^2 + 2x - 1$

Differentiation

Review:Indices

$\longrightarrow ax^n$

$a = ax^0 \longrightarrow$

$ax = ax^1 \longrightarrow$

$(x^m)(x^n) = x^{m+n} \longrightarrow$

$\frac{x^m}{x^n} = x^{m-n} \longrightarrow$

$\frac{1}{x^m} = 1x^{-m} \longrightarrow$

$\sqrt{x} = x^{\frac{1}{2}} \longrightarrow$

$\sqrt[3]{x} = x^{\frac{1}{3}} \longrightarrow$

Differentiation

Example: $y = 3x(2x^2 - 1) + \frac{3}{2x^2}$

Example: $f(x) = 2x(x+2)(x-3)$

Differentiation

Example: $y = 4x\sqrt{x} - 5\sqrt[3]{x}$

Example: $f(x) = \dfrac{3x^2 + 7}{\sqrt{x}}$

Differentiation

8.3) , 8.4) and 8.5) Differentiating:

$$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots$$

$$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots$$

$$\boxed{ax^n \xrightarrow{D} a(n)x^{n-1}}$$

$$3x^5 \xrightarrow{D}$$

$$-4x^{\frac{1}{2}} \xrightarrow{D}$$

$$2x^{-3} \xrightarrow{D}$$

Differentiation

$$\boxed{ax \xrightarrow{D} a}$$

Prove! $ax = ax^1 \xrightarrow{D} a(1)x^{1-1}$

$ax^0 = a$

$5x \xrightarrow{D}$

$-100x \xrightarrow{D}$

$\frac{2}{3}x \xrightarrow{D}$

Differentiation

$$\boxed{a \xrightarrow{D} 0}$$

Prove!

$$a = ax^0 \xrightarrow{D} a(0)\,x^{0-1}$$

$$= 0$$

$$5 \xrightarrow{D}$$

$$-100 \xrightarrow{D}$$

$$\frac{3}{2} \xrightarrow{D}$$

Differentiation

Example:Differentiation

$$y = 3x^2 + 5x - 3x^{\frac{1}{2}} + 4$$

Differentiation

Example:Differentiation

$$f(x) = 3\sqrt{x}\,(2x + 5x^2)$$

Differentiation

Example:Differentiation

$$y = 3x\sqrt{x} + \frac{10}{\sqrt{x}}$$

Differentiation

Example:Differentiation

$$f(x) = \frac{4\sqrt{x} + 5x^2 - 3}{2x}$$

Differentiation

Example:Differentiation

$$y = kx^3 + 5x^2 - k^2x + 3$$

Differentiation

Variables:

$$y = x^2 + 7x - 1 \xrightarrow{D}$$

$$x = 7y^{\frac{1}{2}} - 4 \xrightarrow{D}$$

$$s = 4t^{-1} + 5 \xrightarrow{D}$$

$$A = \frac{7}{4}r + 4r^{\frac{1}{2}} \xrightarrow{D}$$

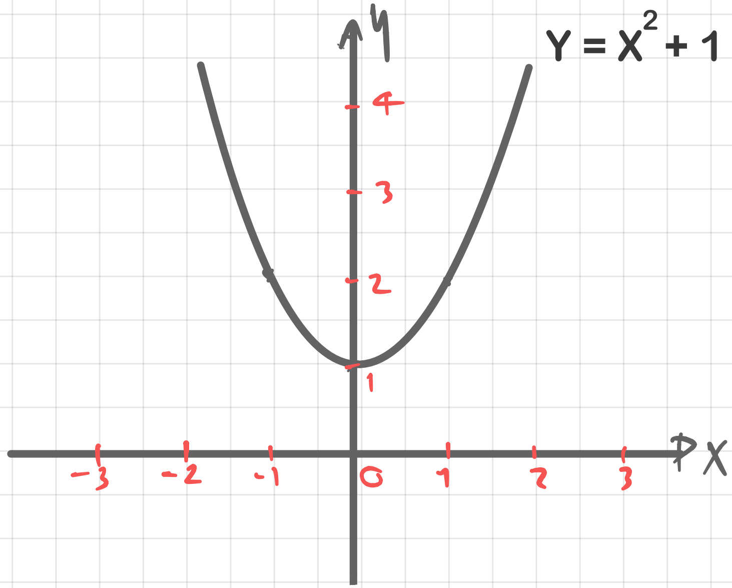
$$V = \frac{1}{3}\pi r^2 \xrightarrow{D}$$

Differentiation

$$V = 4\pi r^2 h^3 \quad \left[\begin{array}{l} V = (4\pi r^2) h^3 \xrightarrow{D} \\ \\ V = (4\pi h^3) r^2 \xrightarrow{D} \end{array} \right.$$

Differentiation

8.6) Gradients , Tangents and Normals.



Differentiation to find a Gradient.

1) Differentiating the equation of curve.

2) Find the gradient of the curve at any point by substituting the value for x into 1)...

Example:Find the gradient of curve at $x = 1$

Differentiation

Example: Find the gradient of the graph $y = x^3 - 12x + 1$ at .....

1) $x = -1$

2) $x = 2$

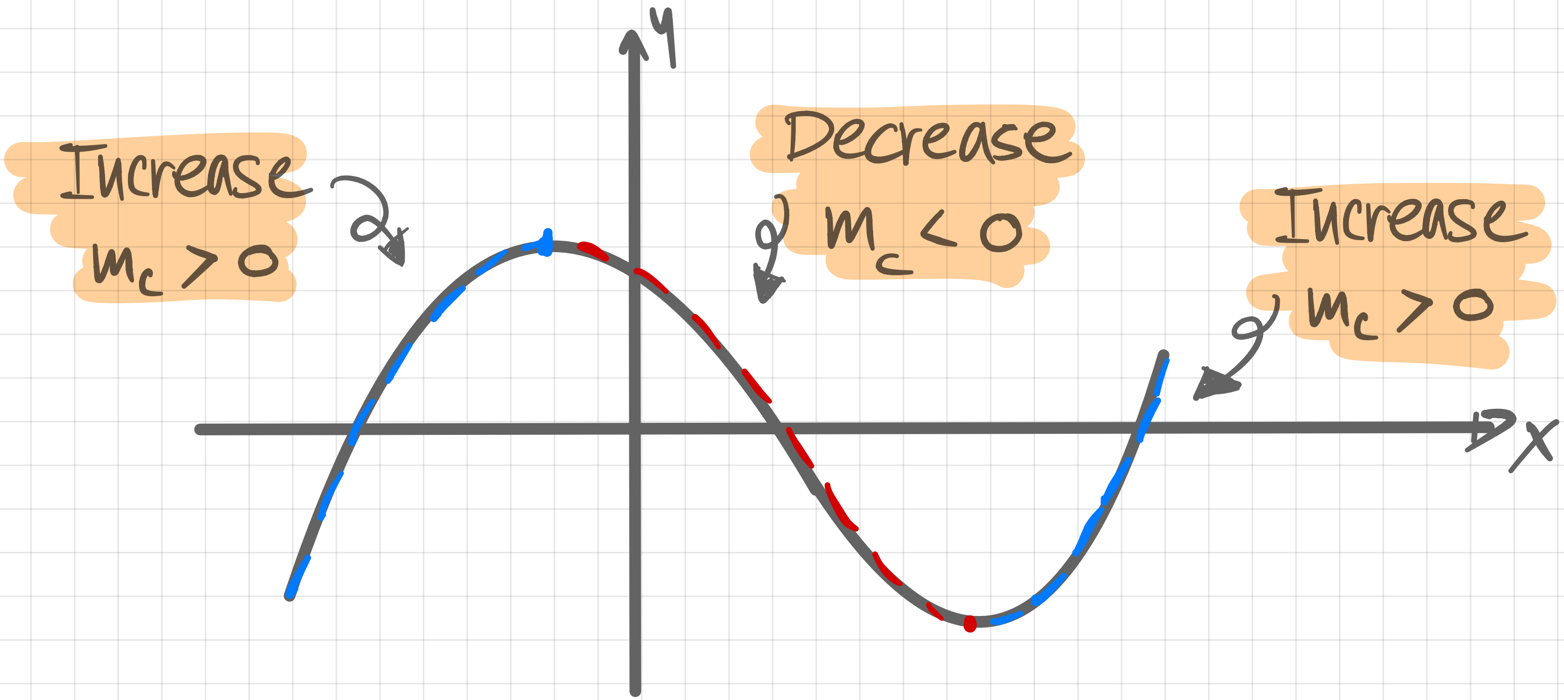
3) $x = \frac{5}{2}$

Differentiation

Example: Find the coordinate for curve $y = 3\sqrt{x}$ has gradient is $\frac{3}{4}$.

Differentiation

Increase function and Decrease function



Differentiation

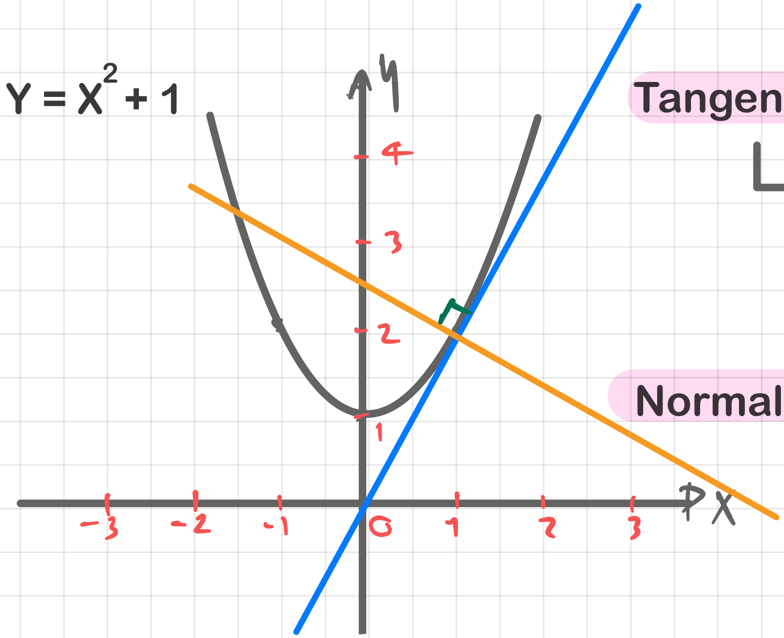
Example: Find values of x for $y = x^2 + 3x - 1$ is increasing function.

Differentiation

Example: Find range of value of x at $f(x)$ is decreasing function for $f(x) = x^3 - 3x^2$

Differentiation

Tangents and Normals



Tangent at (1,2)

→ $m_T = m_c$ (Differentiate)

Normal at (1,2)

→ Normal $\perp$ Tangent

→ $m_T \times m_N = -1$

Differentiation

Find equation of tangent to curve

$y = x^2 + 1$ at point $(1,2)$

Find equation of normal to curve

$y = x^2 + 1$ at point $(1,2)$

Differentiation

Example: Find equation of tangent to curve $y = x^3 - 3x^2 + 2x - 1$ at $x = 3$

Differentiation

Example: Find equation of normal to curve $y = 8 - 3\sqrt{x}$ at $x = 4$.

8.7) Second order derivatives

$$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots \xrightarrow{D} \frac{d^2y}{dx^2} = \ldots$$

$$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots \xrightarrow{D} f''(x) = \ldots$$

$$s = t\ldots \xrightarrow{D} \frac{ds}{dt} = \ldots \xrightarrow{D} \frac{d^2s}{dt^2} = \ldots$$

Displacement	Velocity	Acceleration $\oplus$
Distance	Speed	Deceleration $\ominus$

Differentiation

Example: Find second order derivative.

$$y = 5x^3 + \sqrt{x} - 7$$

Differentiation

Example: Find second order derivative.

$$f(x) = 5x^4 + \frac{7}{x}$$

Differentiation

Example: Find second order derivative.

$$s = (t-2)(t+1)$$

Differentiation

The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\dfrac{dy}{dx}$

Differentiation

The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$ $\longrightarrow 6x^2 + 2kx + 5$

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k,

Differentiation

The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{dy}{dx}$ $= 6x^2 + 2kx + 5$

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k, $= \frac{41}{8}$

(c) the value of the y coordinate of P,

Differentiation

. The curve C has equation $y = 2x^3 + kx^2 + 5x + 6$, where k is a constant.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ $6x^2 + 2kx + 5$

(2)

The point P, where $x = -2$, lies on C.

The tangent to C at the point P is parallel to the line with equation $2y - 17x - 1 = 0$

Find

(b) the value of k, $= \frac{41}{8}$

(4)

(c) the value of the y coordinate of P, $= \frac{1}{2}$

(2)

(d) the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)