PURE 1

INTEGRATION

By Kru Sie

Integration

Integration is the reverse process of differentiation

Consider...

Note...

Integrate with respect to x

a x^2

b x^6

c x

d x^{-4}

Integrate with respect to x

$3x^2$

$4x^7$

$6x^{-2}$

Integrate with respect to x

$\frac{3}{4}x^{-3}$

$\frac{1}{3}x$

Integration

TOP Maths A level

Integrate with respect to x

*You need to rearrange expression in form of ax^n

$6\sqrt{x}$

$\frac{2}{3x^2}$

$\frac{3}{\sqrt{x}}$

Integrate with respect to y

$y^{\frac{1}{2}}$

$4y^{\frac{1}{3}}$

$\sqrt[4]{y}$

When integrate of constant without x

$$\int 5 \, dx$$

Note...

Try... with many terms

$$\int (2x + 3)\ \mathrm{d}x$$

$$\int (3x^4 + x^2 - 6)\ \mathrm{d}x$$

Note...

Hard integration

$$\int x(x^2 - 3)\ \mathrm{d}x$$

$$\int (x - 2)^2\ \mathrm{d}x$$

Hard integration

$$\int x^2\left(\frac{2}{x^4} - 3\right)\ \mathrm{d}x$$

Hard integration

$$\int (x - \frac{4}{x})^2 \, dx$$

Hard integration

$$\int \frac{1+6x^2}{3x^2}\,\mathrm{d}x$$

Hard integration

$$\int \frac{4p^4 - p^2}{2p} \, \mathrm{d}p$$

Hard integration

$$\int \frac{2t+3}{\sqrt{t}}\,\mathrm{d}t$$

Finding function

To find the constant "c"

1. Integrate the function
2. Substitute the value x and y of a point on the curve , or the value of the function at a given point f(x) = k into the integrated function
3. Solve the equation to find c

Integration

TOP Maths A level

Example

a Find $\int (3x + 2)\,dx$.

b Given that $\frac{dy}{dx} = 3x + 2$ and that $y = 5$ when $x = 1$, find an expression for y in terms of x.

Integration

TOP Maths A level

Question 1

The curve $y = f(x)$ passes through the point $(1,5)$.

Given that $f'(x) = 3 + 2x - x^2$, find an expression for $f(x)$.

Question 2

The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3x^2 + kx,$$

where k is a constant.

Given that C passes through the points $(1,7)$ and $(2,17)$,

a find the value of k,

b find an equation of the curve.

Question 3

The curve C has the equation $y = f(x)$ and crosses the x-axis at the point $P\ (-2, 0)$.

Given that

$$f'(x) = 3x^2 + 3x - 1$$

find an expression for $f(x)$, and the tangent to the curve at the point where $x = 1$

Question 4

The curve $y = f(x)$ passes through the point $(1,2)$

Given that

$$f'(x) = 1 + \frac{4}{x^3}$$

a find an expression for $f(x)$.

The point A on the curve $y = f(x)$ has x-coordinate 2.

b find the normal to the curve $y = f(x)$ at A

Question 5

A curve has equation $y = f(x)$.

The point $P\left(4,-5\right)$ lies on the curve.

Given that

- $f''(x) = 1 - \frac{3}{2\sqrt{x}}$
- $f'(x) = -1$ at P

find

(a) the equation of the tangent to the curve at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found,

(b) $f(x)$.