Top Math IGCSE

Intensive Maths IGCSE -Oct Nov 2023

Day:2

Topic📚

Straight line graphs

Harder Graph

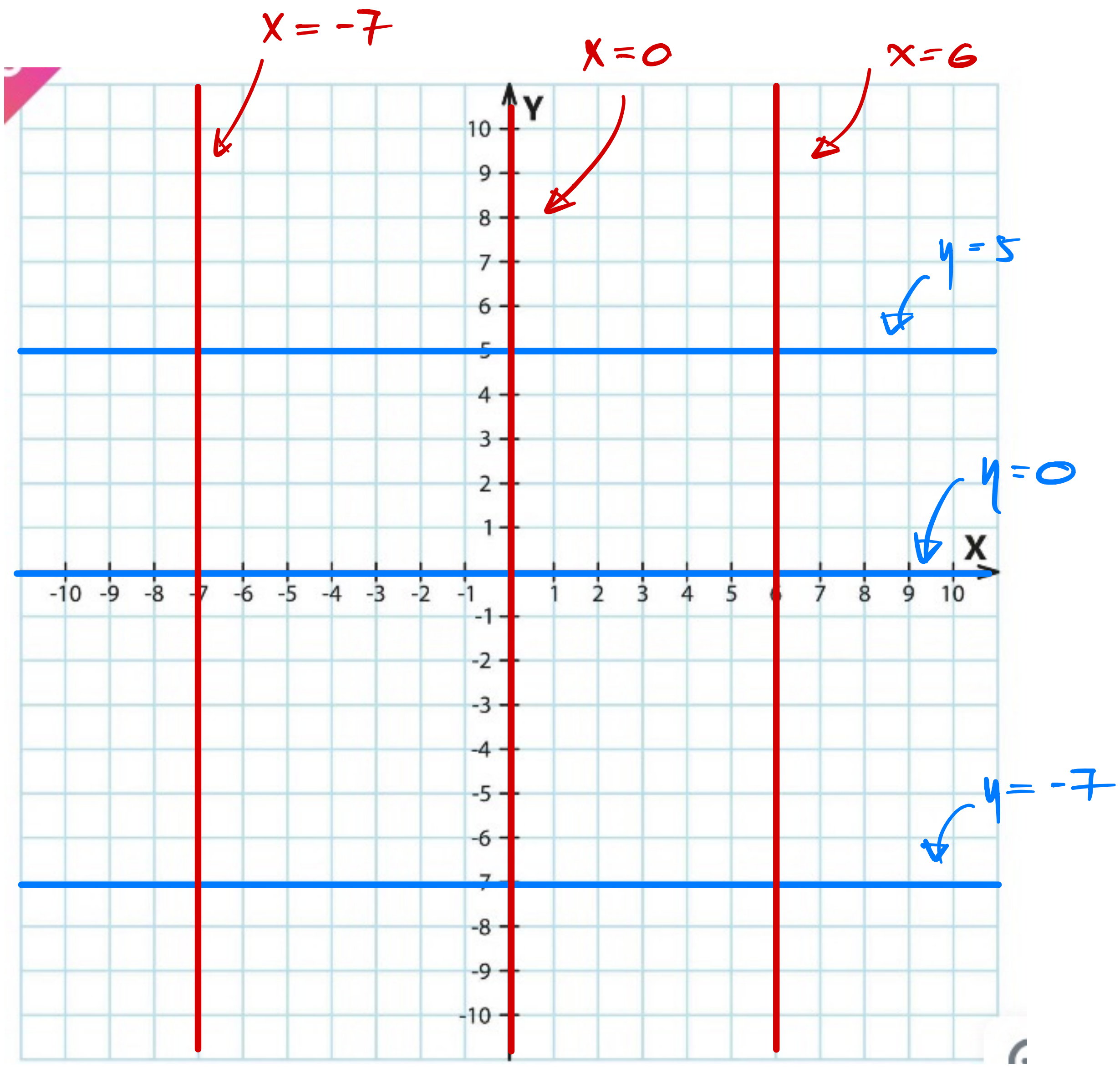
Function and Graph

Differentiation

Part - ครูส้ม🍊

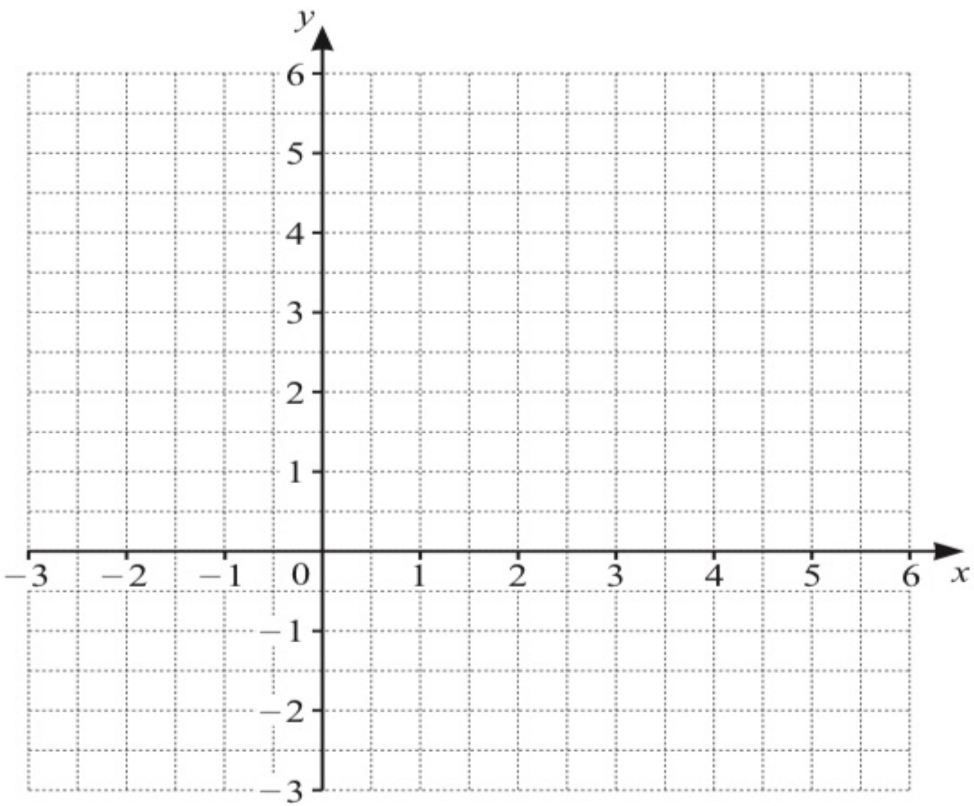
Graphs:Straight Lines.

Lines:Basic



6 **(a)** In the square $ABCD$, A has coordinates $(-2, 1)$ and B has coordinates $(1, 5)$.
C has coordinates (a, b), where a and b are both positive integers.

Find the coordinates of C and the coordinates of D.
You may use the grid to help you.



C (...................... , ...)

D (...................... , ...) [4]

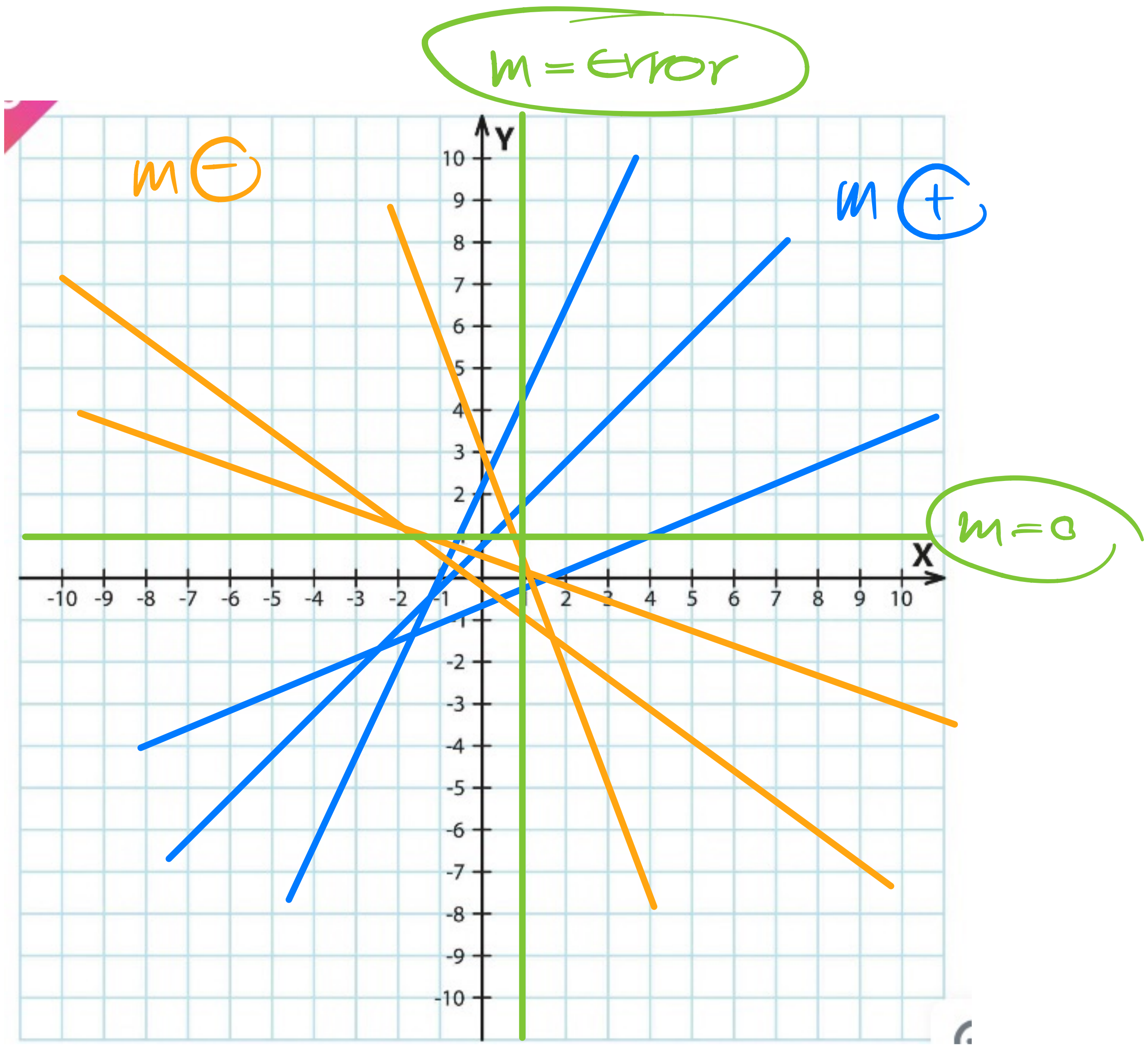
1) Gradient (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: The point A (5, 13) and the point B (−1, 1)

Work out the gradient of AB.

Graphs:Straight Lines.



Graphs:Straight Lines.

2) Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example: The point A (5, 13) and the point B (−1, 1)

Work out the midpoint of AB.

Graphs:Straight Lines.

3) Distance between point (Length)

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: The point A (5, 13) and the point B (−1, 1)

Work out the length of AB.

Graphs:Straight Lines.

4) Equation of line

gradient (m)
point (x_1, y_1) }

$y = mx + c$

$y_1 = m(x_1) + c$

$\varepsilon 3 = c$

$y = mx + c$

Graphs:Straight Lines.

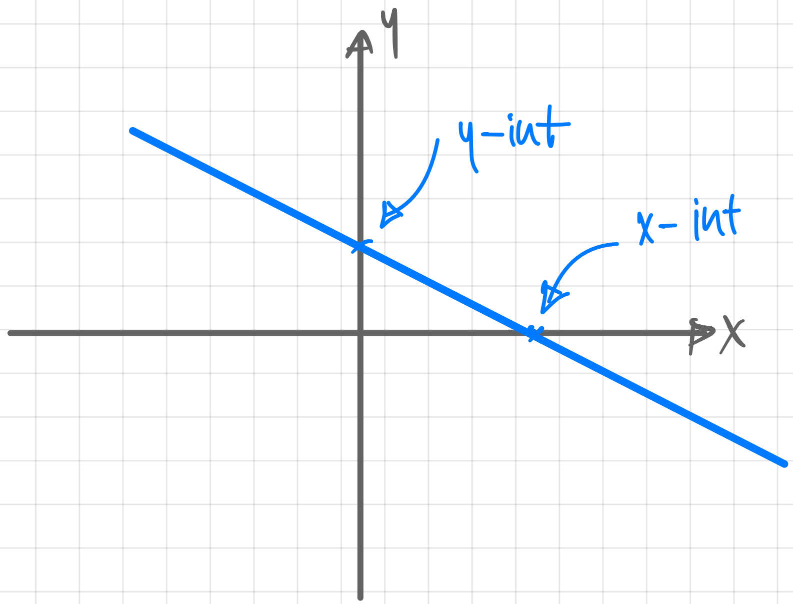
- Intercept

Cross the x-axis

x-intercept ⟶ $y = 0$; $x = ?$

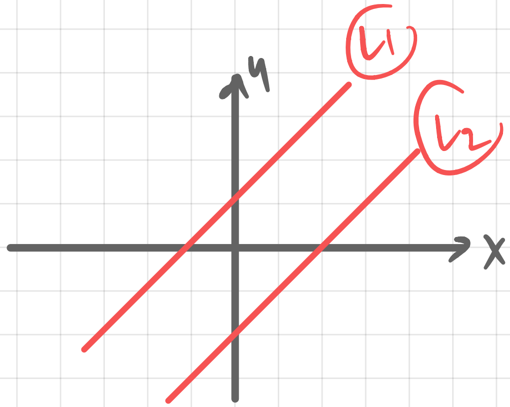
Cross the y-axis

y-intercept ⟶ $x = 0$; $y = ?$



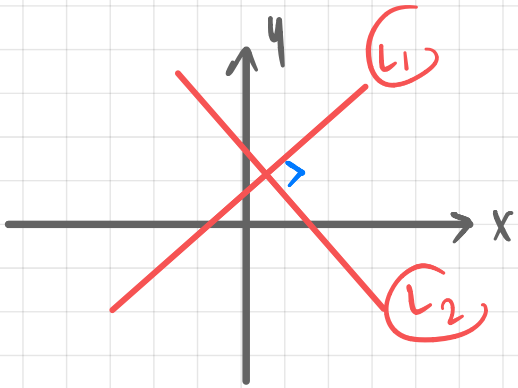
Graphs:Straight Lines.

4) Parallel lines



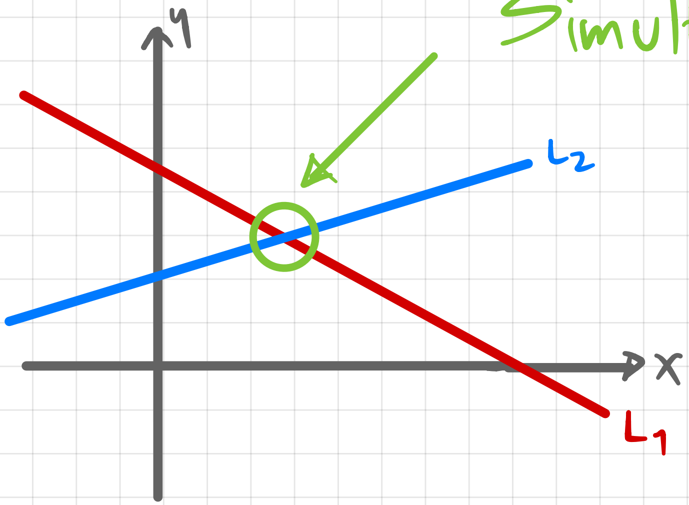
$L_1 \parallel L_2 \rightarrow$ $m_1 = m_2$

5) Perpendicular lines

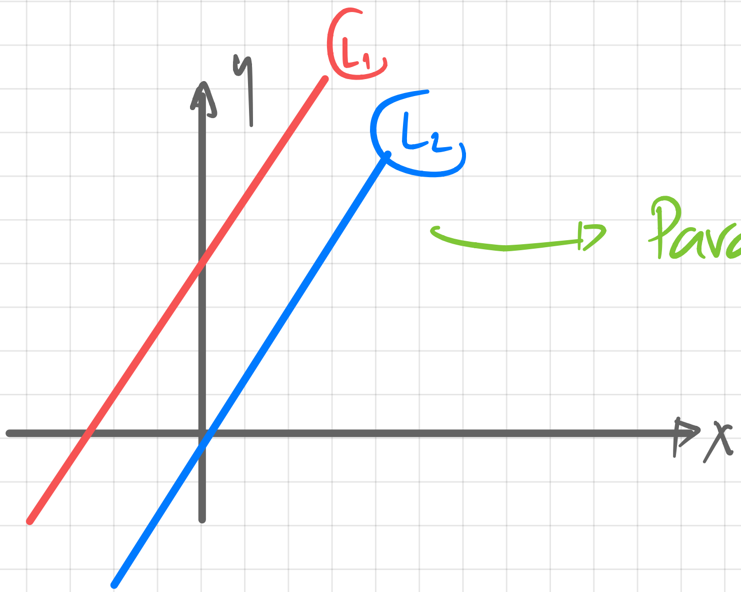


$L_1 \perp L_2 \rightarrow$ $m_1 \times m_2 = -1$

6) Intersection point



Simultaneous equations



→ Parallel (Don't have intersection point)

15 C is the point $(5, -1)$ and D is the point $(13, 15)$.

(a) Find the midpoint of CD.

(...................... , ...) [2]

(b) Find the gradient of CD.

.... [2]

(c) Find the equation of the perpendicular bisector of CD.
Give your answer in the form $y = mx + c$.

$y =$ .... [3]

(b) P has coordinates $(-1, 3)$ and Q has coordinates $(6, 4)$.

(i) Find the coordinates of the midpoint of PQ.

(.................... , ...) [2]

(ii) Find the length PQ.

.... [3]

(iii) Find the gradient of PQ.

.... [2]

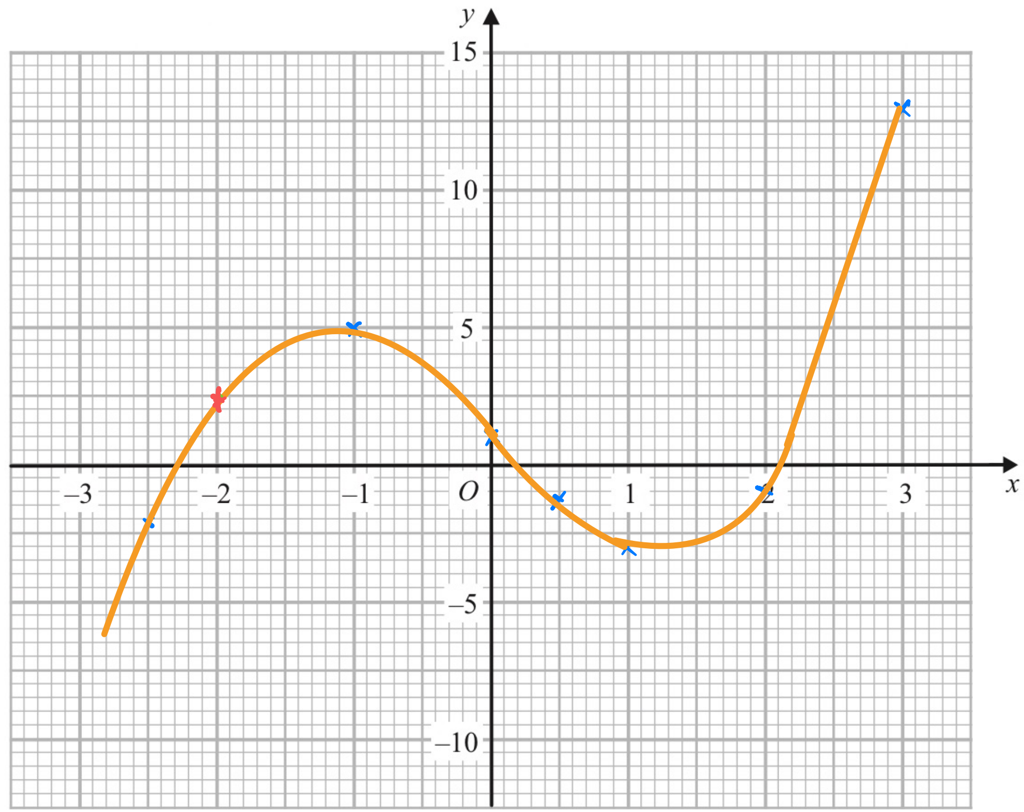
(iv) Find the equation of the line parallel to PQ that crosses the x-axis at $x = 2$.

.... [3]

Functions and Graphs

Gradient of graph at x=....

Find gradient of graph at x=-2



(2)

1) Tangent of curve at point x=-2

2) Coordinates of two any points from the tangent

3) Find gradient of curve from gradient of two points

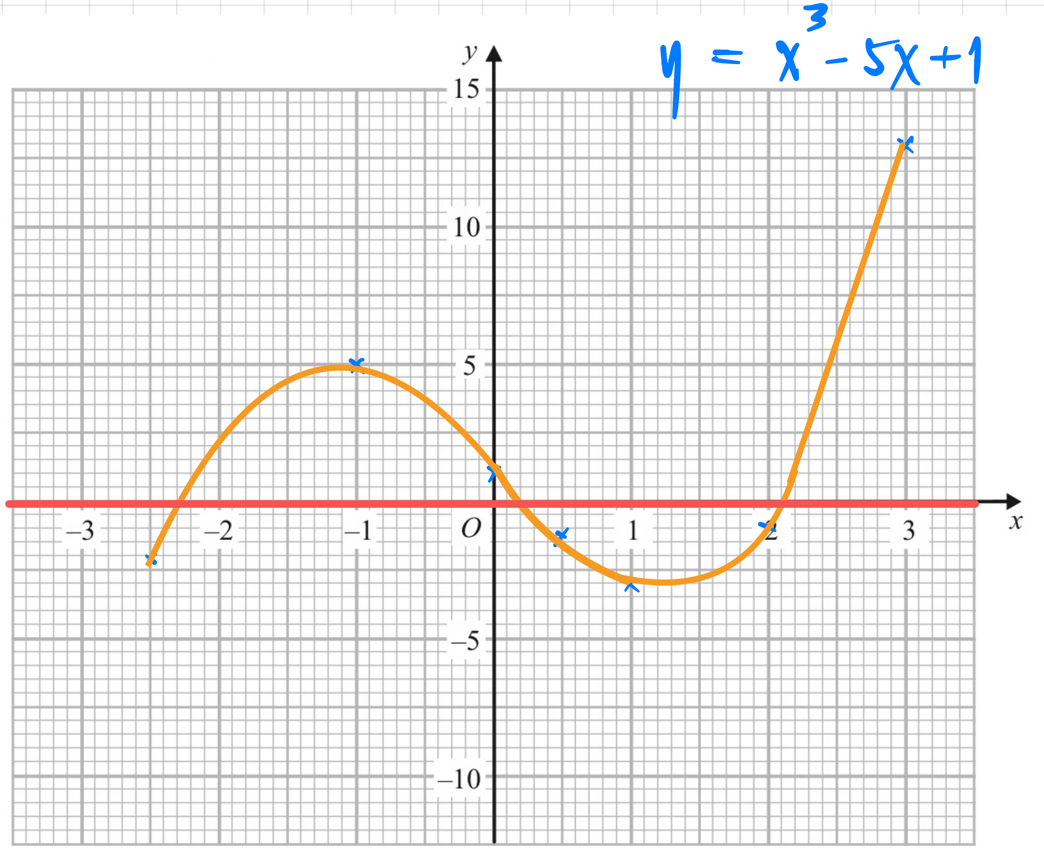
Use your graph to solve equation

Example

$y = x^3 - 5x + 1$

Use your graph to solve equation

$x^3 - 5x + 1 = 0$

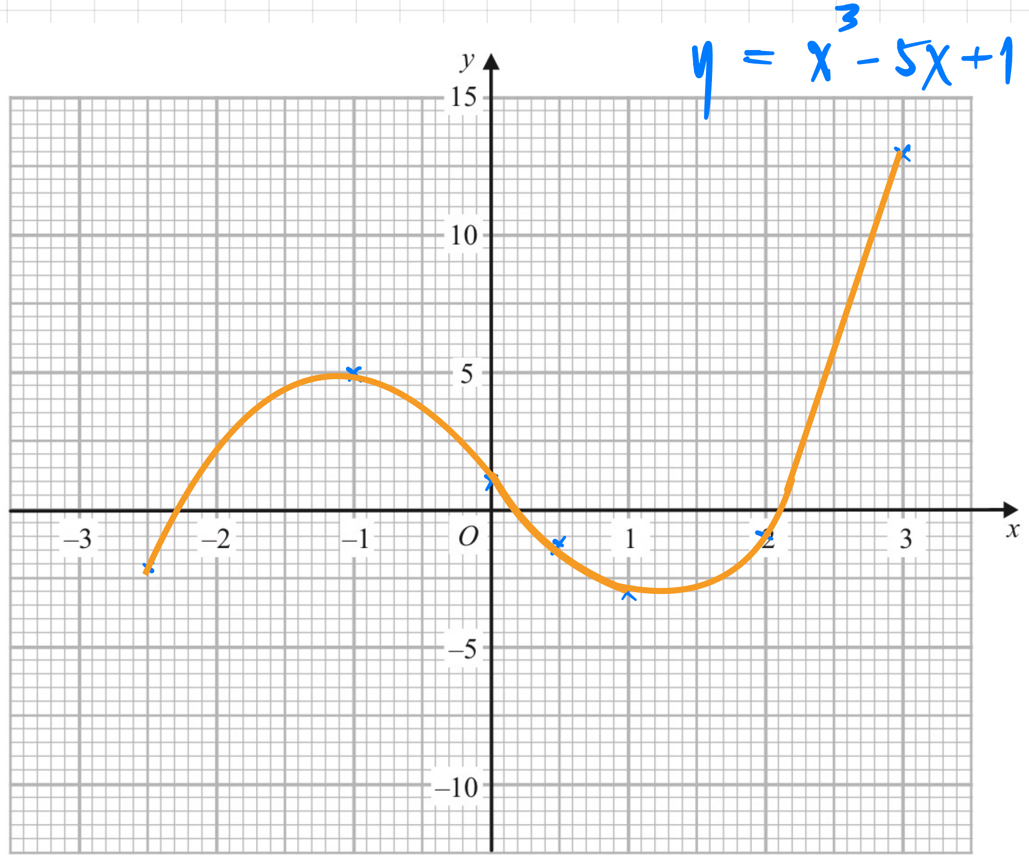


(2)

Example $y = x^3 - 5x + 1$

Use your graph to solve equation

$x^3 - 5x + 1 = 5$

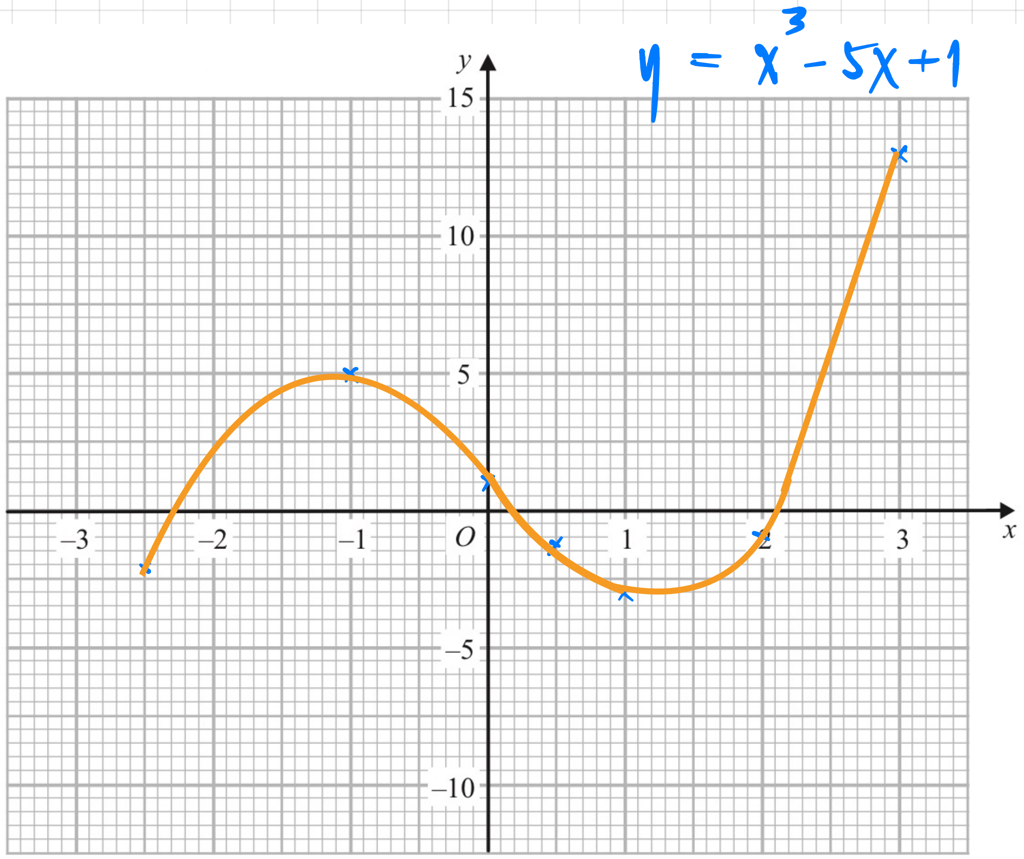


(2)

Example $y = x^3 - 5x + 1$

Use your graph to solve equation

$x^3 - 5x = 7$

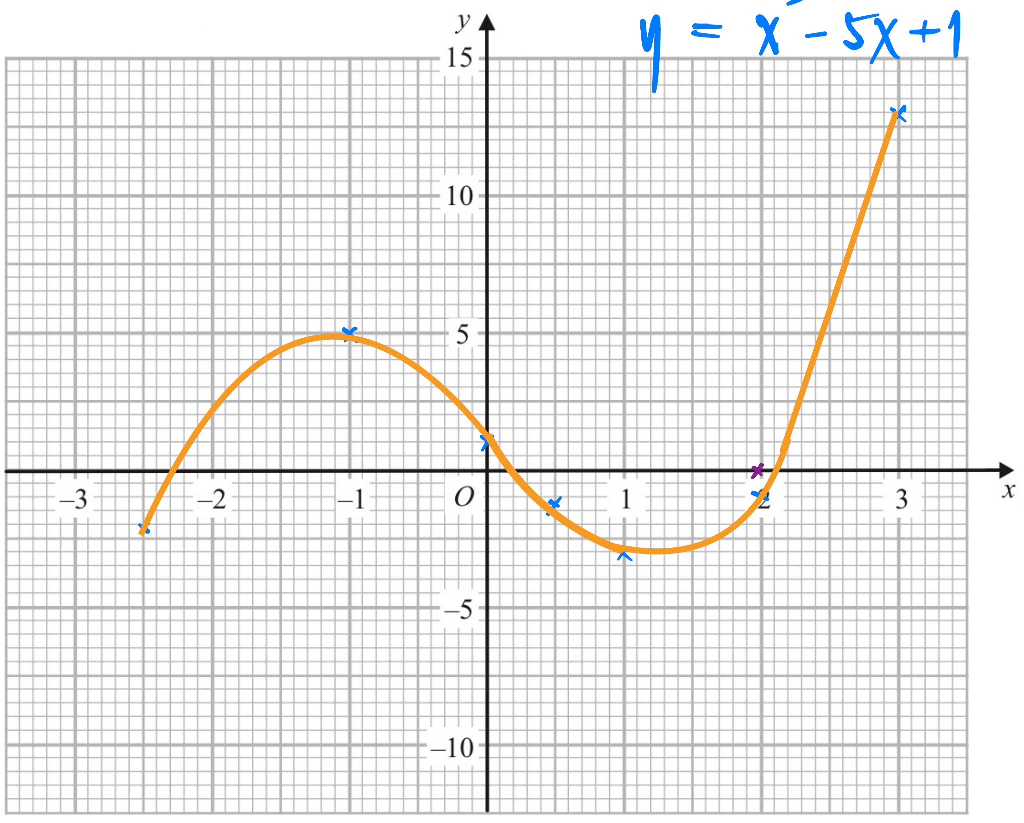


(2)

Example $y = x^3 - 5x + 1$

Use your graph to solve equation

$x^3 - 6x + 3 = 0$

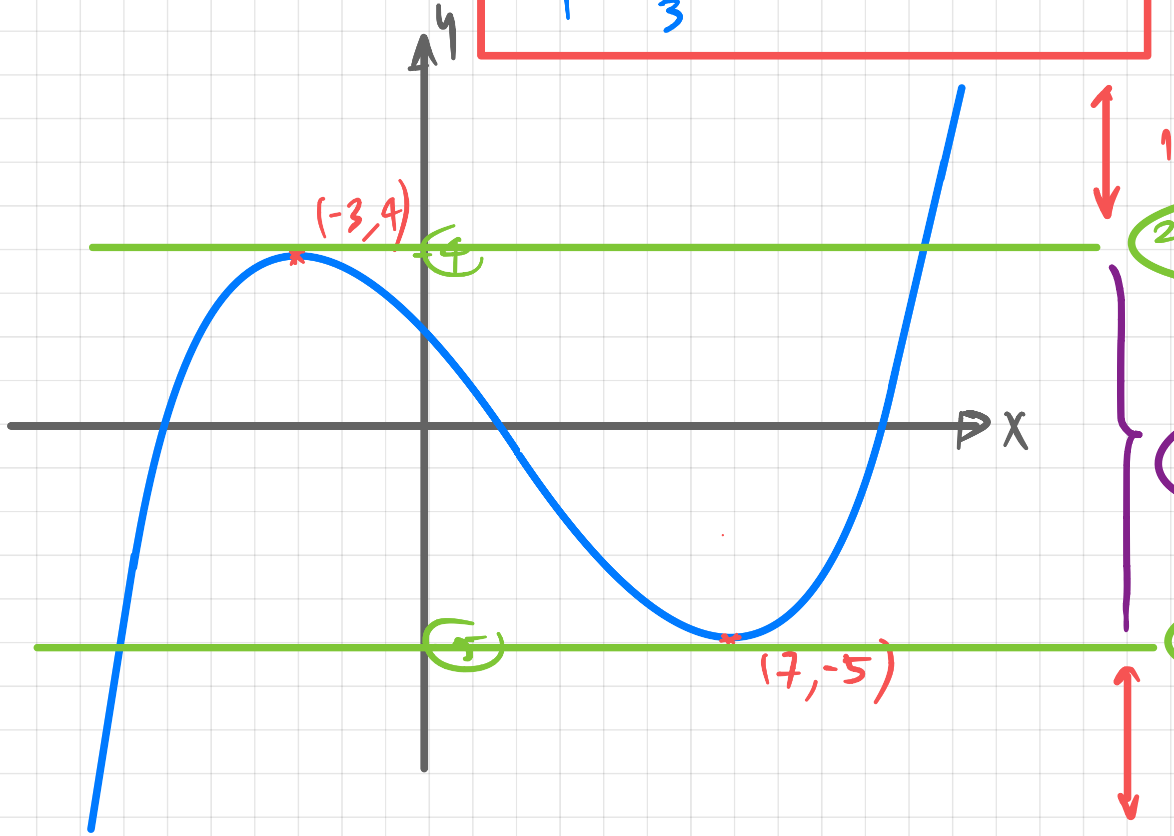


(2)

Functions and Graphs

Number of solutions (roots) → **Number of intersection**

$$y = \frac{1}{3}x^3 - 2x^2 - 3x + 2$$



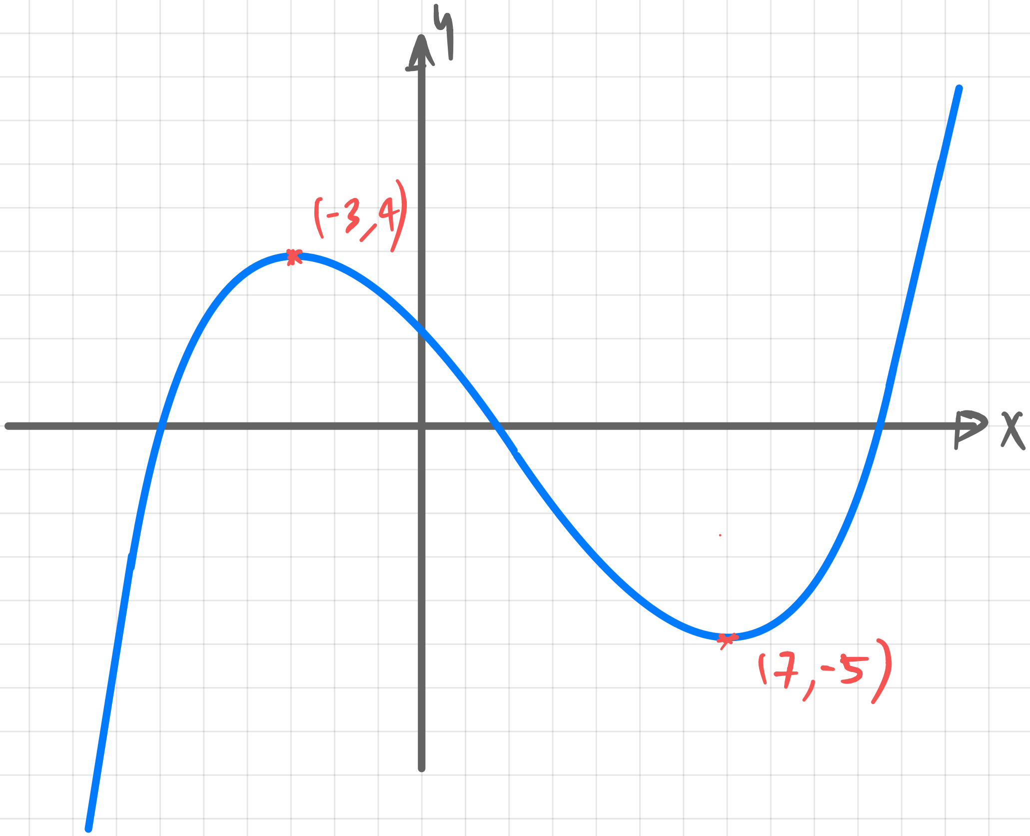
- 1 solutions → $y > 4$
- 2 solutions → $y = 4$
- 3 solutions → $-5 < y < 4$
- 2 solutions → $y = -5$
- 1 solutions → $y < -5$

Functions and Graphs

$$y = \frac{1}{3}x^3 - 2x^2 - 3x + 2$$

$\longrightarrow$ $\frac{1}{3}x^3 - 2x^2 - 3x + 2 = k$

has one solution ; find values of k

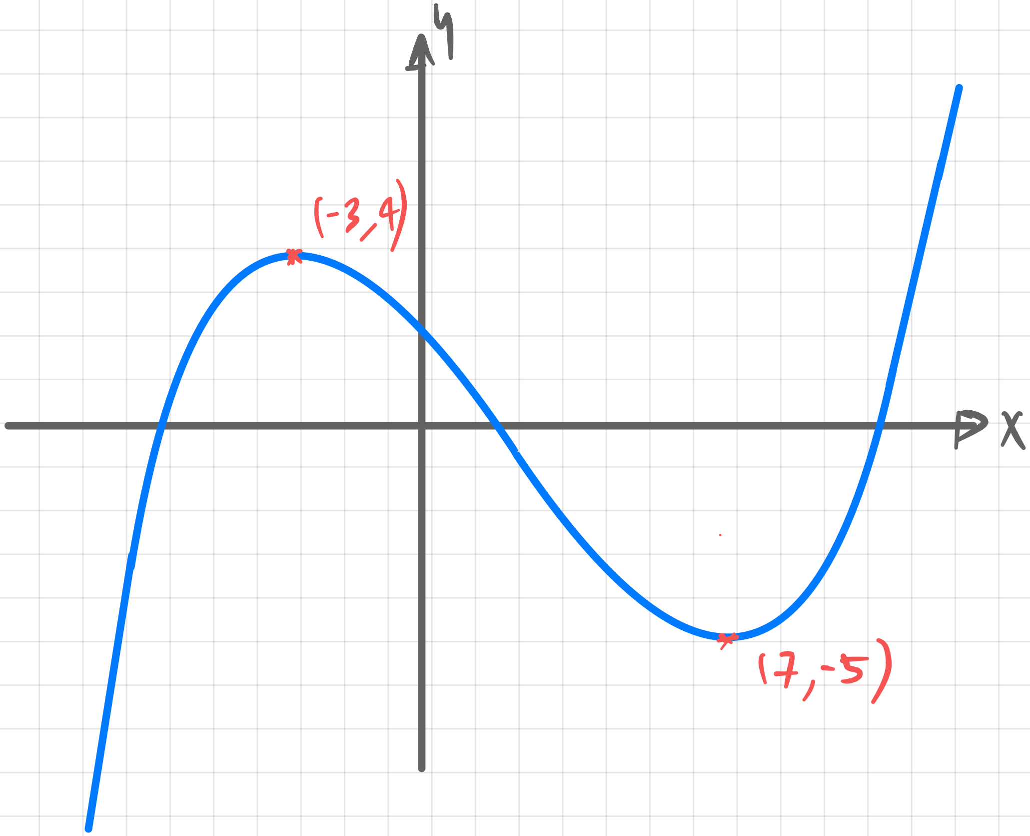


Functions and Graphs

$$y = \frac{1}{3}x^3 - 2x^2 - 3x + 2$$

$$\frac{1}{3}x^3 - 2x^2 - 3x + 2 = k$$

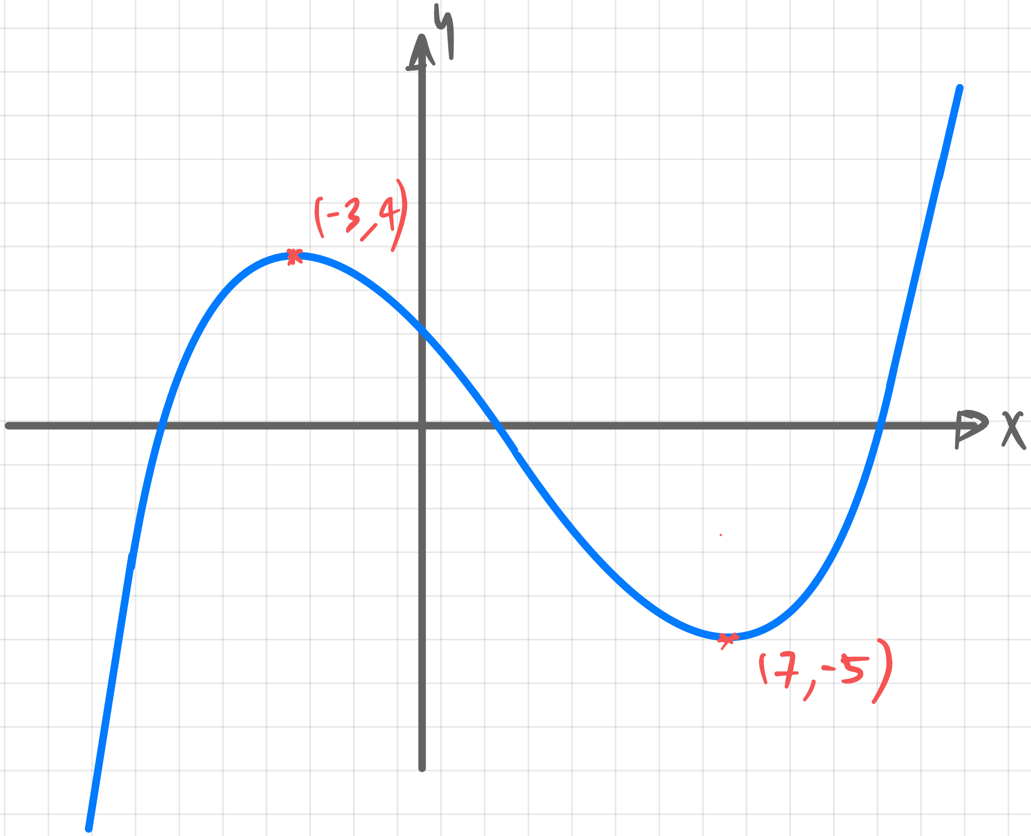
has two solutions ; find values of k



$$y = \frac{1}{3}x^3 - 2x^2 - 3x + 2$$

$\longrightarrow$ $\frac{1}{3}x^3 - 2x^2 - 3x + 2 = k$

has three solutions; find values of k

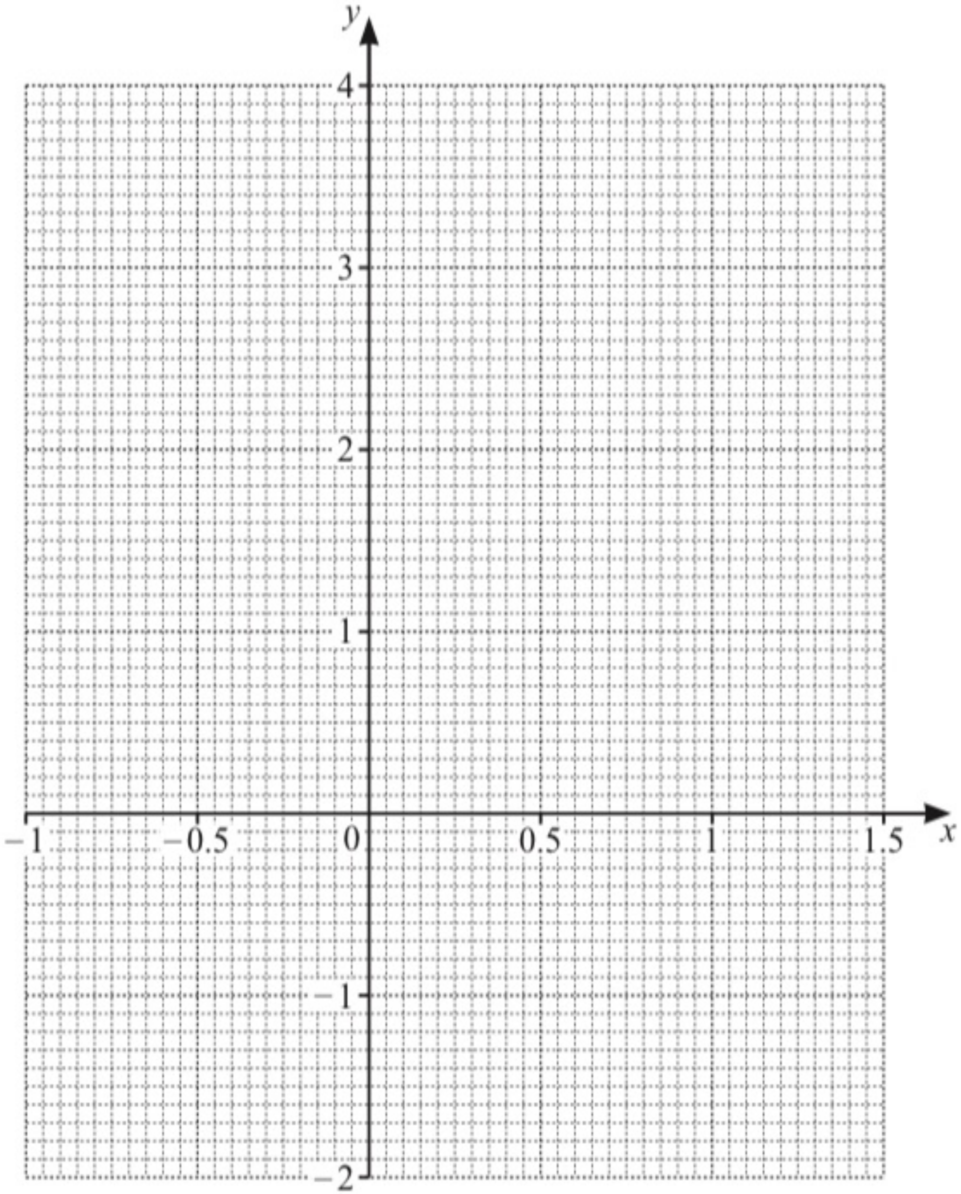


20 The table shows some values for $y = 3x^2 - 2x - 1$.

x	-1	-0.5	0	0.5	1	1.5
y	4		-1		0	2.75

(a) Complete the table. [1]

(b) On the grid, draw the graph of $y = 3x^2 - 2x - 1$ for $-1 \leqslant x \leqslant 1.5$.



[3]

(c) By drawing a suitable straight line, solve the equation $3x^2 - 4x - 2 = 0$ for $-1 \leqslant x \leqslant 1.5$.

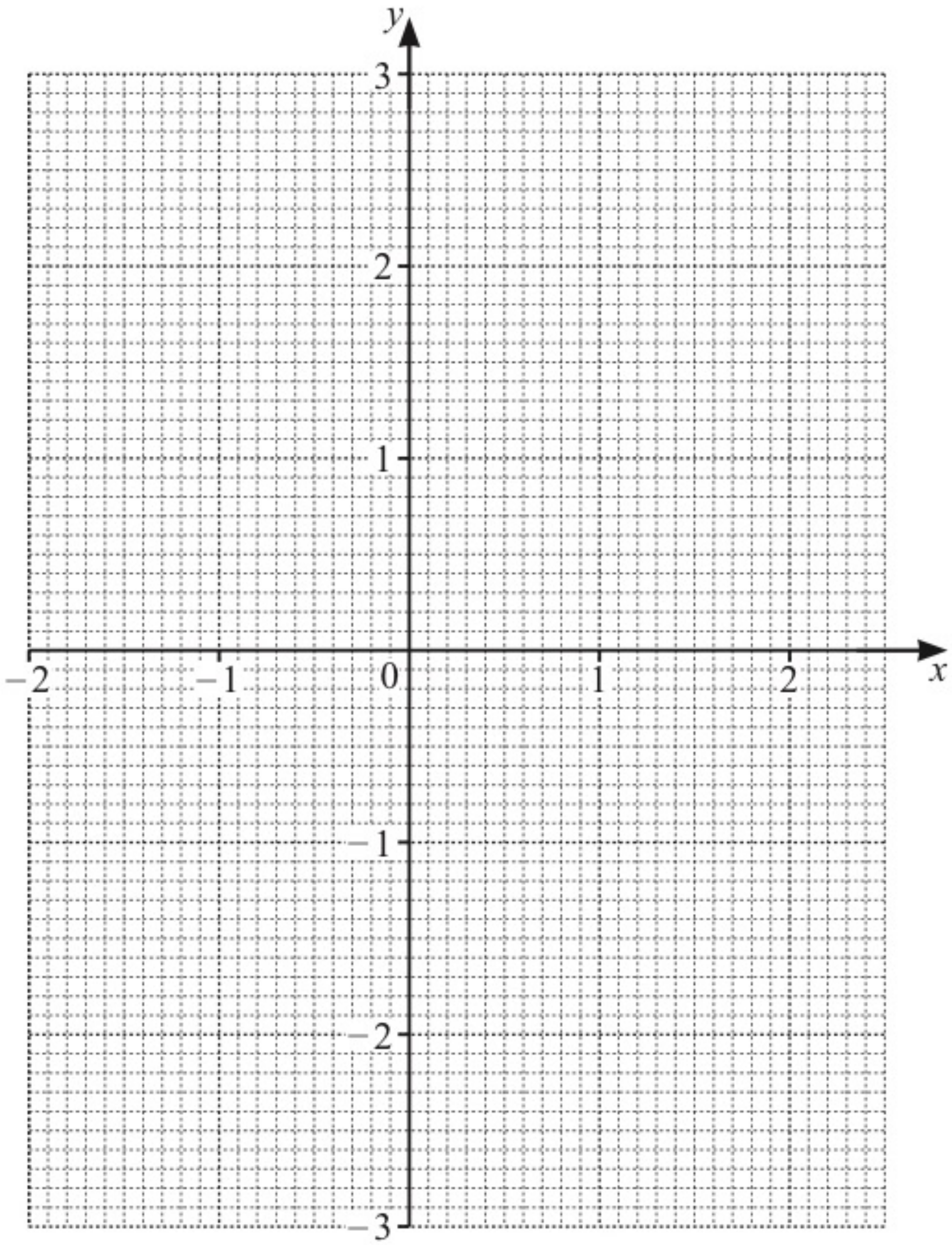
$x =$ .... [3]

10 The table shows some values for $y = 2^x - 3$.

x	−2	−1	0	0.5	1	1.5	2	2.5
y	−2.75			−1.58		−0.17	1	2.66

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2^x - 3$ for $-2 \leqslant x \leqslant 2.5$.



[4]

(c) Use your graph to solve the equation $2^x - 3 = 2$.

$x =$ .... [1]

(d) By drawing a suitable straight line, solve the equation $2^x - x - 1.5 = 0$.

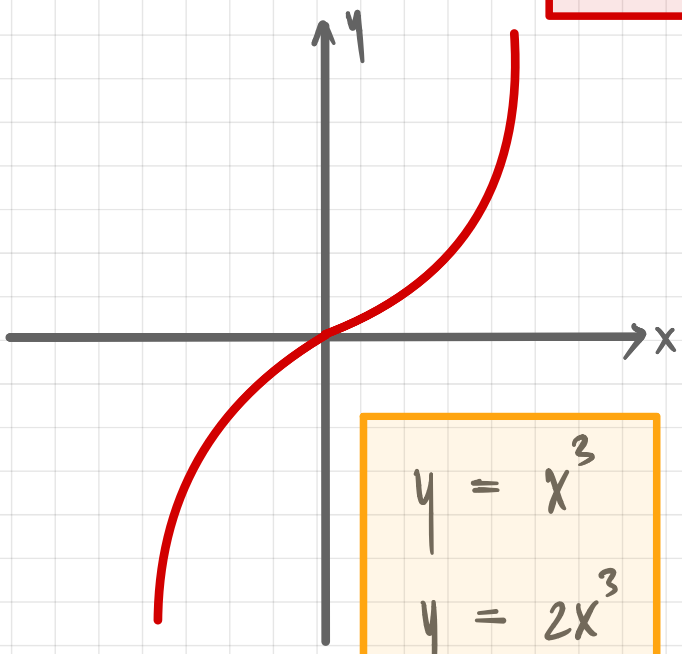
$x =$ .... or $x =$ .... [4]

Graphs: Harder Graphs.

Cubic Graphs

$$y = ax^3$$

$a > 0$

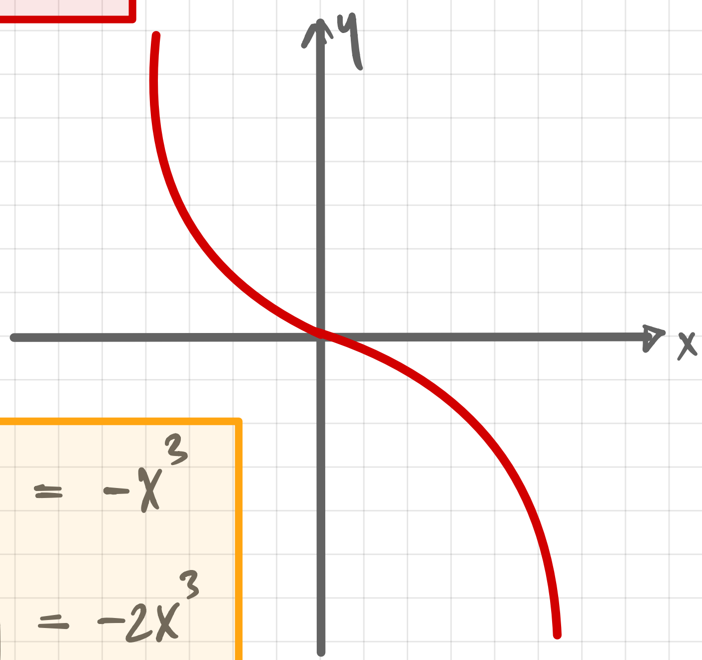


$y = x^3$

$y = 2x^3$

$y = \frac{1}{2}x^3$

$a < 0$



$y = -x^3$

$y = -2x^3$

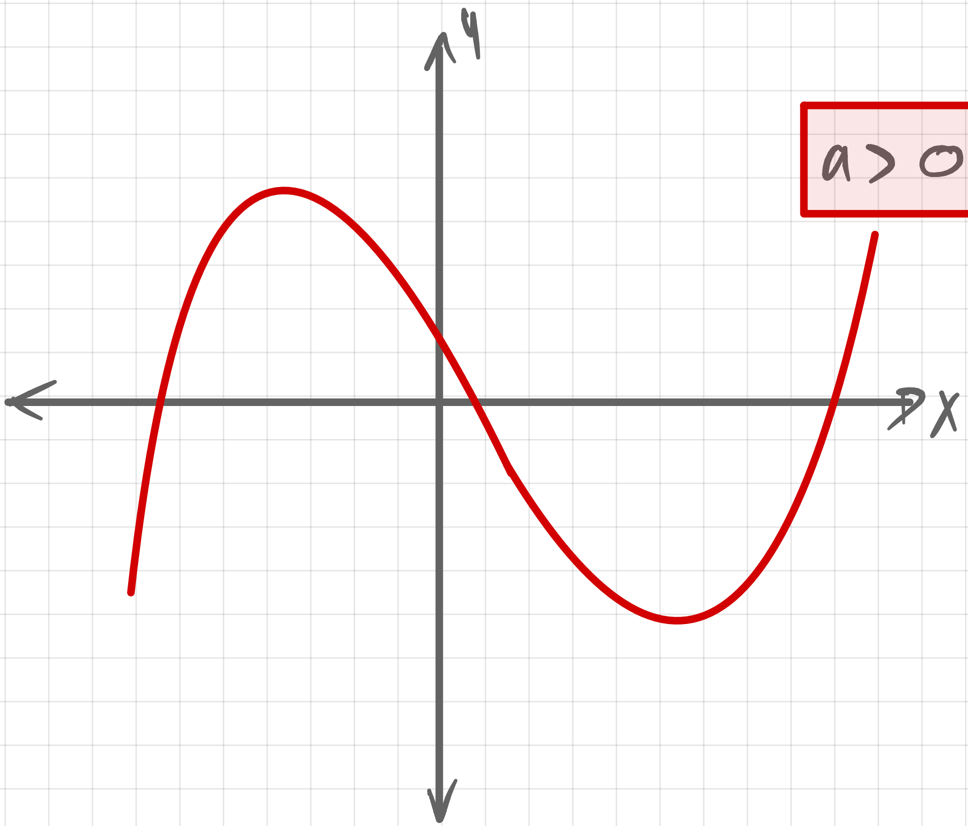
$y = -\frac{1}{2}x^3$

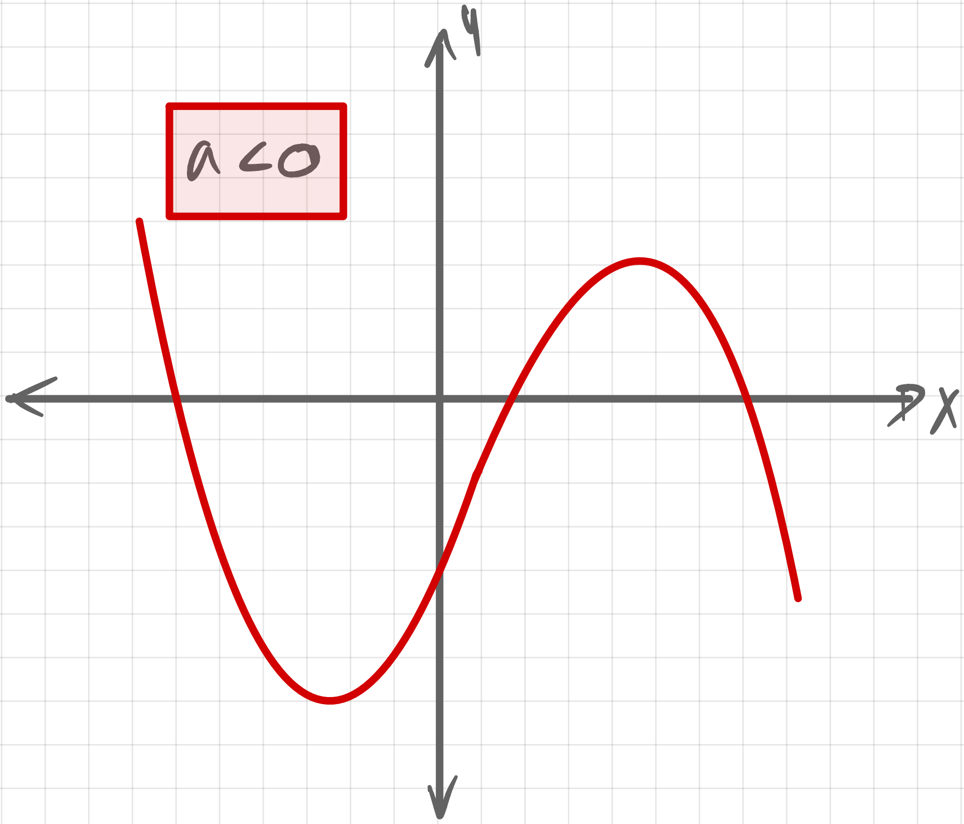
Graphs:Harder Graphs.

Cubic Graphs

$$y = ax^3 + bx^2 + cx + d$$

$$y = (\quad)(\quad)(\quad)$$





Graphs:Harder Graphs.

Example: Sketch $y = (x-2)(x+1)(x-1)$

Graphs:Harder Graphs.

Example: Sketch $y = (x+2)(3-x)(x-1)$

Graphs:Harder Graphs.

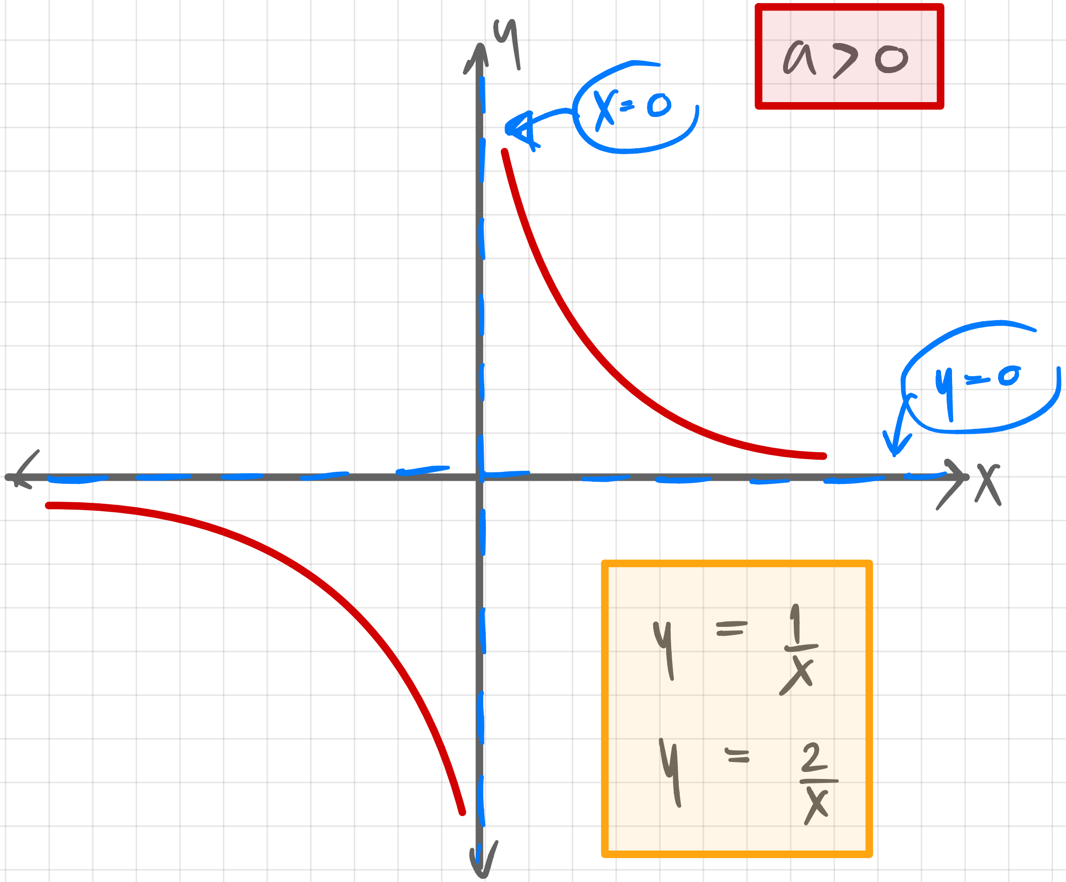
TOP Math IGCSE

Reciprocal Graphs

$y = \frac{a}{x}$ → Asymptote → $x = 0$, $y = 0$ → $xy = a$

- $xy = 5$
- $xy = -3$

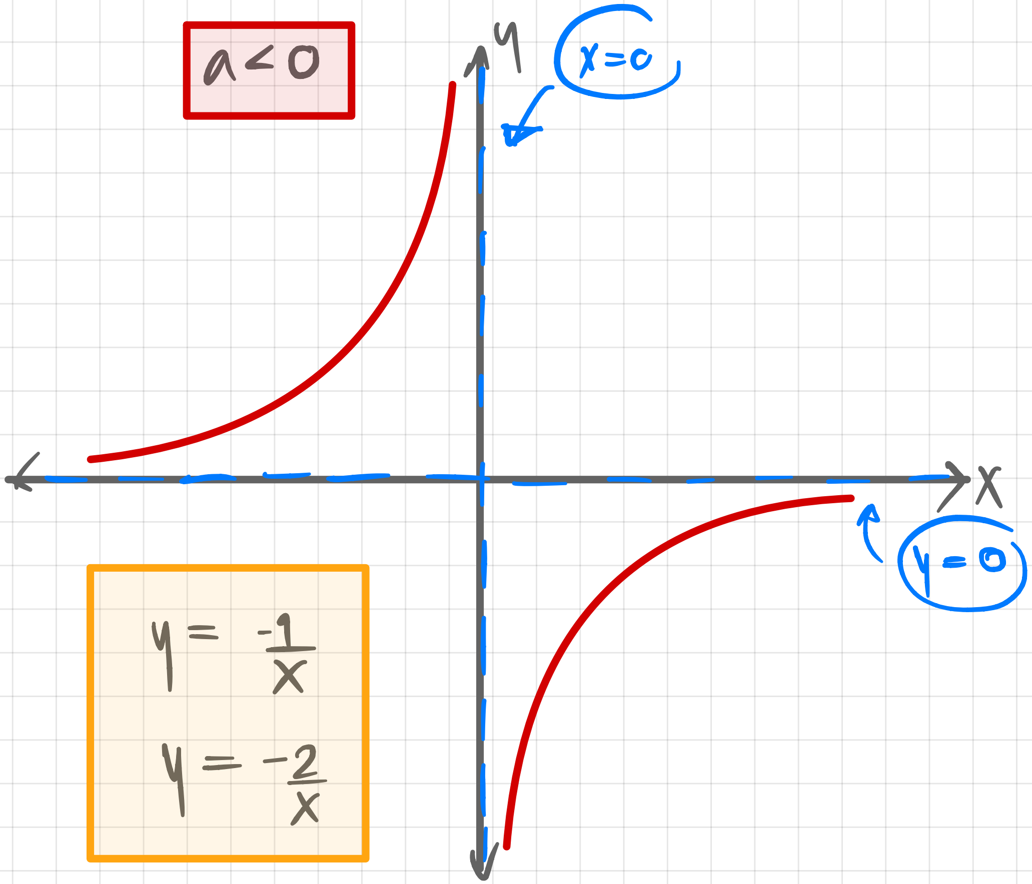
$a > 0$



$y = \frac{1}{x}$

$y = \frac{2}{x}$

$a < 0$



$y = \frac{-1}{x}$

$y = \frac{-2}{x}$

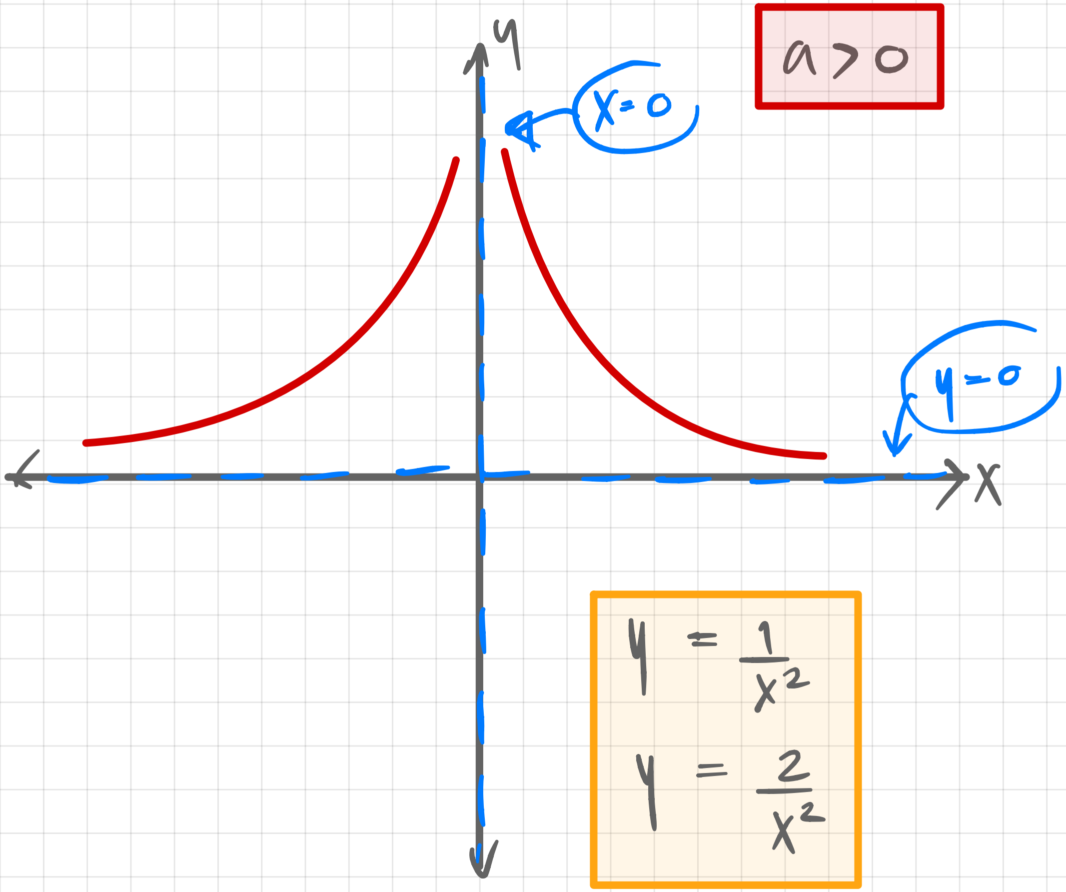
Graphs: Harder Graphs.

$$y = \frac{a}{x^2} \rightarrow \text{Asymptote} \rightarrow x = 0,\ y = 0 \rightarrow yx^2 = a$$

$yx^2 = 3$

$yx^2 = -5$

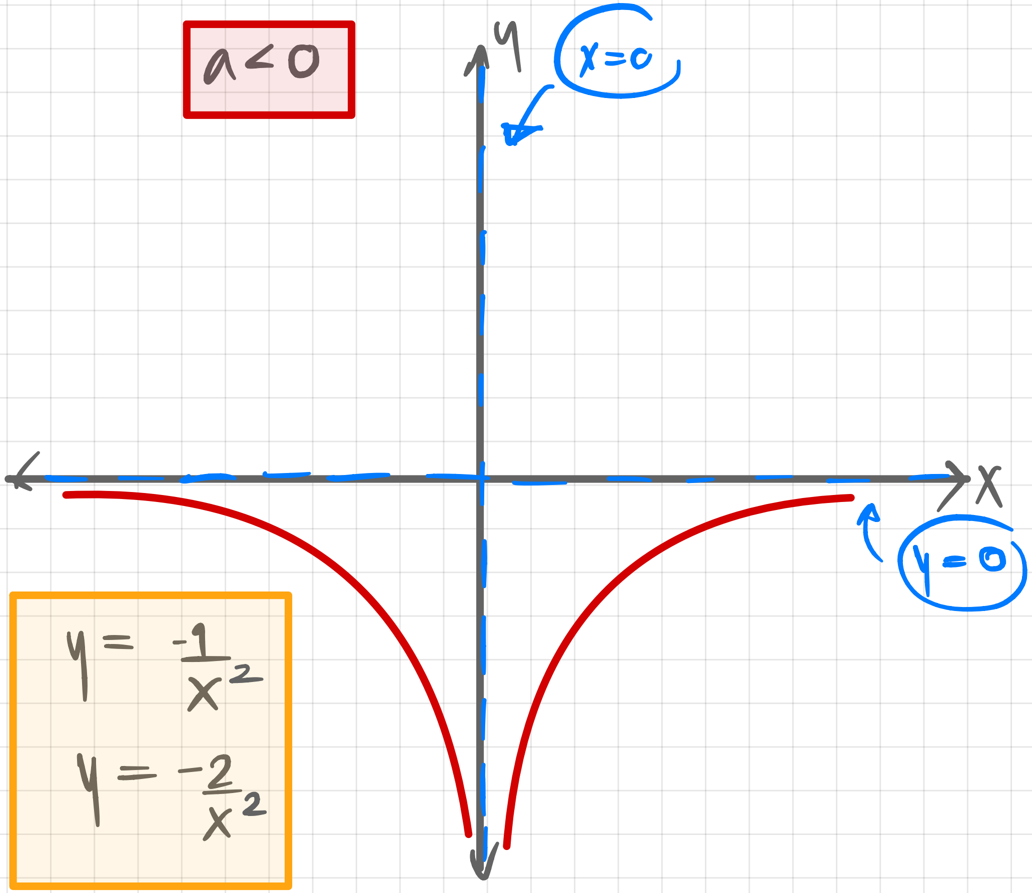
$a > 0$



$y = \frac{1}{x^2}$

$y = \frac{2}{x^2}$

$a < 0$



$y = \frac{-1}{x^2}$

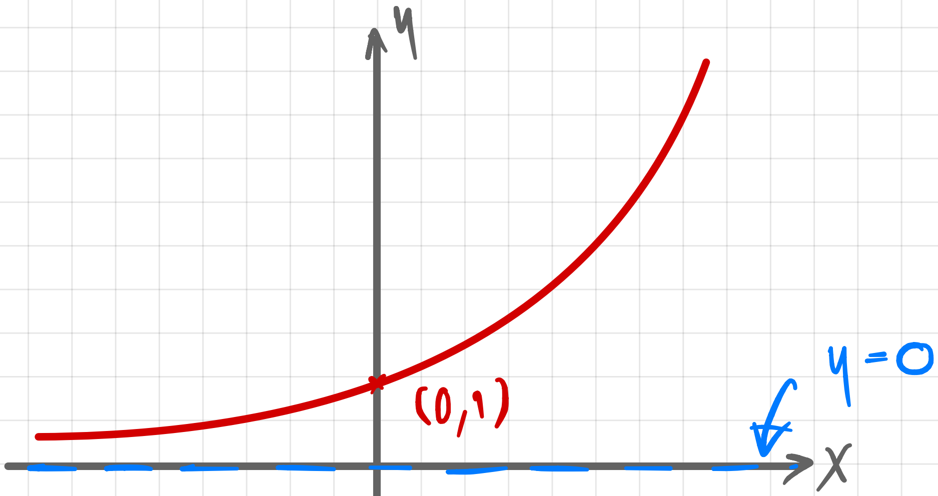
$y = \frac{-2}{x^2}$

Graphs:Harder Graphs.

Exponential Graphs

$y = a^x$ → Asymptote → $y = 0$

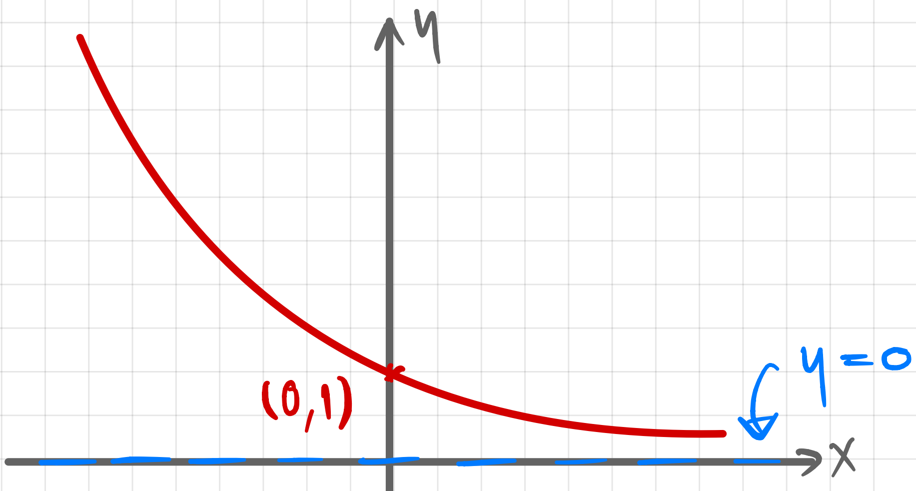
$a > 1$



$y = 2^x$

$y = e^x$

$0 < a < 1$



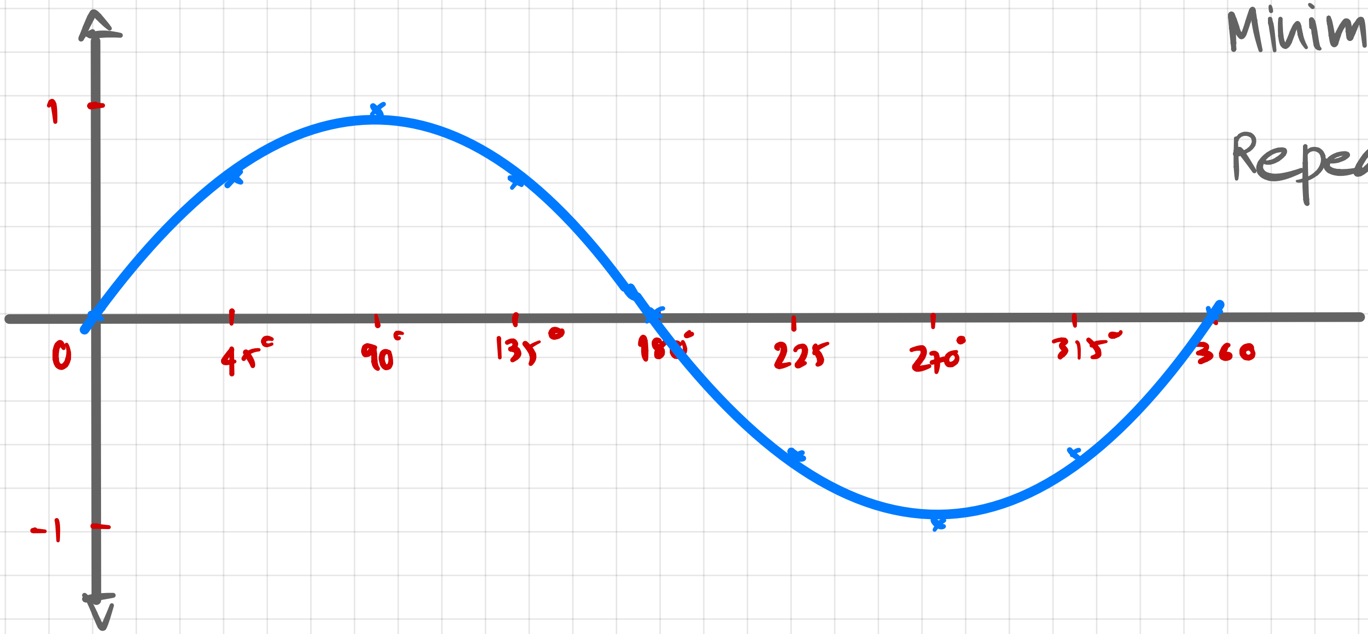
$y = \left(\frac{1}{2}\right)^x$

$y = 3^{-x}$ → $(3^{-1})^x = \left(\frac{1}{3}\right)^x$

Trigonometry Graphs

$y = \sin(x)$ for $0 \le x \le 360$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	0.7	1	0.7	0	-0.7	-1	-0.7	0



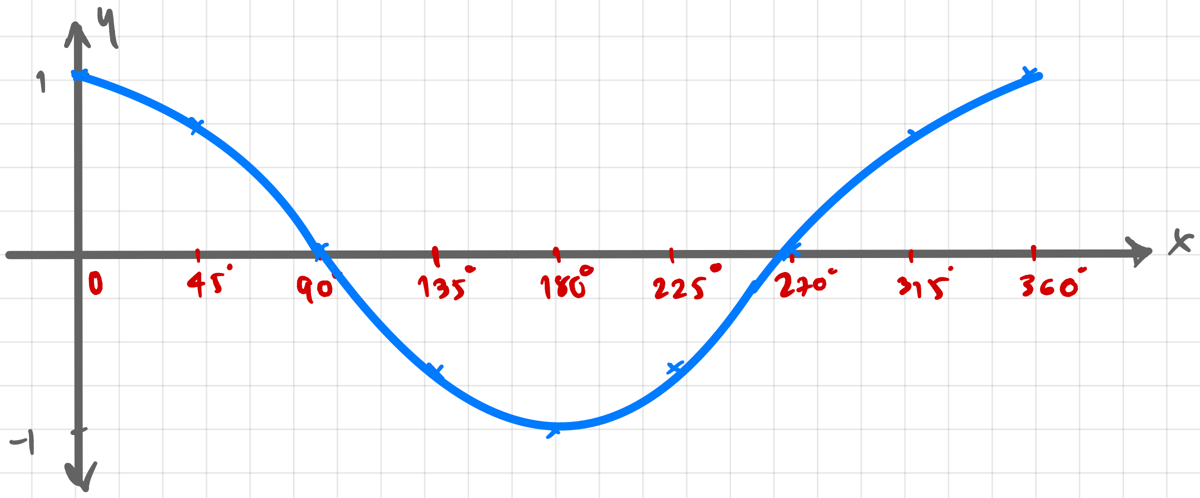
Maximum = 1
Minimum = -1
} Amplitude = 1

Repeats every 360°
↳ Period = 360°

Graphs:Harder Graphs.

$y = \cos(x)$ for $0 \leq x \leq 360$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	1	0.7	0	-0.7	-1	-0.7	0	0.7	1



Maximum = 1
Minimum = -1

} Amplitude = 1

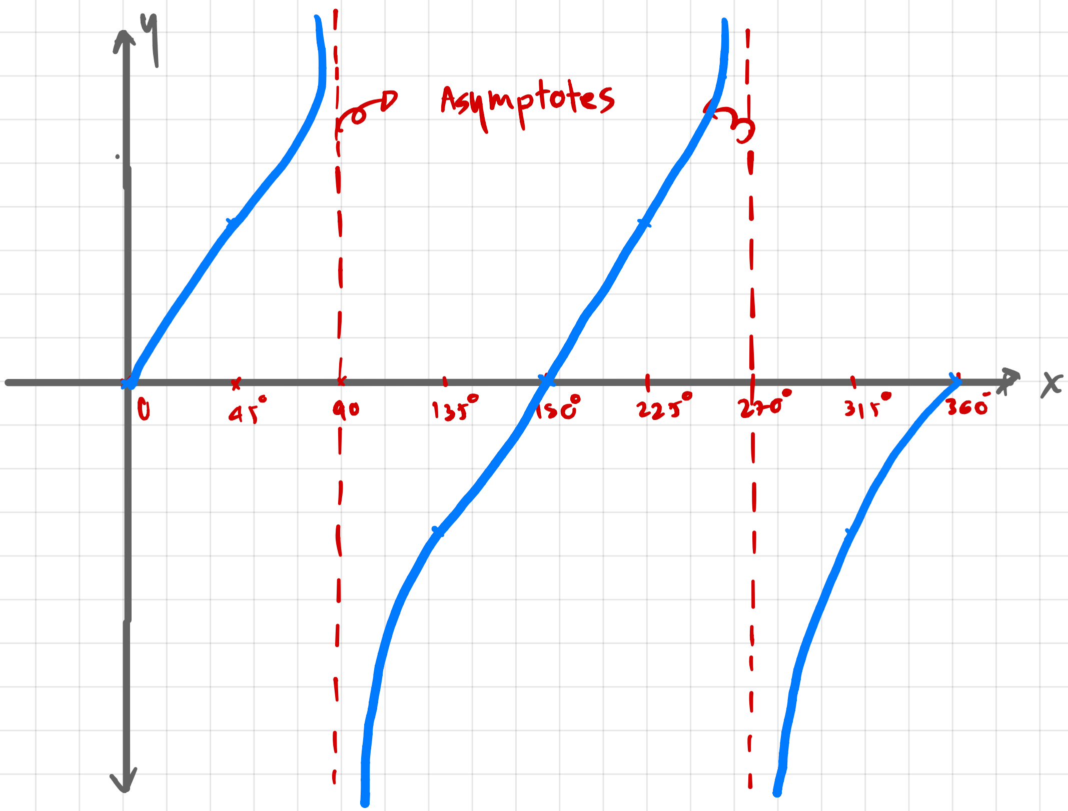
Repeats every 360°

↳ Period = 360°

Graphs:Harder Graphs.

$y = \tan(x)$ for $0 \le x \le 360$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	1	–	-1	0	1	–	-1	0



Maximum –

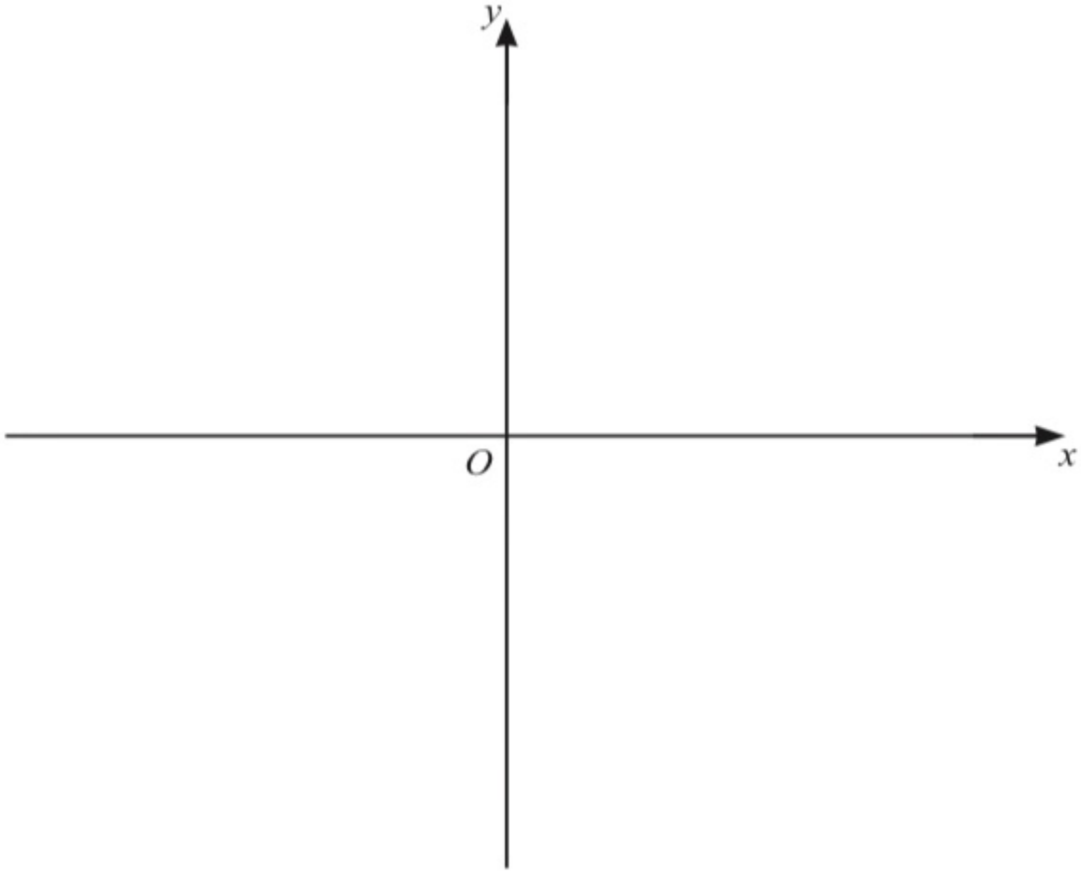
Minimum –

Repeats every = 180°

Asymptotes $x = 90$, $x = 270$

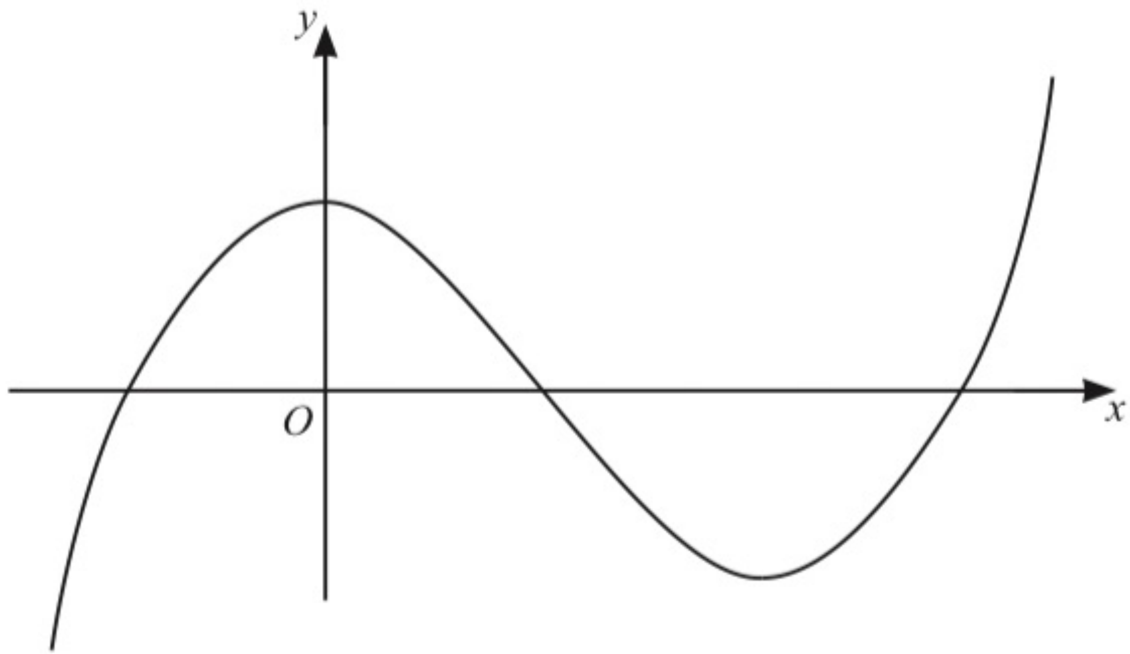
8 **(a)** **(i)** Show that the equation $y = (x-4)(x+1)(x-2)$ can be written as $y = x^3 - 5x^2 + 2x + 8$.

(ii) On the diagram, sketch the graph of $y = x^3 - 5x^2 + 2x + 8$, indicating the values where the graph crosses the axes.



[4]

7 **(a)** The diagram shows the graph of a function.

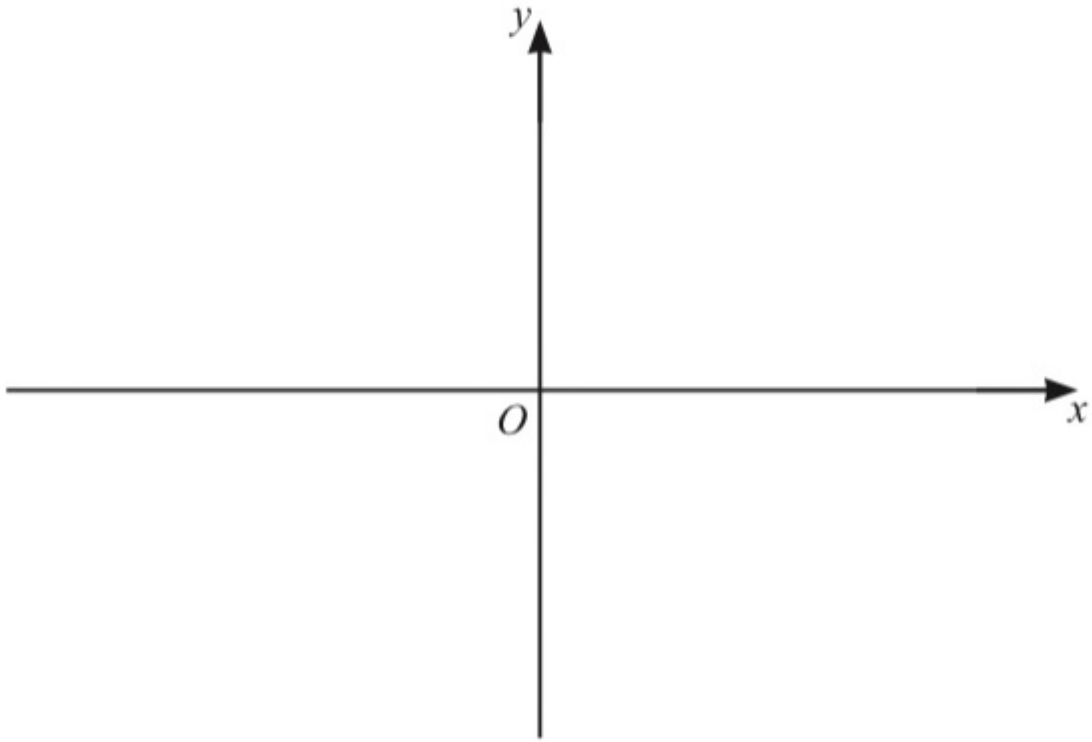


Put a ring around the word which correctly identifies the type of function.

reciprocal quadratic cubic exponential linear

[1]

(b) **(i)**

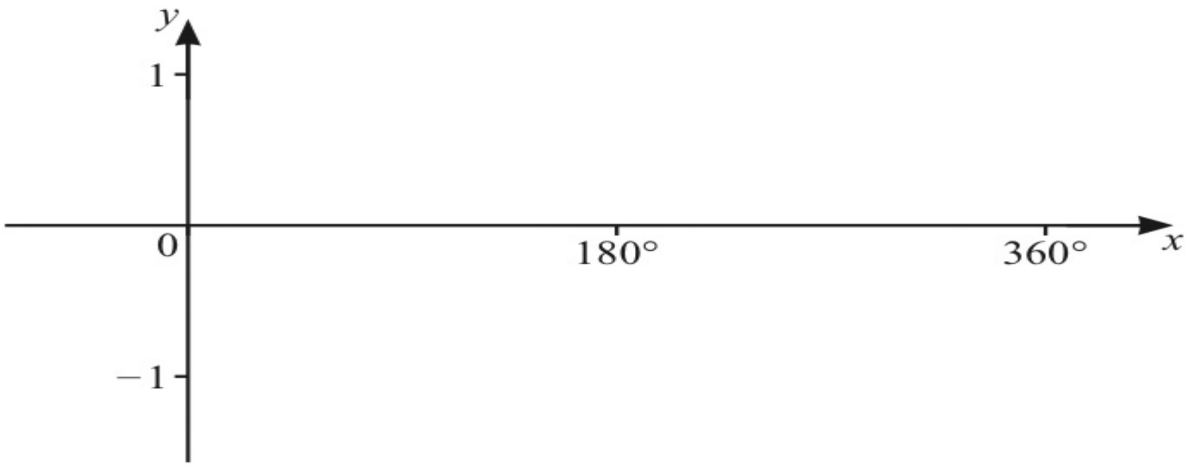


On the diagram, sketch the graph of $y = \dfrac{1}{2x}$, $x \neq 0$. [2]

(ii) Solve the equation $\dfrac{1}{2x} = 2x$.

$x =$ .... and $x =$ .... [2]

(c) (i)



On the diagram, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$. [2]

(ii) Solve the equation $3\sin x + 1 = 0$ for $0° \leqslant x \leqslant 360°$.

$x =$ .... and $x =$ .... [3]

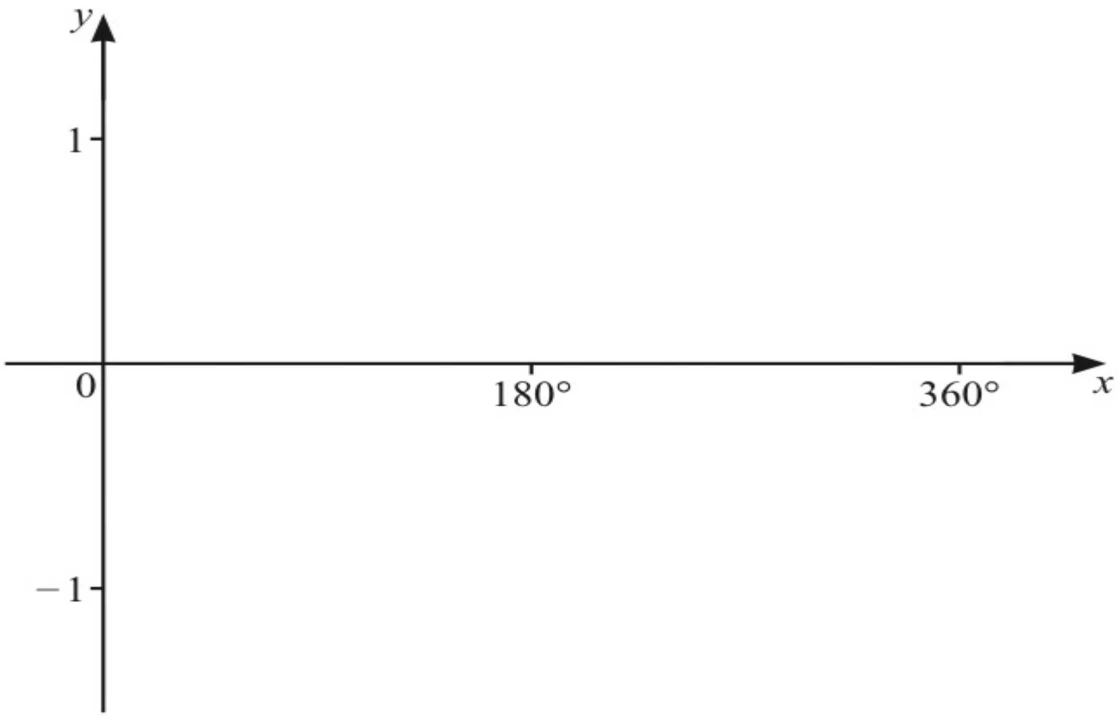
17 **(a)** Sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

[2]

(b) Solve the equation $3\sin x + 1 = 0$ for $0° \leqslant x \leqslant 360°$.

$x =$ .... or $x =$ .... [3]

19 **(a)** On the diagram, sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.



[2]

(b) Solve the equation $5\cos x + 3 = 0$ for $0° \leqslant x \leqslant 360°$.

$x =$ .... or $x =$ .... [3]

21 Solve the equation $5\sin x = -3$ for $0° \leqslant x \leqslant 360°$.

.... [3]

23 Solve the equation $3\sin x + 3 = 1$ for $0° \leqslant x \leqslant 360°$.

$x =$ .... or $x =$ .... [3]

Differentiation.

Review:Indices

$\longrightarrow ax^n$

$$a = ax^0 \longrightarrow$$

$$ax = ax^1 \longrightarrow$$

$$(x^m)(x^n) = x^{m+n} \longrightarrow$$

$$\frac{x^m}{x^n} = x^{m-n} \longrightarrow$$

$$\frac{1}{x^m} = 1x^{-m} \longrightarrow$$

$$\sqrt{x} = x^{\frac{1}{2}} \longrightarrow$$

$$\sqrt[3]{x} = x^{\frac{1}{3}} \longrightarrow$$

Differentiation.

Differentiation:

$$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots$$

$$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots$$

$$ax^{n} \xrightarrow{D} a(n)x^{n-1}$$

$$3x^{5} \xrightarrow{D}$$

$$-4x^{\frac{1}{2}} \xrightarrow{D}$$

$$2x^{-3} \xrightarrow{D}$$

Differentiation.

$ax \xrightarrow{D} a$

Prove! $ax = ax^1 \xrightarrow{D} a(1)x^{1-1}$

$ax^0 = a$

$5x \xrightarrow{D}$

$-100x \xrightarrow{D}$

$\frac{2}{3}x \xrightarrow{D}$

Differentiation.

$a \xrightarrow{D} 0$

$ax^n \xrightarrow{D} a(n)x^{n-1}$

Prove ! $a = ax^0 \xrightarrow{D} a(0)x^{0-1}$

$= 0$

$5 \xrightarrow{D}$

$-100 \xrightarrow{D}$

$\frac{3}{2} \xrightarrow{D}$

Differentiation.

Example:Differentiation

$$y = 3x^4 + 5x - 4x^{\frac{1}{2}} + 1 - 2x^{-1}$$

Differentiation.

Example:Differentiation

$$f(x) = (3x - 5)^2$$

Differentiation.

Example:Differentiation

$$y = (3x+1)(x-1)(x+2)$$

Differentiation.

Example:Differentiation

$$f(x) = 4\sqrt{x} + \frac{5}{2x}$$

Differentiation.

Example:Differentiation

$$y = \frac{3x^2 + 4x - 6}{2x^3}$$

Differentiation.

Second order derivative

$$y = x\ldots \xrightarrow{D} \frac{dy}{dx} = \ldots \xrightarrow{D} \frac{d^2y}{dx^2} = \ldots$$

$$f(x) = x\ldots \xrightarrow{D} f'(x) = \ldots \xrightarrow{D} f''(x) = \ldots$$

Differentiation.

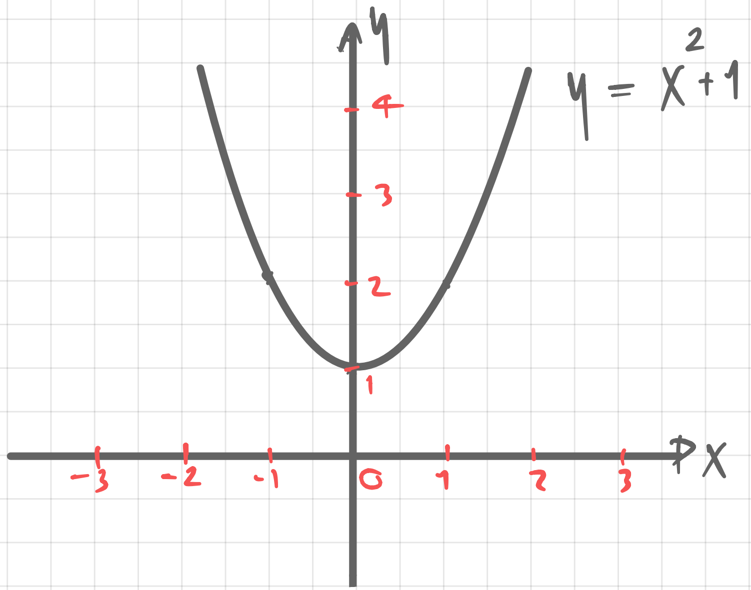
Example: Find second order derivative.

1) $y = 5x^2 + 3x - 4$

2) $y = x^3 - 4x^2 + 5$

Differentiation.

Differentiation to find a Gradient.



1) Differentiating the equation of curve.

2) Find the gradient of the curve at any point by substituting the value for x into 1)...

Find the gradient of curve at $x = 1$

Differentiation.

Example: Find the gradient of the graph $y = x^3 - 12x + 1$ at .....

1) $x = -1$

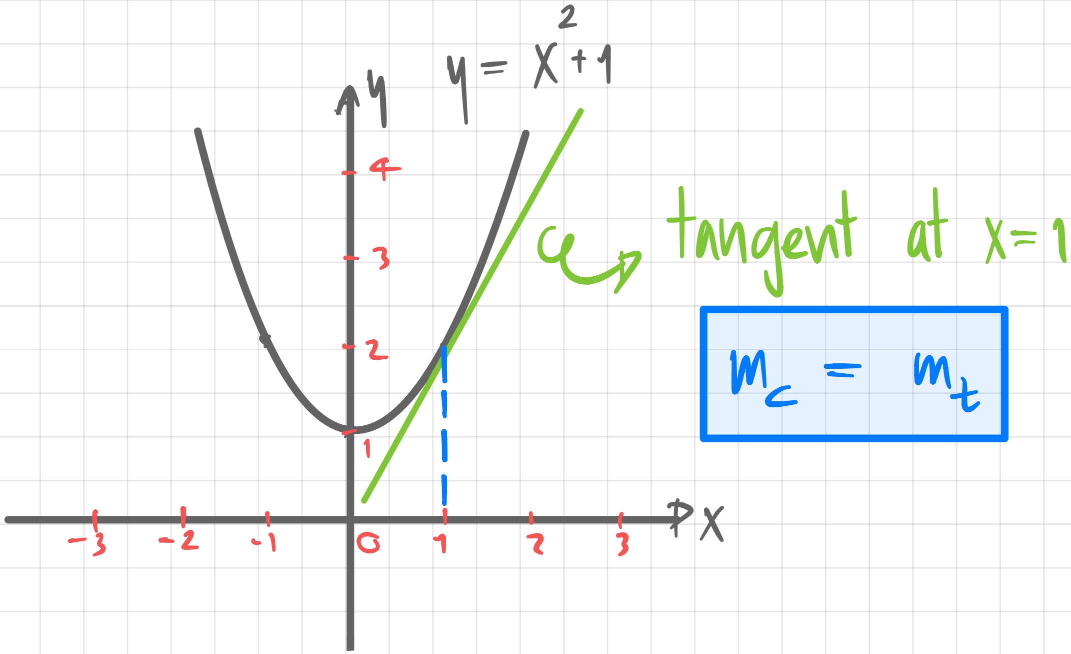
2) $x = 2$

3) $x = \frac{5}{2}$

Example Find coordinate of point of curve $y = x^2 + 1$ has gradient is -4.

Differentiation.

Gradient of the curve = Gradient of the tangent.



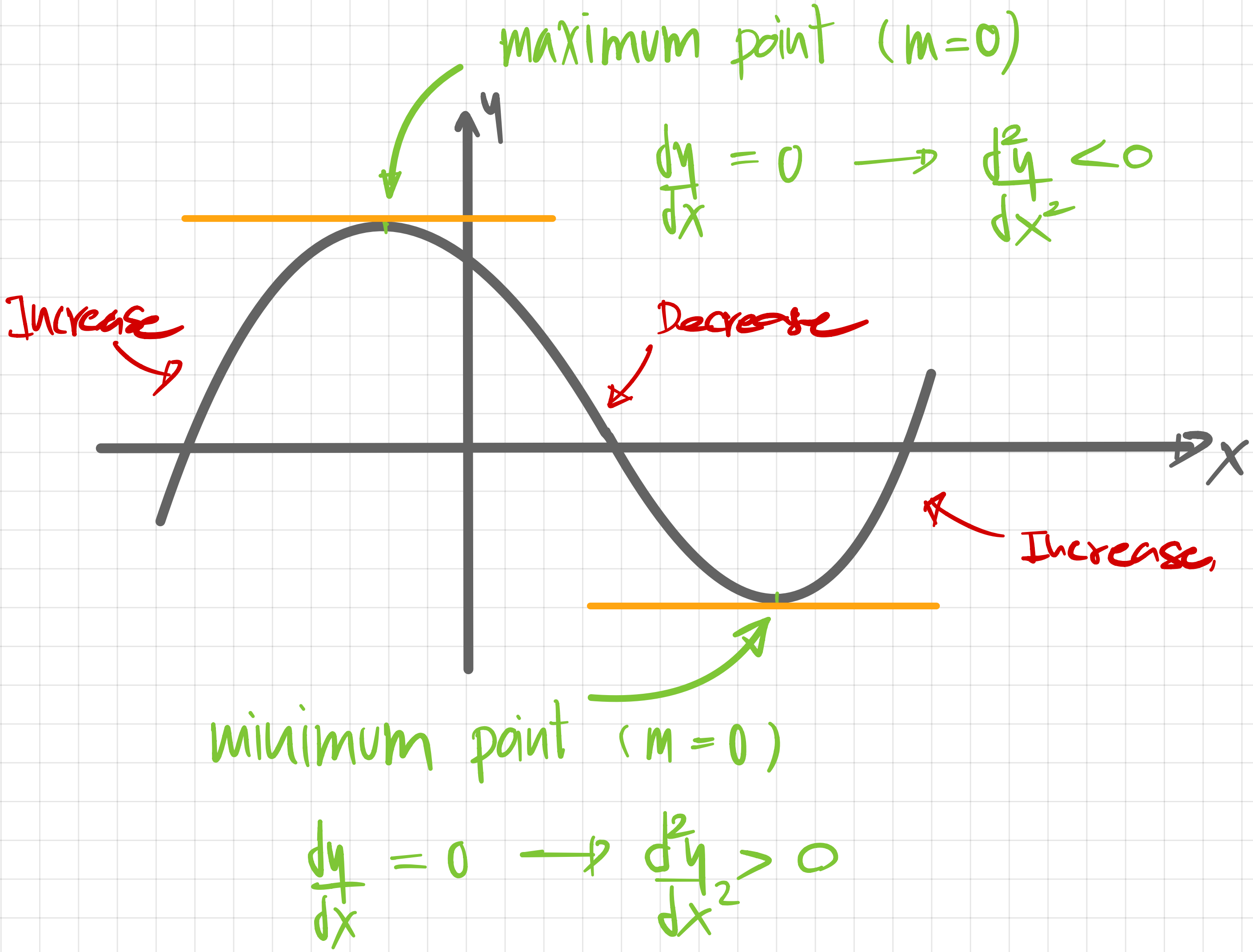
$m_c = m_t$

Find equation of tangent to curve $y = x^2 + 1$ at point $(1, 2)$

Differentiation.

Example: Find equation of tangent to curve $y = \frac{4}{x} + 1$ at $x = 2$

Differentiation for Stationary points (Turning points)



Differentiation.

Example Find the stationary point of curve $y = x^2 + 4x - 3$ and determine.

1)Differentiate

2)Differentiate=0 ; Solve for x

3)Substitute x to Original equation to find y for coordinates

4)Second order derivative for Max or Min

Differentiation.

Example Find the stationary point of curve $y = -x^2 - 4x + 1$ and determine.

1)Differentiate

2)Differentiate=0 ; Solve for x

3)Substitute x to Original equation to find y for coordinates

4)Second order derivative for Max or Min

21 A curve has equation $y = x^3 - 12x$.

(a) Find the gradient of the curve at the point (1,−11).

.... [3]

(b) Find the coordinates of the turning points of the curve.

(............... , ...) and (............... , ...) [3]

12 The equation of a curve is $y = x^4 - 8x^2 + 5$.

(a) Find the derivative, $\left(\frac{dy}{dx}\right)$, of $y = x^4 - 8x^2 + 5$.

.... [2]

(b) Find the coordinates of the three turning points.
You must show all your working.

(.......... , ...) and (.......... , ...) and (.......... , ...) [4]

(c) Determine which one of these turning points is a maximum.
Justify your answer.

[2]

(b) The graph of $y = x^3 - 5x^2 + 2x + 8$ has two tangents with a gradient of 10.

Find the equations of these two tangents.
You must show all your working and give your answers in the form $y = mx + c$.

$y =$ ....

$y =$ .... [7]

12 A curve has equation $y = x^3 - kx^2 + 1$.
When $x = 2$, the gradient of the curve is 6.

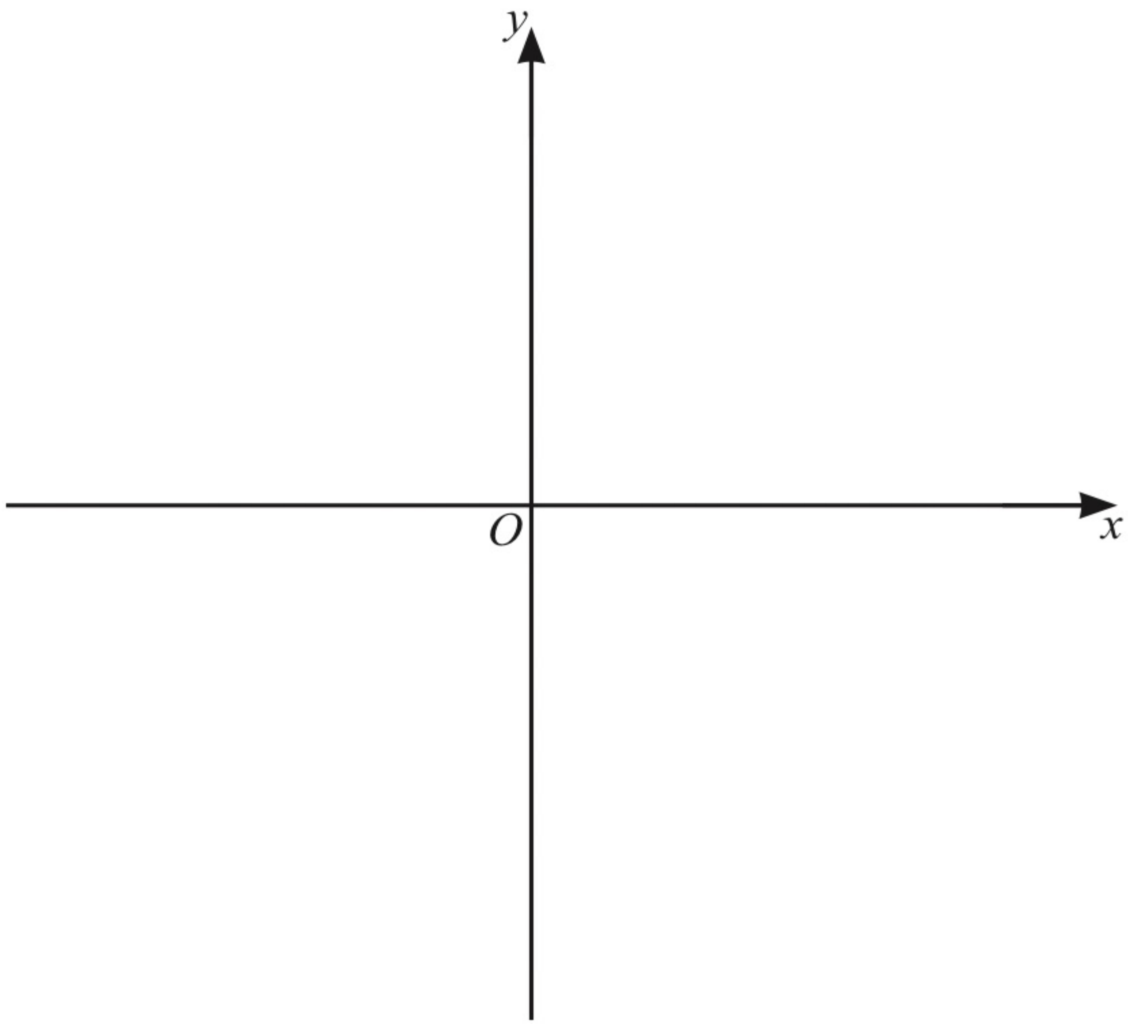
(a) Show that $k = 1.5$.

[5]

(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

(................, ...) and (................, ...) [4]

(c) Sketch the curve $y = x^3 - 1.5x^2 + 1$.



[2]

20 $\qquad f(x) = 6x - 7 \qquad\qquad g(x) = x^{-3}$

(a) Find $f(x+2)$.
Give your answer in its simplest form.

.... [2]

(b) Find $f^{-1}(x)$.

$f^{-1}(x) =$ .... [2]

20 $f(x) = 6x - 7$ $g(x) = x^{-3}$

(c) Find x when $g(x) = f(22)$.

$x =$ .... [2]

10 $f(x) = x - 4$ $g(x) = 2x + 5$ $h(x) = 3^x$

(a) Find

(i) $f(-3)$

.... [1]

(ii) $g^{-1}(x)$

$g^{-1}(x) =$ .... [2]

(iii) $f(x) \times g(x) \times f(x)$.

.... [4]

(b) Find x when $h(x) = g(f(2))$.

$x =$ .... [2]

19 $\mathrm{f}(x) = 7x - 8$ $\mathrm{g}(x) = \dfrac{4}{x} + 5$ $\mathrm{h}(x) = 2^x + 1$

(a) Find $\mathrm{f}^{-1}(x)$.

$\mathrm{f}^{-1}(x) =$ .... [2]

(b) Find the value of x when $\mathrm{h}(x) = \mathrm{g}\left(\dfrac{1}{3}\right)$.

$x =$ .... [2]

4 $\quad f(x) = 2x - 1 \qquad g(x) = 3x - 2 \qquad h(x) = \frac{1}{x},\ x \neq 0 \qquad j(x) = 5^x$

(a) Find

(i) $f(2)$,

.... [1]

(ii) $gf(2)$.

.... [1]

4 $f(x) = 2x - 1$ $g(x) = 3x - 2$ $h(x) = \frac{1}{x}, \; x \neq 0$ $j(x) = 5^x$

(b) Find $g^{-1}(x)$.

$g^{-1}(x) =$ .... [2]

4 $f(x) = 2x - 1$ $g(x) = 3x - 2$ $h(x) = \frac{1}{x},\ x \neq 0$ $j(x) = 5^x$

(c) Find x when $h(x) = j(-2)$.

$x =$ .... [2]

4 $\quad f(x) = 2x - 1 \qquad g(x) = 3x - 2 \qquad h(x) = \frac{1}{x},\ x \neq 0 \qquad j(x) = 5^x$

(d) Write $f(x) - h(x)$ as a single fraction.

.... [2]

4 $f(x) = 2x - 1$ $g(x) = 3x - 2$ $h(x) = \frac{1}{x},\ x \neq 0$ $j(x) = 5^x$

(e) Find the value of $jj(2)$.

.... [1]

4 $f(x) = 2x - 1$ $g(x) = 3x - 2$ $h(x) = \frac{1}{x},\ x \neq 0$ $j(x) = 5^x$

(f) Find x when $j^{-1}(x) = 4$.

$x =$ .... [2]

Functions.

Extra:

Domain $\longrightarrow x$

Range $\longrightarrow y$

$$\left.\begin{matrix} \text{Domain} \longrightarrow x \\ \text{Range} \longrightarrow y \end{matrix}\right\} \quad \boxed{f(x)} = \boxed{3x+1}$$

$f(A) = 18 \longrightarrow$

$f(2) = B \longrightarrow$

Functions.

Inverse Function

$f(x) = 3x + 1$

Range of $f(x) \longrightarrow$ Domain of $f^{-1}(x)$

Domain of $f(x) \longrightarrow$ Range of $f^{-1}(x)$

Inverse function when can't find inverse function:

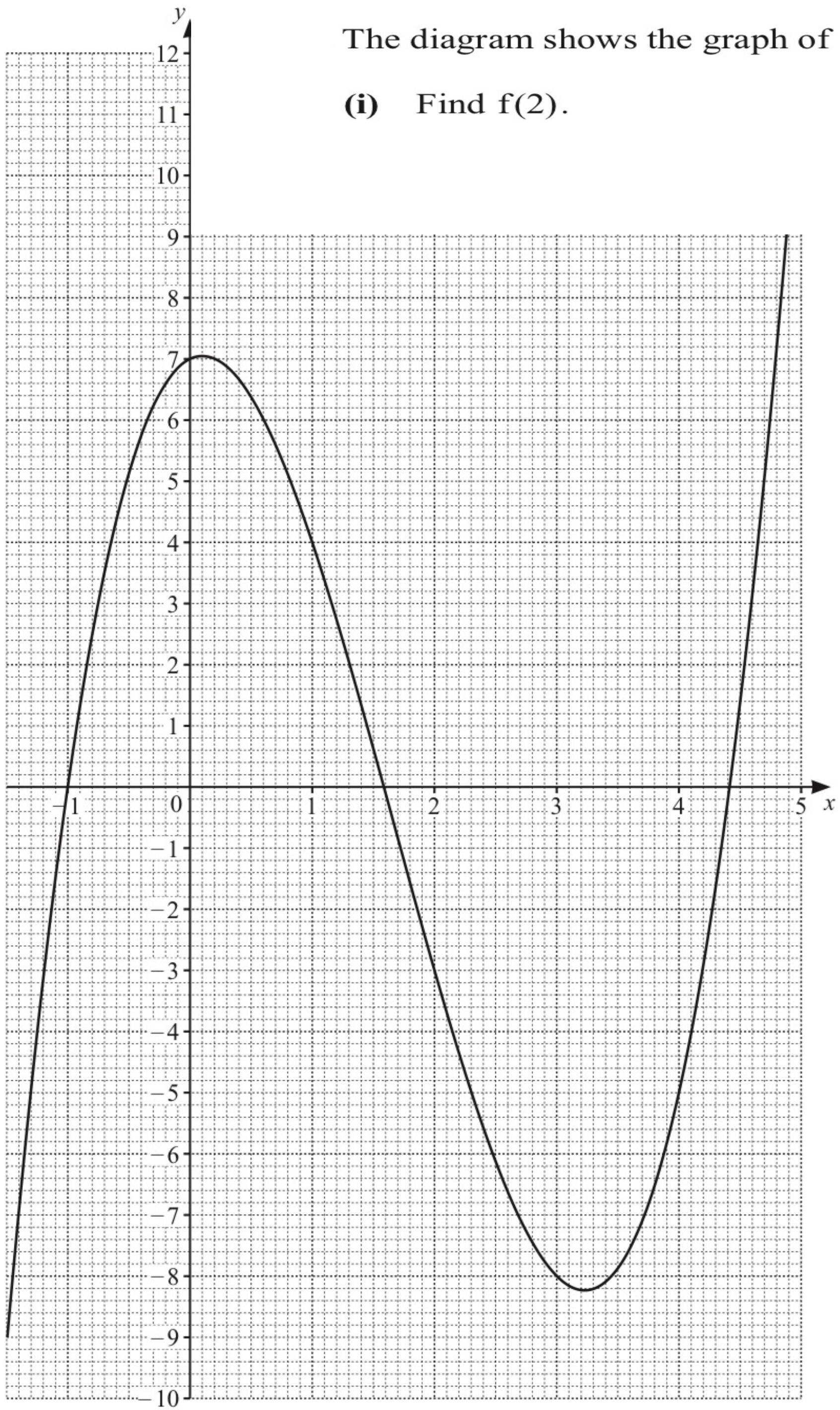
Example: $f(x) = 5^x$

Find value of x when $f^{-1}(x) = 2$

Example: $h(x) = 2^{x+1}$

Find value of $h^{-1}(32)$

6 (a)

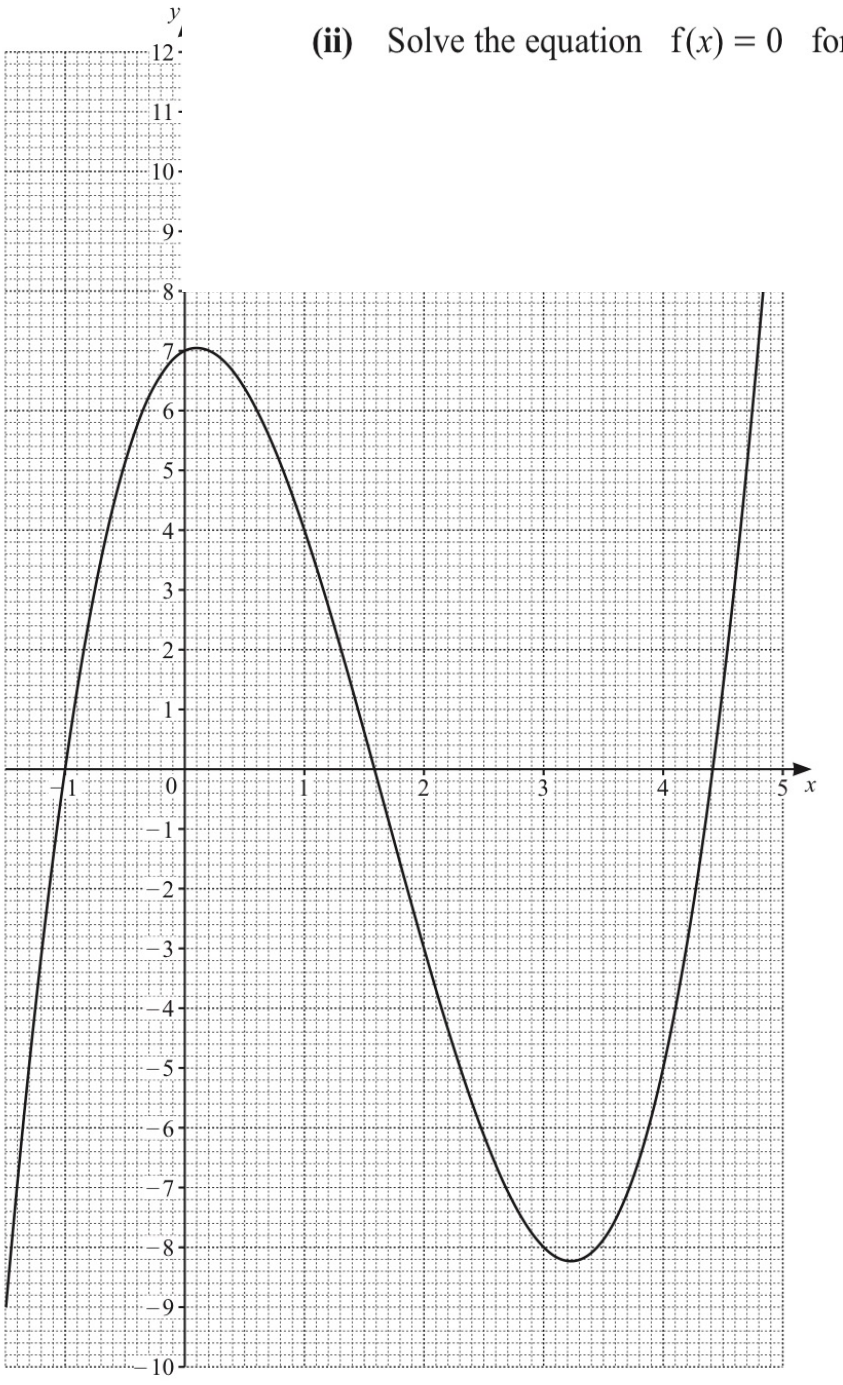


The diagram shows the graph of $y = \mathrm{f}(x)$ for $-1.5 \leqslant x \leqslant 5$.

(i) Find f(2).

.... [1]

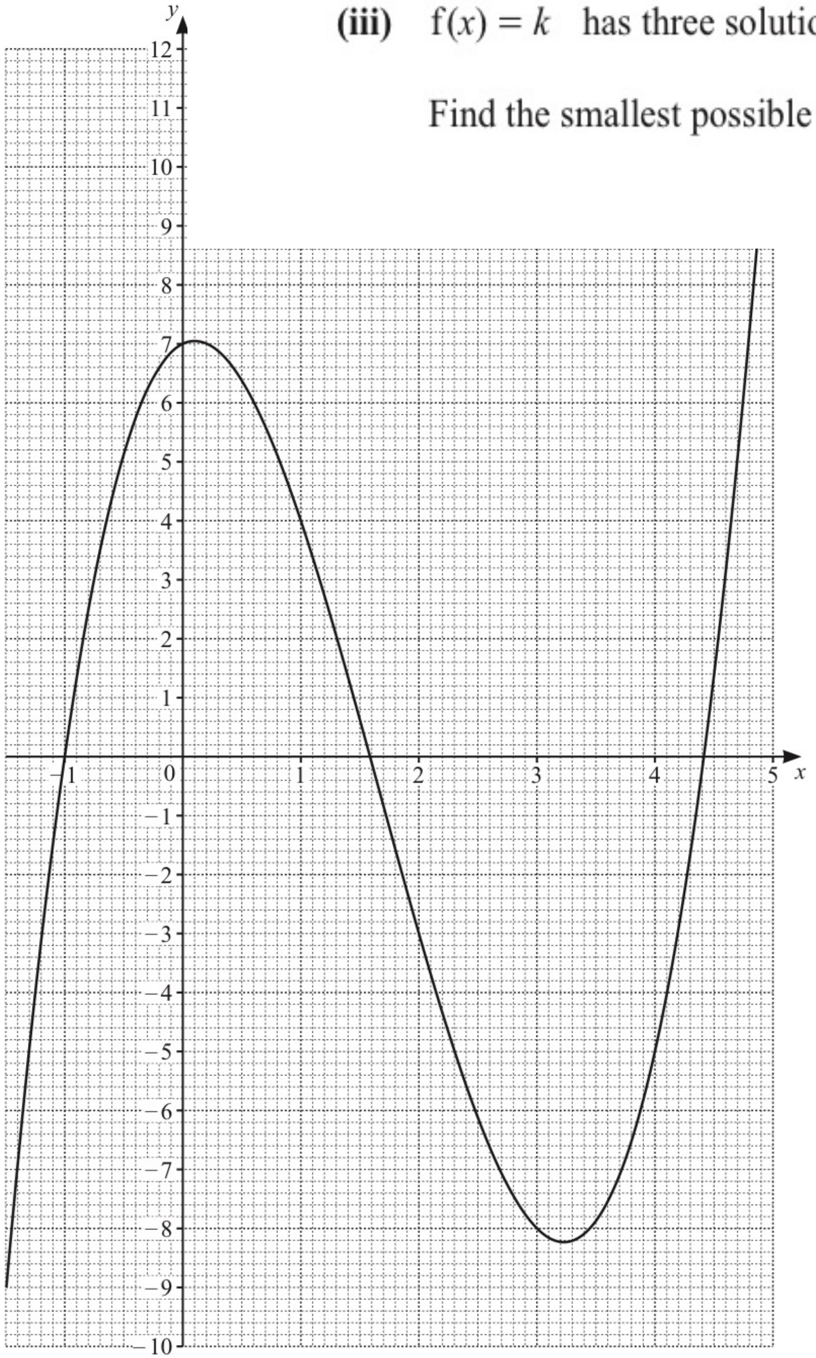
6 **(a)**



(ii) Solve the equation $f(x) = 0$ for $-1.5 \leqslant x \leqslant 5$.

$x =$ .... or $x =$ .... or $x =$ .... [3]

6 (a)



(iii) $f(x) = k$ has three solutions for $-1.5 \leqslant x \leqslant 5$ where k is an integer.

Find the smallest possible value of k.

$k =$ .... [1]

(b) $y = 3x^2 - 12x + 7$

(i) Find the value of $\frac{dy}{dx}$ when $x = 5$.

.... [3]

(ii) Find the coordinates of the point on the graph of $y = 3x^2 - 12x + 7$ where the gradient is 0.

(........................ , ...) [2]

(c) When $y = 2x^p + qx^2$, $\dfrac{dy}{dx} = 14x^6 + 6x$.

Find the value of p and the value of q.

$p =$ ....

$q =$ .... [2]

3 A line, l, joins point F (3, 2) and point G (− 5, 4).

(a) Calculate the length of line l.

.... [3]

(b) Find the equation of the perpendicular bisector of line l in the form $y = mx + c$.

$y =$ .... [5]

(c) A point H lies on the y-axis such that the distance $GH = 13$ units.

Find the coordinates of the two possible positions of H.

(................ , ...) and (................ , ...) [4]

8 **(a)** A has coordinates $(-2,7)$, B has coordinates $(1,-5)$ and C has coordinates $(5,4)$.

(v) Find the equation of the line perpendicular to AB that passes through C.
Give your answer in the form $y = mx + c$.

$y =$ .... [3]

11 M has coordinates (4, 1) and N has coordinates (−2, −7).

(a) Find the length of MN.

.... [3]

(b) Find the gradient of MN.

.... [2]

(c) Find the equation of the perpendicular bisector of MN.

.... [4]