

MATHS ONLINE



By: Kru Tar

9/A*
TOP MATHS



Kru Tar
Day 2

คอร์สตะลุยโจทย์
Intensive Maths IGCSE

BOOSTER
OCT/NOV 2023

TOPMaths
A* Level



1.8 Set and Venn diagram

6.1 Probability

6.2 Probability from Venn diagrams

6.10 Tree diagrams

4.5 Congruence

4.6 Similarity

6.3 Averages and range

6.4 Mean of frequency table

6.5 Quartiles, Cumulative frequency and Box plots

6.6 Histograms

6.7 Pie charts

6.8 Stem and Leaf diagrams

6.9 Scatter diagrams

5.4 Vectors

TOPICS



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6.4 Mean of frequency table

Ex.

x	frequency
15	3
15.5	17
16	29
16.5	34
17	12

1) Mean

5) LQ

2) Median

6) UQ

3) Mode

7) IQR

4) Range

8) The 60th percentile



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6.4 Mean of frequency table

Class interval

Ex.

x	frequency
30-31	2
32-33	25
34-36	30
37-39	13

1) Mean

5) LQ

2) Median

6) UQ

3) Mode

7) IQR

4) Range

8) The 60th percentile



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6.4 Mean of frequency table

- 17 Some students were asked how many books they each had in their school bags. The table shows some of this information.

Number of books	5	6	7	8	9	10
Frequency	4	5	x	11	7	5

The mean number of books is 7.6 .

Calculate the value of x .

$$x = \dots\dots\dots [3]$$



- 6 (b) Roberto records the value of each of the coins he has at home.
The table shows the results.

Value (cents)	1	2	5	10	20	50
Frequency	3	1	3	2	4	2

- (iv) Work out the total value of Roberto's coins.

..... cents [2]

- (v) Work out the mean.

..... cents [1]



- 3 Kai and Ann carry out a survey on the distances travelled, in kilometres, by 200 cars.

Kai completes this frequency table for the data collected.

Distance (d km)	$80 < d \leq 100$	$100 < d \leq 150$	$150 < d \leq 200$	$200 < d \leq 300$	$300 < d \leq 400$
Frequency	7	33	76	52	32

- (a) (i) Calculate an estimate of the mean.

..... km [4]



- 3 Kai and Ann carry out a survey on the distances travelled, in kilometres, by 200 cars.

Kai completes this frequency table for the data collected.

Distance (d km)	$80 < d \leq 100$	$100 < d \leq 150$	$150 < d \leq 200$	$200 < d \leq 300$	$300 < d \leq 400$
Frequency	7	33	76	52	32

- (ii) Ann uses this frequency table for the same data.
There is a different interval for the final group.

Distance (d km)	$80 < d \leq 100$	$100 < d \leq 150$	$150 < d \leq 200$	$200 < d \leq 300$	$300 < d \leq 360$
Frequency	7	33	76	52	32

Without calculating an estimate of the mean for this data, find the difference between Ann's and Kai's estimate of the mean.
You must show all your working.

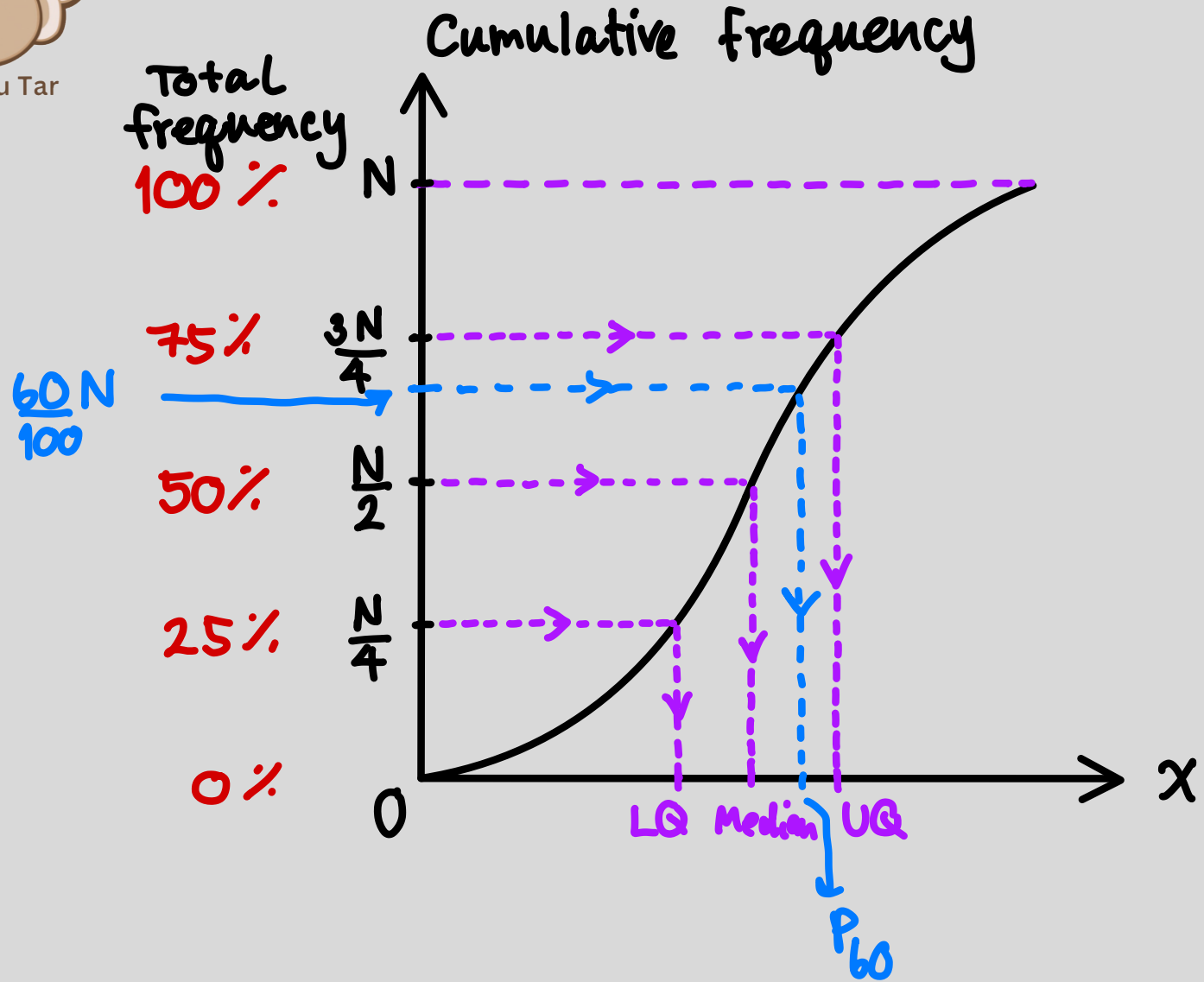
..... km [2]





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6.5 Quartiles, Cumulative frequency and Box plots



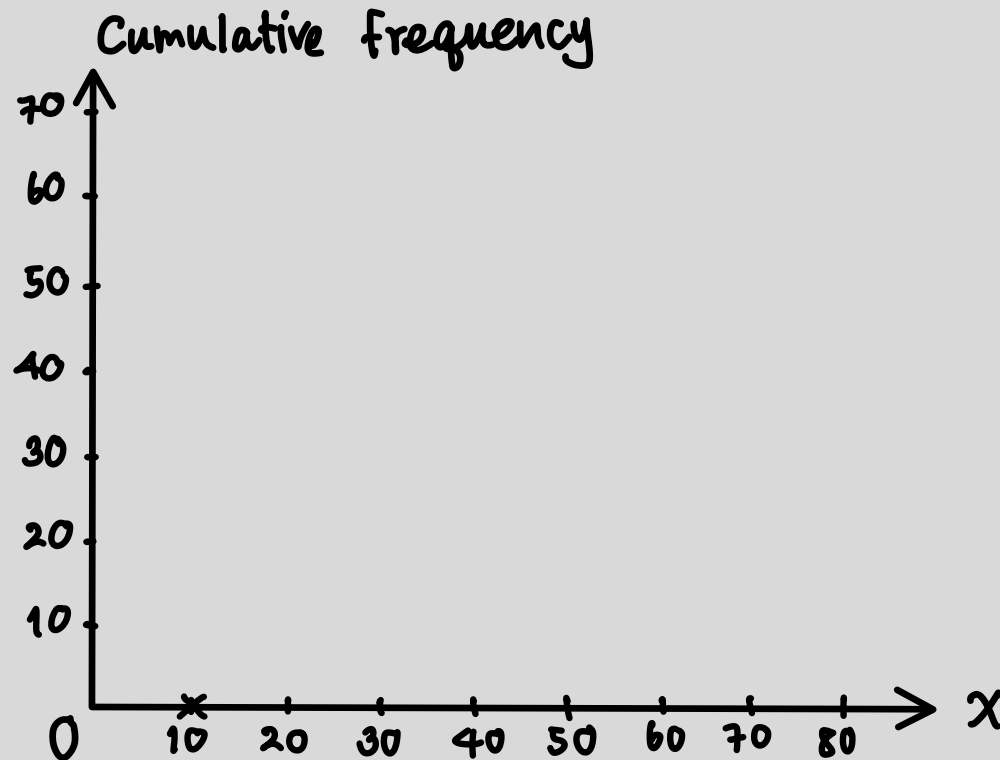


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6.5 Quartiles, Cumulative frequency and Box plots

x	frequency
10 - 20	2
20 - 35	25
35 - 60	30
60 - 80	13

x	Cumulative frequency
$x \leq 20$	
$x \leq 35$	
$x \leq 60$	
$x \leq 80$	



1) Median

2) LQ

3) UQ

4) IQR

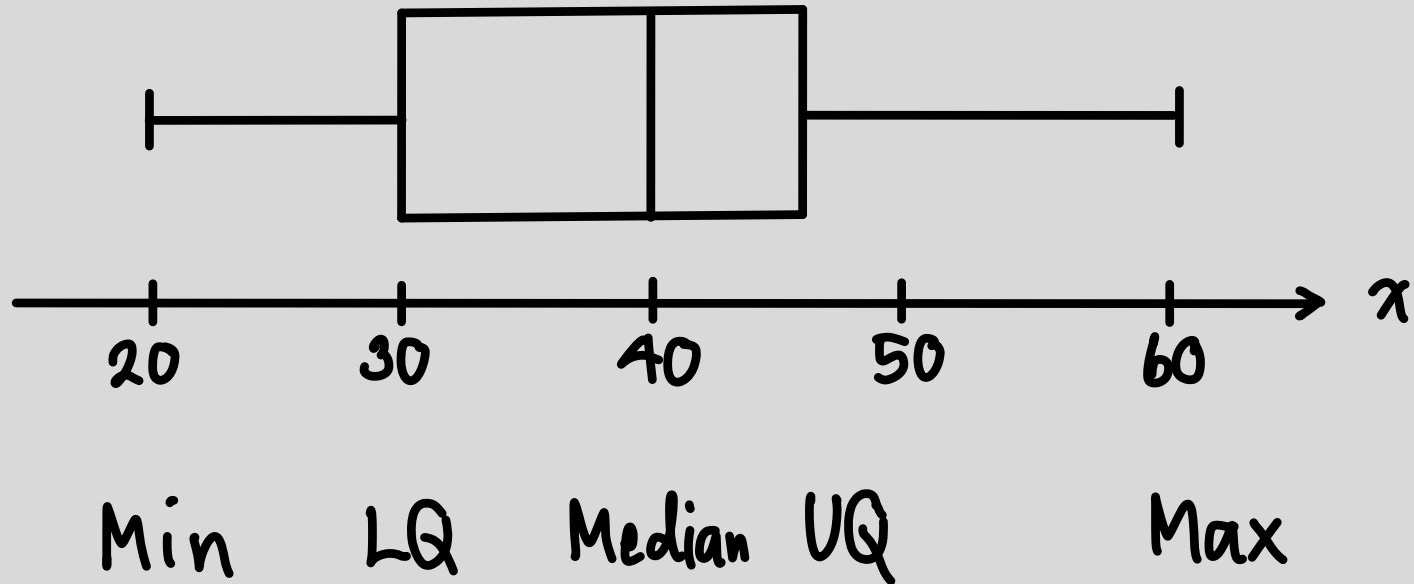
5) The 60th percentile



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6.5 Quartiles, Cumulative frequency and Box plots

Box plots





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6.5 Quartiles, Cumulative frequency and Box plots



5 The table shows information about the mass, m grams, of each of 120 letters.

Mass (m grams)	$0 < m \leq 50$	$50 < m \leq 100$	$100 < m \leq 200$	$200 < m \leq 500$
Frequency	43	31	25	21

(c) Complete the cumulative frequency table.

Mass (m grams)	$m \leq 50$	$m \leq 100$	$m \leq 200$	$m \leq 500$
Cumulative frequency				

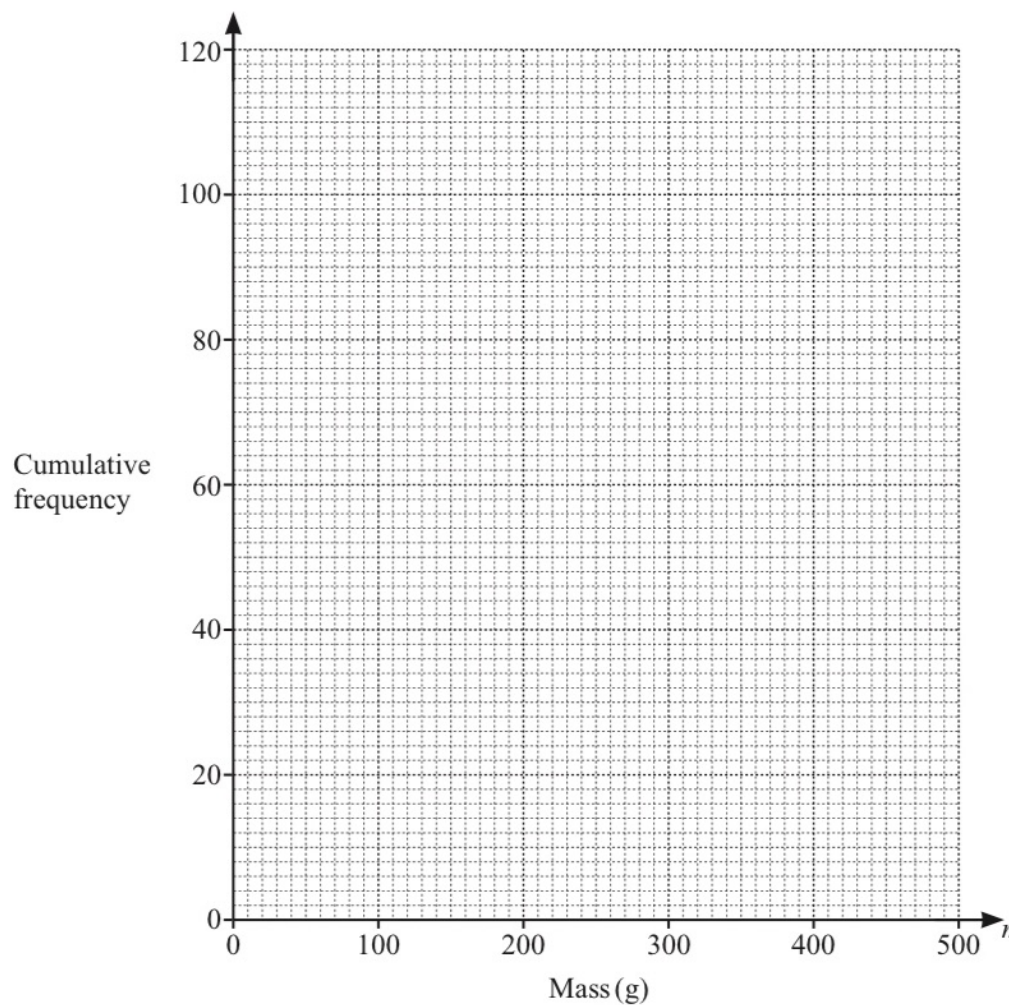
[2]

5 (c) Complete the cumulative frequency table.

Mass (m grams)	$m \leq 50$	$m \leq 100$	$m \leq 200$	$m \leq 500$
Cumulative frequency	43	74	99	120

(d) Draw a cumulative frequency diagram.

[2]



[3]



5



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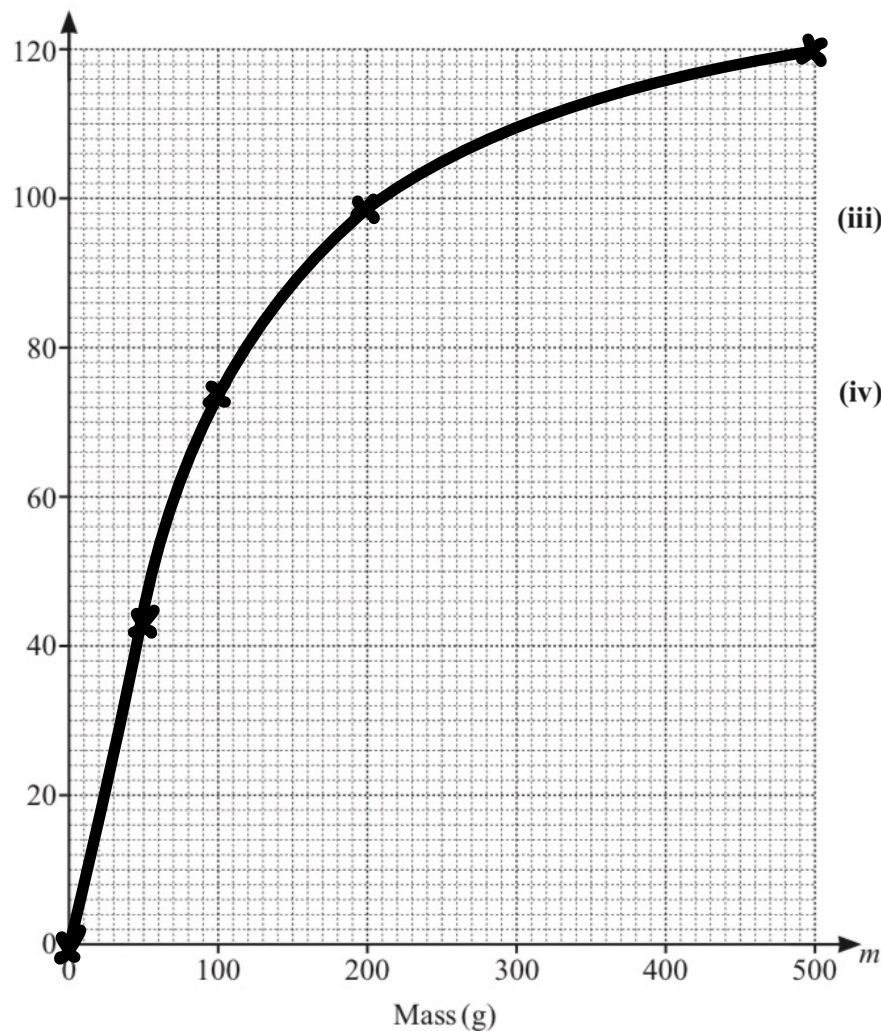
(e) Use the cumulative frequency diagram to find an estimate for

(i) the median,

..... g [1]

(ii) the upper quartile,

..... g [1]



(iii) the 40th percentile,

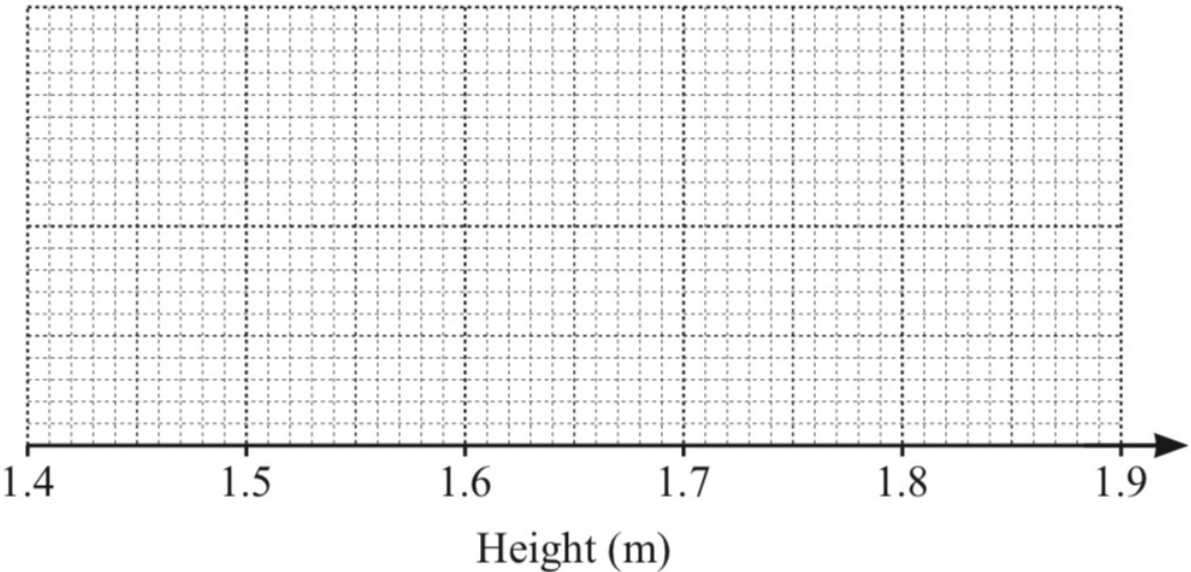
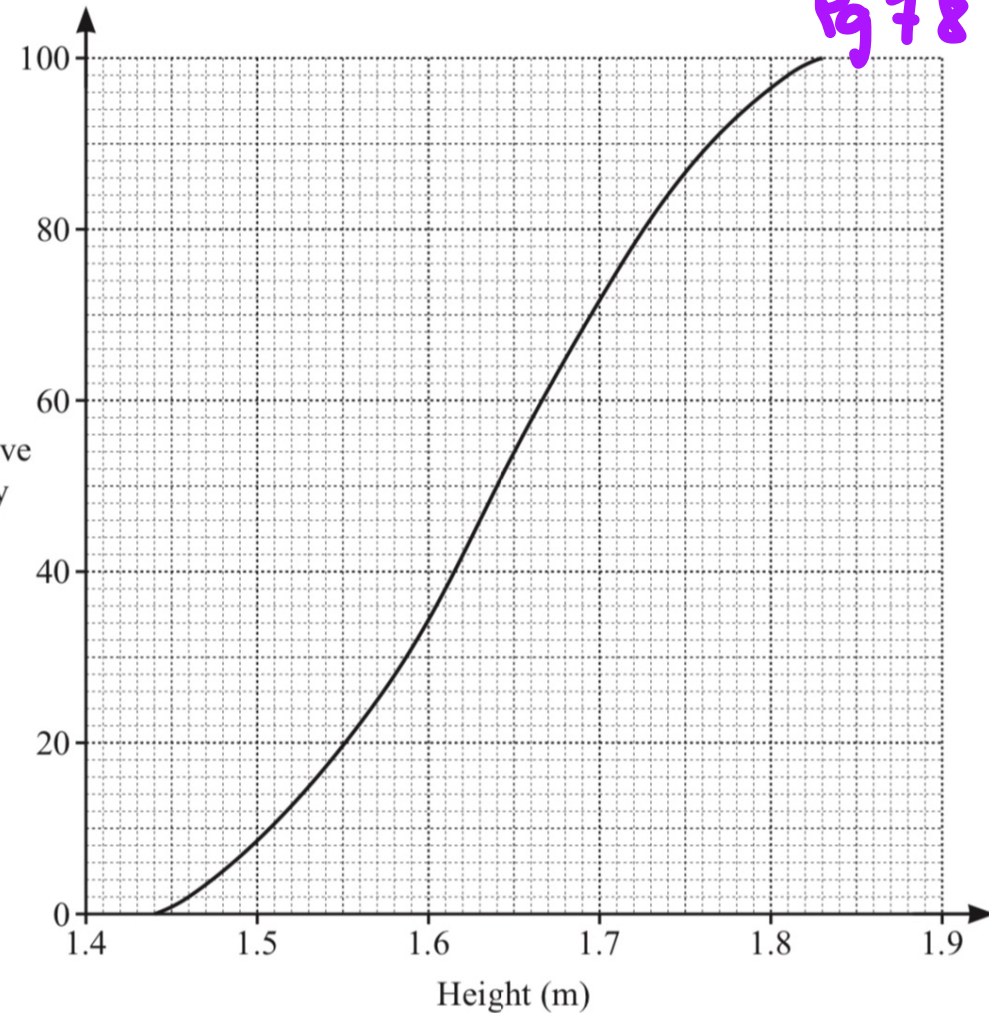
..... g [2]

(iv) the number of letters with a mass m where $250 < m \leq 400$.

..... [2]

8 (b) This cumulative frequency diagram shows information about the height, in metres, of each of 100 students.

The height of the tallest student is 1.83 metres.
The height of the shortest student is 1.45 metres.

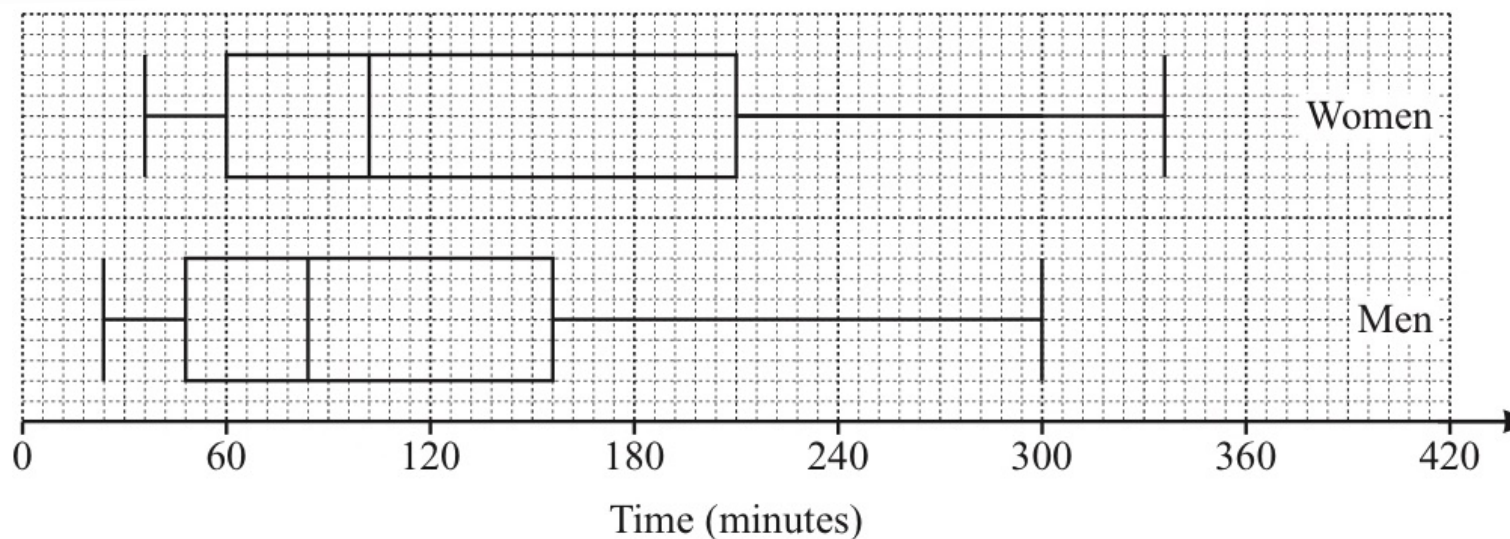


On this grid, draw a box-and-whisker plot for the heights of the 100 students.

[4]



3 (a)



The box-and-whisker plots show the times spent exercising in one week by a group of women and a group of men.

Below are two statements comparing these times.

For each one, write down whether you agree or disagree, giving a reason for your answer.

Statement	Agree or disagree	Reason
On average, the women spent less time exercising than the men.		
The times for the women show less variation than the times for the men.		





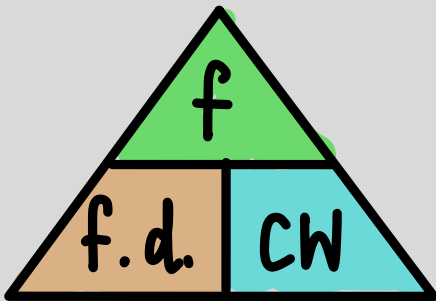
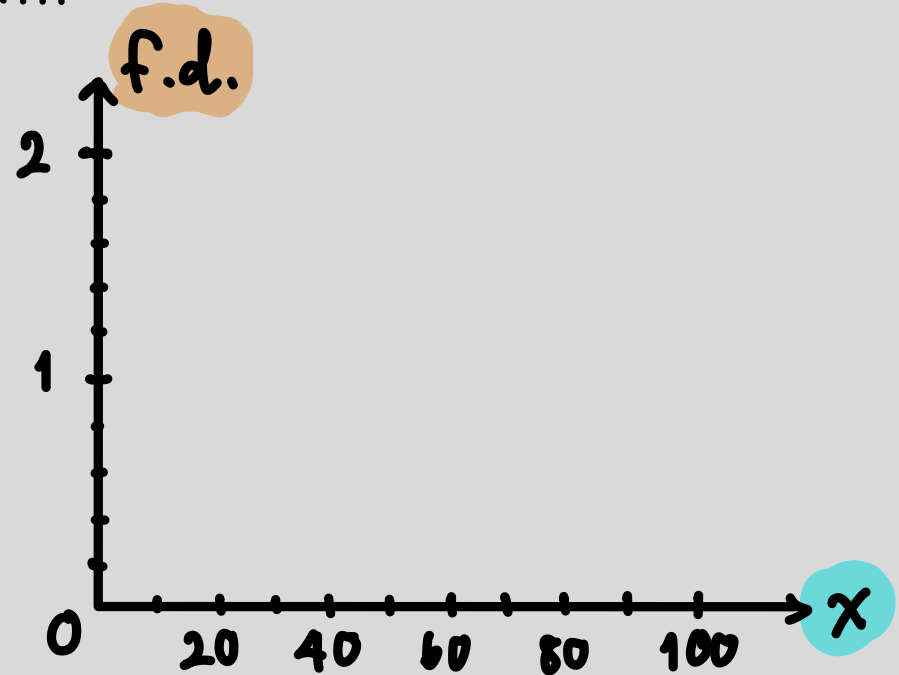
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6.6 Histograms

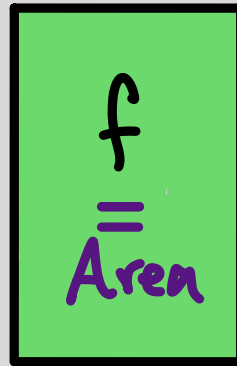
Area of each bar represents frequency.

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

x	frequency	CW	f.d.
0-20	28		
20-40	36		
40-50	20		
50-70	30		
70-100	18		



f.d.



CW



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6.6 Histograms



6. (b) The frequency table shows information about the mass of each of 50 trucks.

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Pg 85

Mass (m kg)	$2000 < m \leq 2600$	$2600 < m \leq 3500$	$3500 < m \leq 5000$	$5000 < m \leq 5700$
Frequency	12	15	16	7

(ii) In a histogram showing this information, the height of the first block is 6 cm.

Calculate the heights of the remaining three blocks.

Height of block for $2600 < m \leq 3500$ cm

Height of block for $3500 < m \leq 5000$ cm

Height of block for $5000 < m \leq 5700$ cm [3]

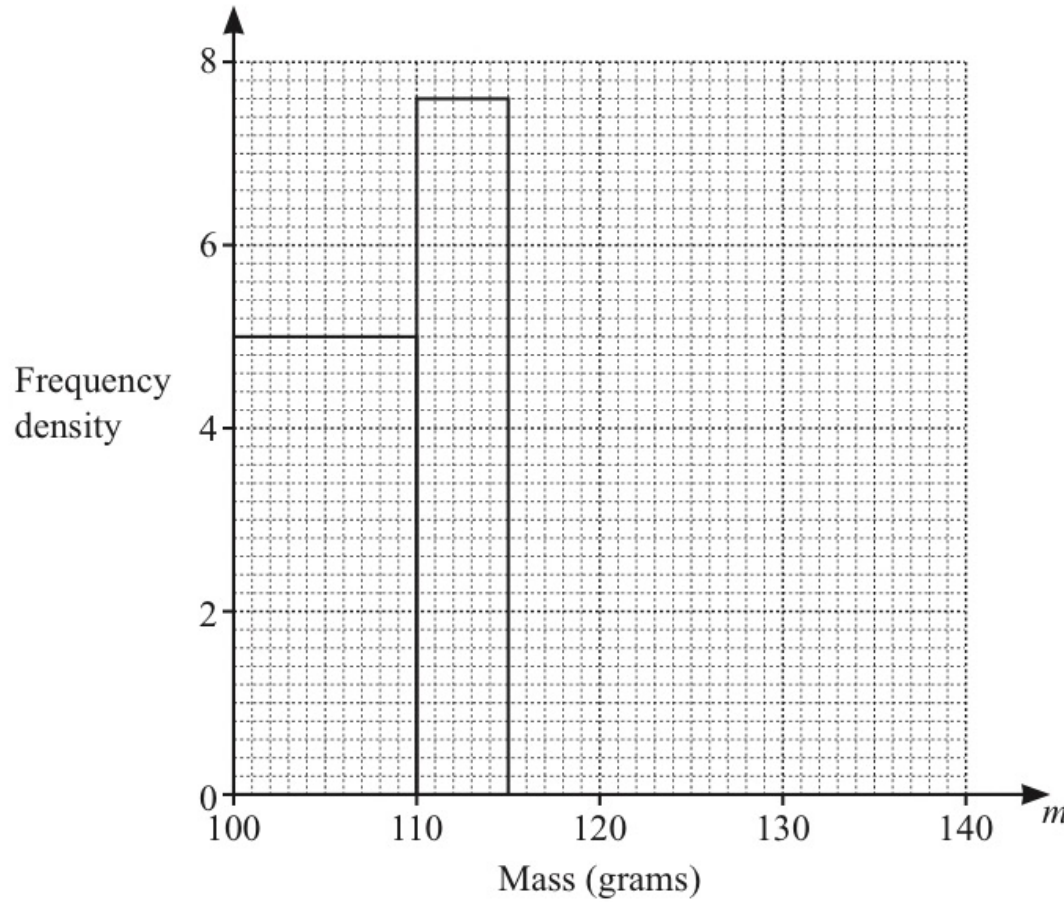


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7 (c) Some apples are weighed and the mass, m grams, of each apple is recorded.
The table shows the results.

Mass (m grams)	$100 < m \leq 110$	$110 < m \leq 115$	$115 < m \leq 125$	$125 < m \leq 140$
Frequency	50	x	44	51

The histogram shows some of the information from the table.



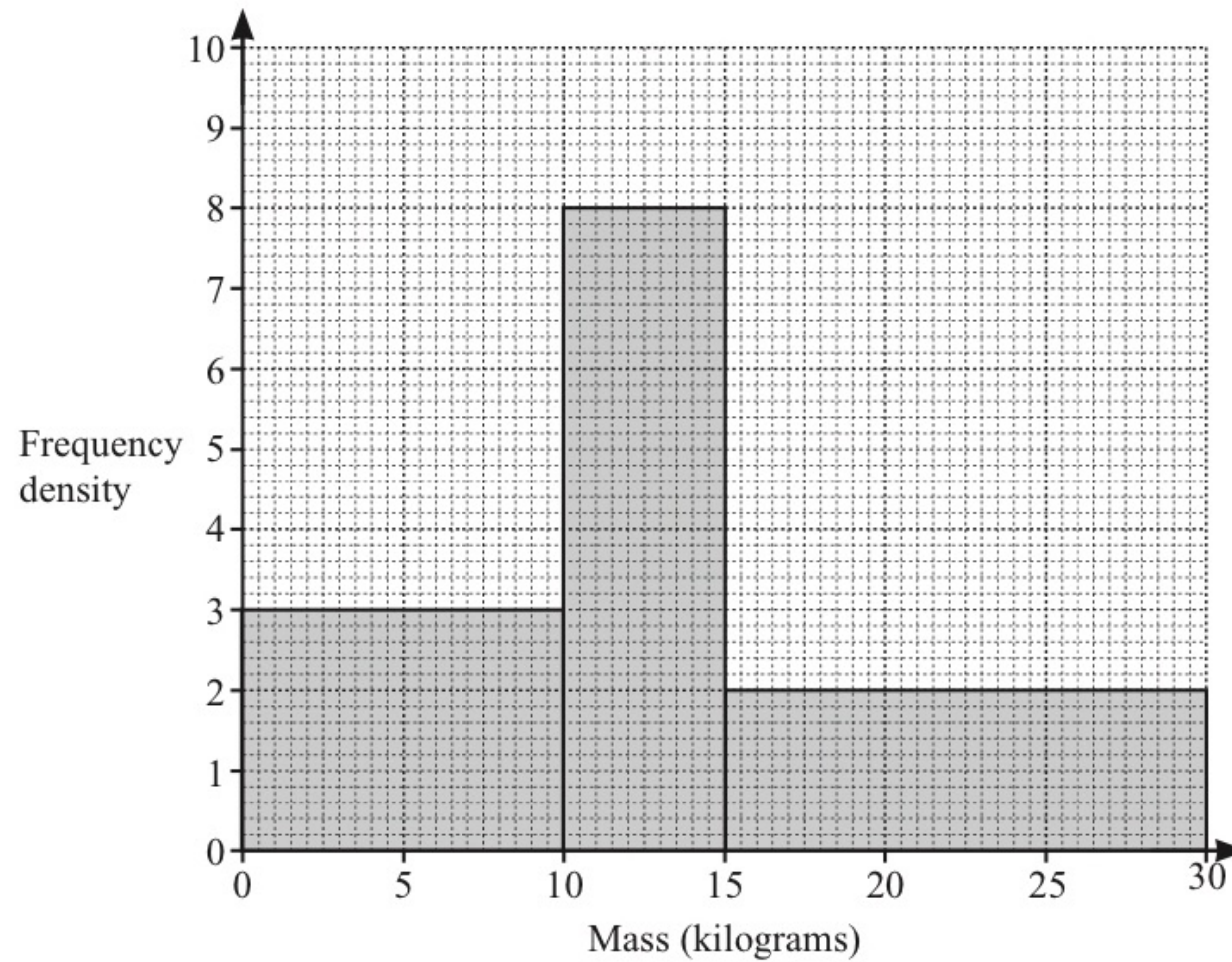
(i) Work out the value of x .

$x = \dots\dots\dots$ [1]

(ii) Complete the histogram.

[2]

- 6 (c) The histogram shows information about the masses of 100 boxes.



Calculate an estimate of the mean.

..... kg [6]



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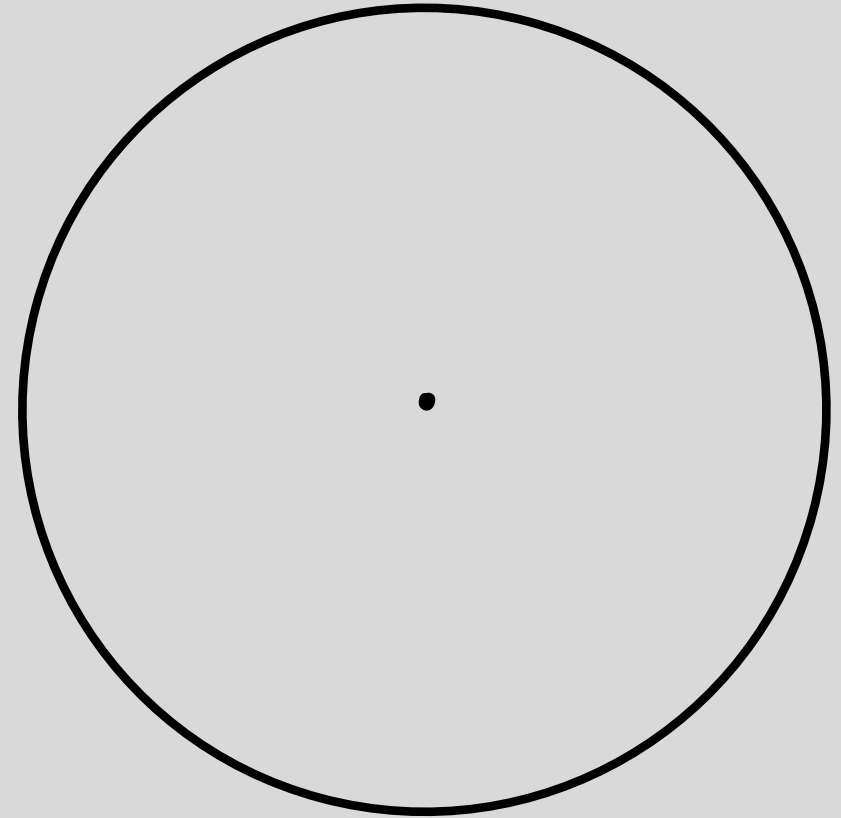
6.7 Pie charts

$$\text{Angle} = \frac{\text{frequency}}{N} \times 360^\circ$$

$$\% = \frac{\text{frequency}}{N} \times 100$$

Ex. The table shows the number of cars of different makes in a car park. Illustrate this data on a pie chart.

Make	A	B	C	D	E	F
Frequency	14	23	37	5	42	18

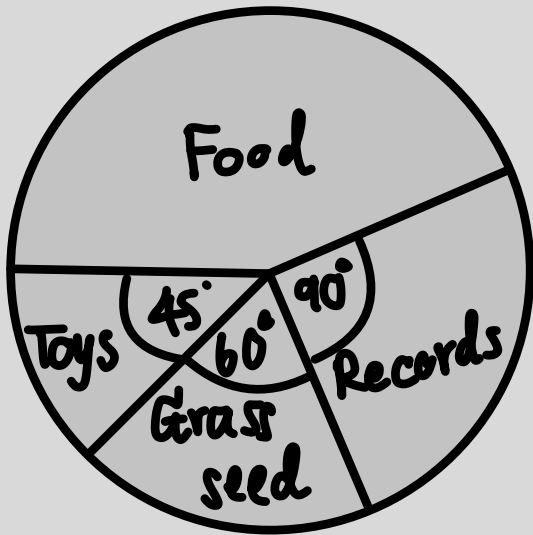




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6.7 Pie charts

Ex. The pie chart illustrates the values of various goods sold by a certain shop. If the total value of the sales was \$24000, find the sales value of
a) toys b) grass seed c) records d) food.





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6.7 Pie charts

- 2 Thibault records the number of cars of each colour in a car park.

Colour	Black	White	Silver	Red
Number of cars	8	5	4	3

He draws a pie chart to show this information.

Calculate the sector angle for the red cars.

..... [2]



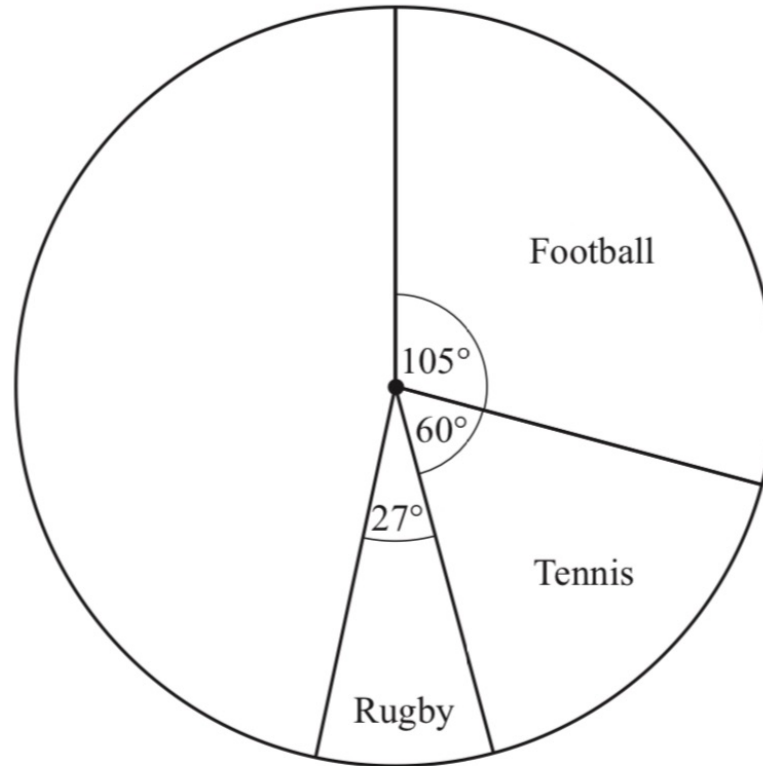


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- 8 (a) Jean asks 600 people to choose their favourite sport.
The pie chart shows some of this information.

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Fig 92



- (i) Show that 100 people choose tennis.

[1]

- (ii) Work out how many people choose rugby.

..... [2]

- (iii) 125 people choose cricket and the rest choose swimming.

Complete the pie chart to show this information.

[2]



18 120 students choose what they want to do when they leave school.
Their choices are shown in the table.

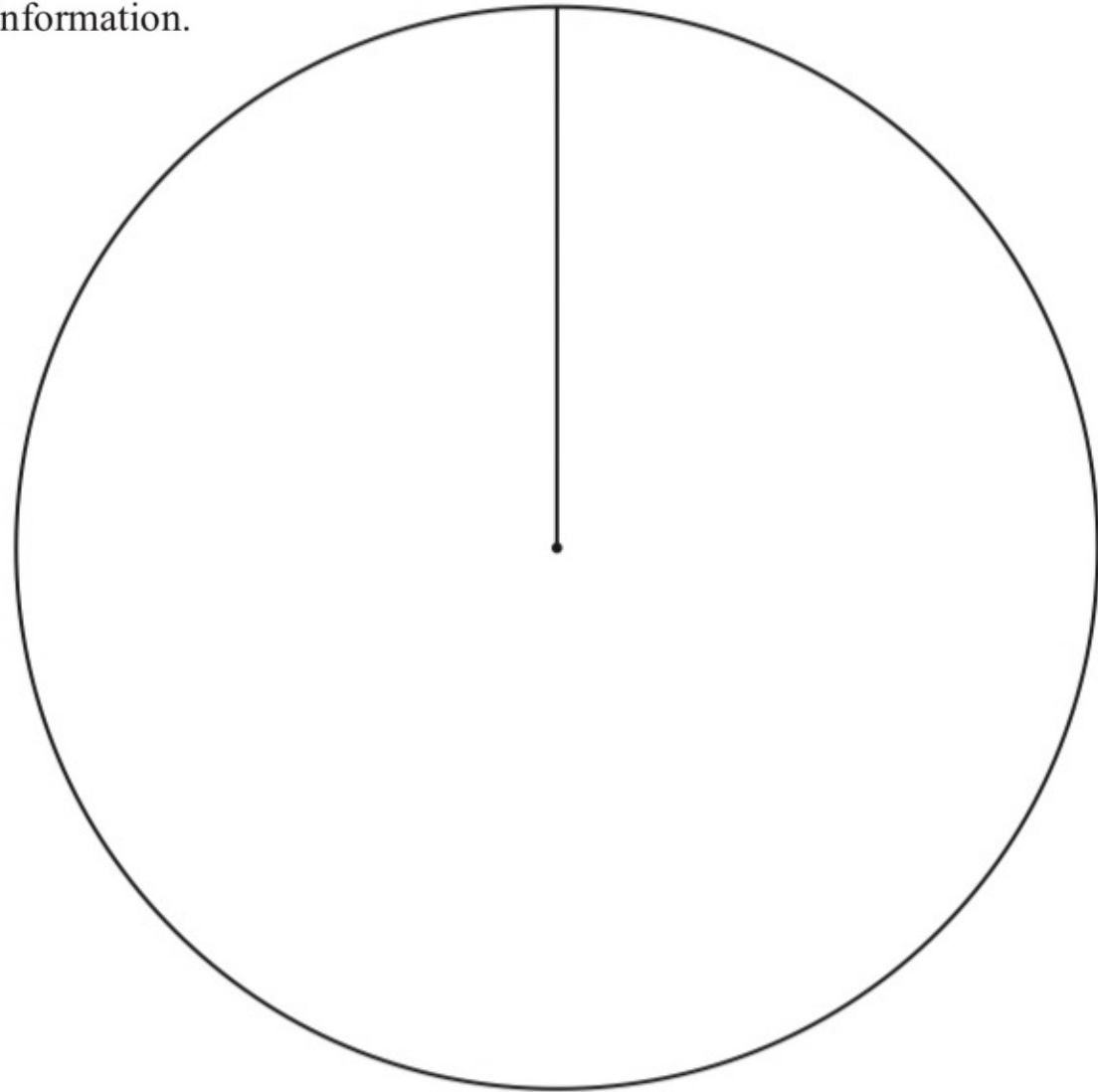
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Fig 93

Complete the pie chart to show this information.
Label each sector clearly.

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Choice	Number of students
University	57
Training	45
Work	18





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6.8 Stem and Leaf diagrams

- 1) It's used to order and present data given to 2 or 3 significant figures
- 2) It keeps the detail of the data but can be time consuming to do.
- 3) The median and quartiles can easily be found.

Ex. The blood pressure of a set of patients are recorded. The results are given below in mmHG.

12 31 24 18 21 34 40 19 23 17 16

Stem	Leaf	key :
1		1) Median
2		2) LQ
3		3) UQ
4		



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6.8 Stem and Leaf diagrams



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2 (a) Anna records the number of text messages she receives for 14 days.

17	15	31	38	31	22	13
18	21	27	28	21	31	29

(i) Complete the stem-and-leaf diagram.

1	
2	
3	

Key:

[3]

(ii) Find the median.

..... [1]

(iii) Find the mode.

..... [1]

(iv) Find the range.

..... [1]



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- 1 (a) The list shows 15 midday temperatures, in degrees Celsius, in Suntown.

17 21 21 18 23 22 25 19

21 17 19 18 21 24 23

- (i) Complete the stem-and-leaf diagram to show this information.

1	7
2	

Key: 1|7 represents 17°C

[2]

- (ii) Find the median.

..... °C [1]

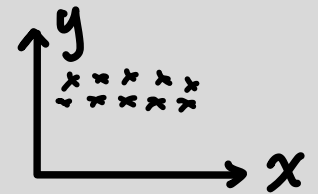
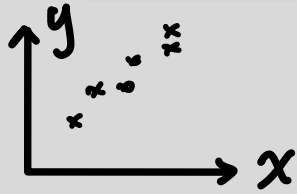
- (iii) Find the upper quartile.

..... °C [1]

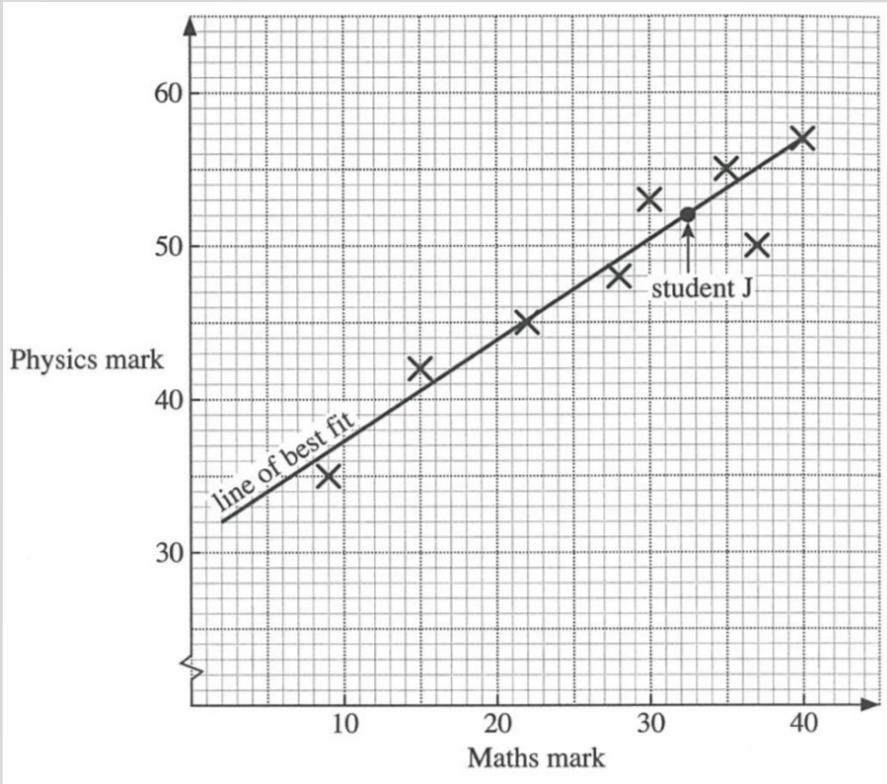


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6.9 Scatter diagrams



Positive correlation — Negative correlation No correlation No correlation No correlation



Student	A	B	C	D	E	F	G	H	I
Maths mark	28	22	9	40	37	35	30	23	?
Physics mark	48	45	34	57	50	55	53	45	52

Mean



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6.9 Scatter diagrams

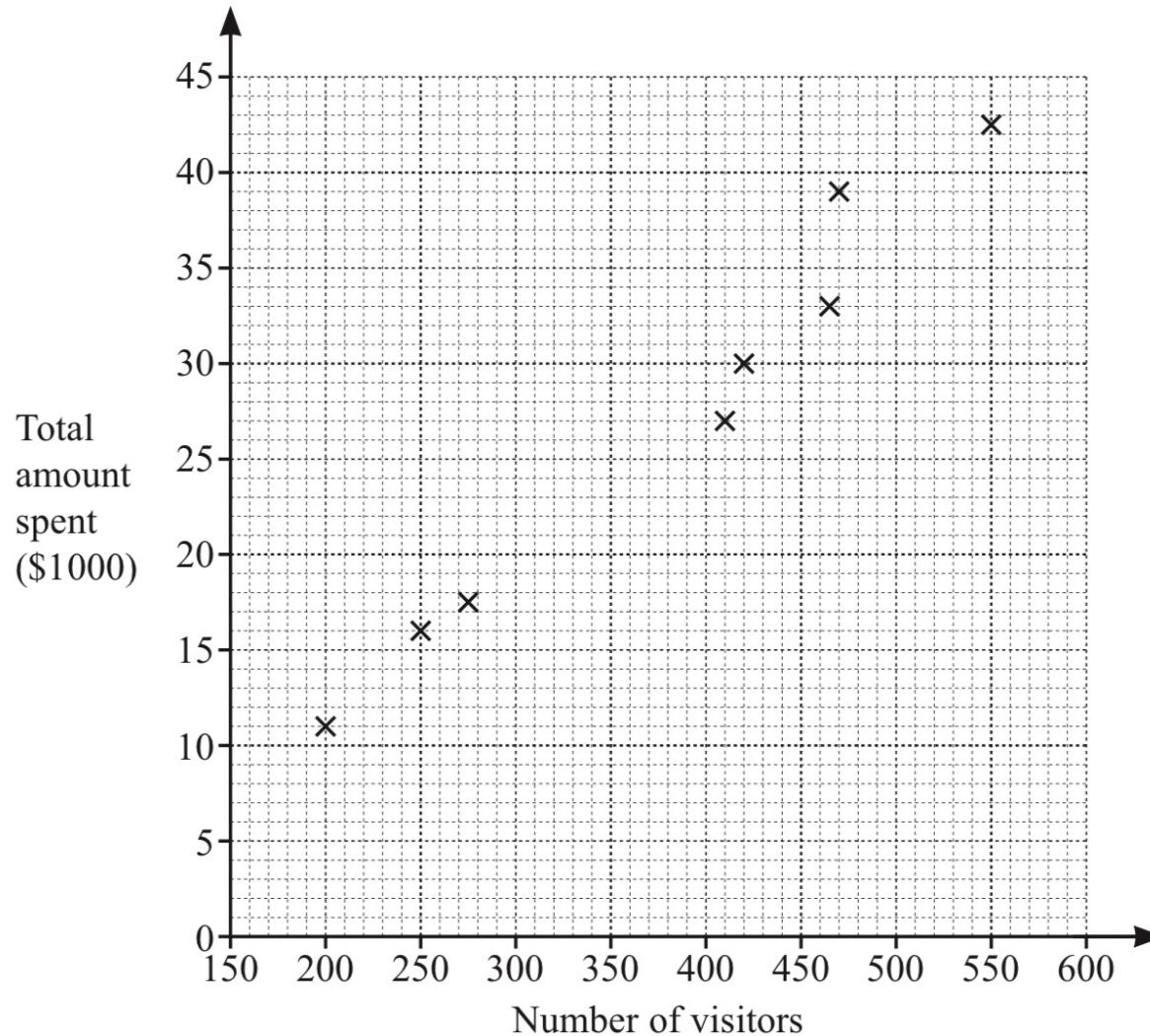


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7 The scatter diagram shows the number of visitors and the total amount spent, in thousands of dollars, at a zoo on each of eight days.

0580/23/M/J/22

Fig 101



(a) On one of the eight days there are 410 visitors.

Find the total amount spent by visitors during this day.

\$ [1]

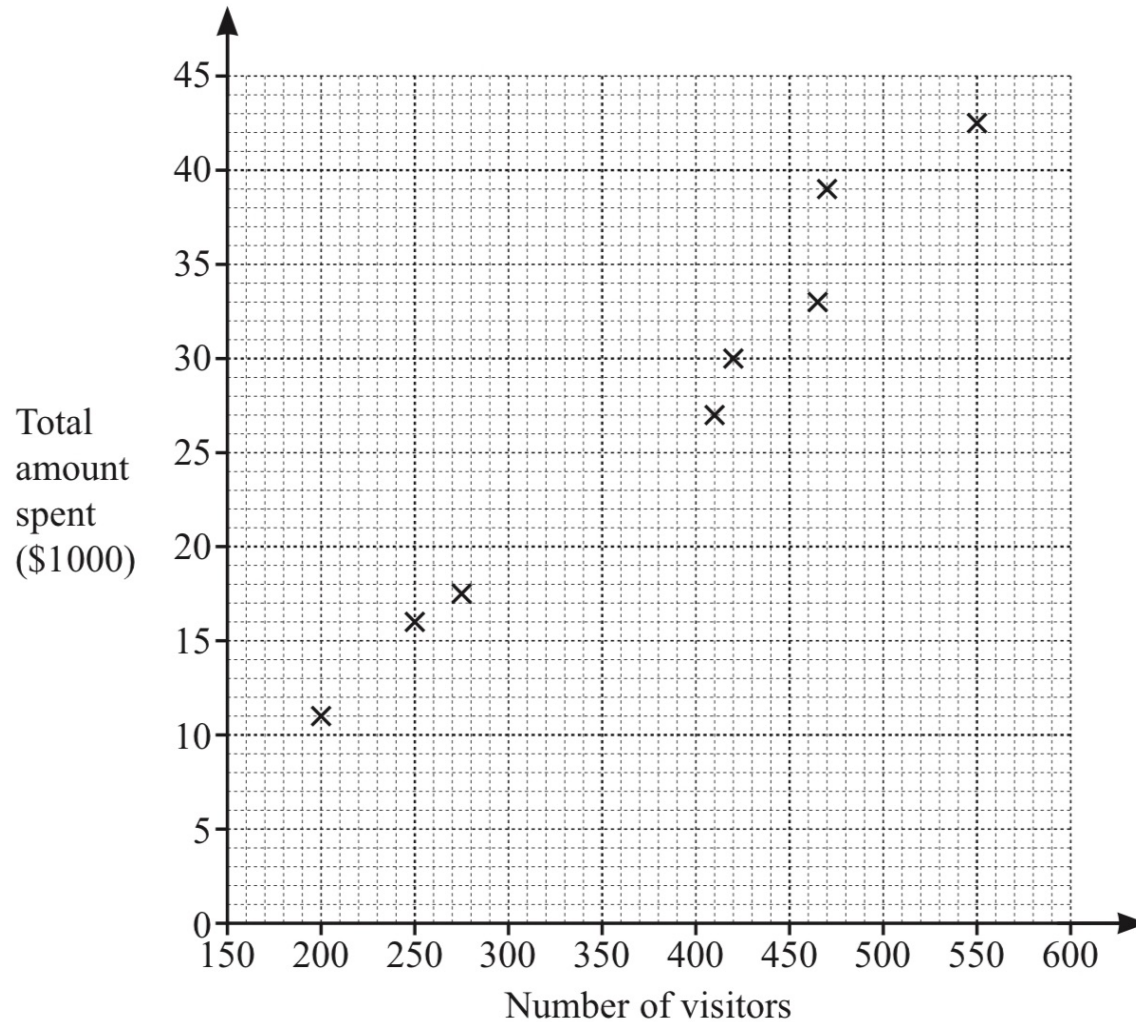


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- 7 The scatter diagram shows the number of visitors and the total amount spent, in thousands of dollars, at a zoo on each of eight days.

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Fig 101



- (b) Information for the ninth day is shown in the table.

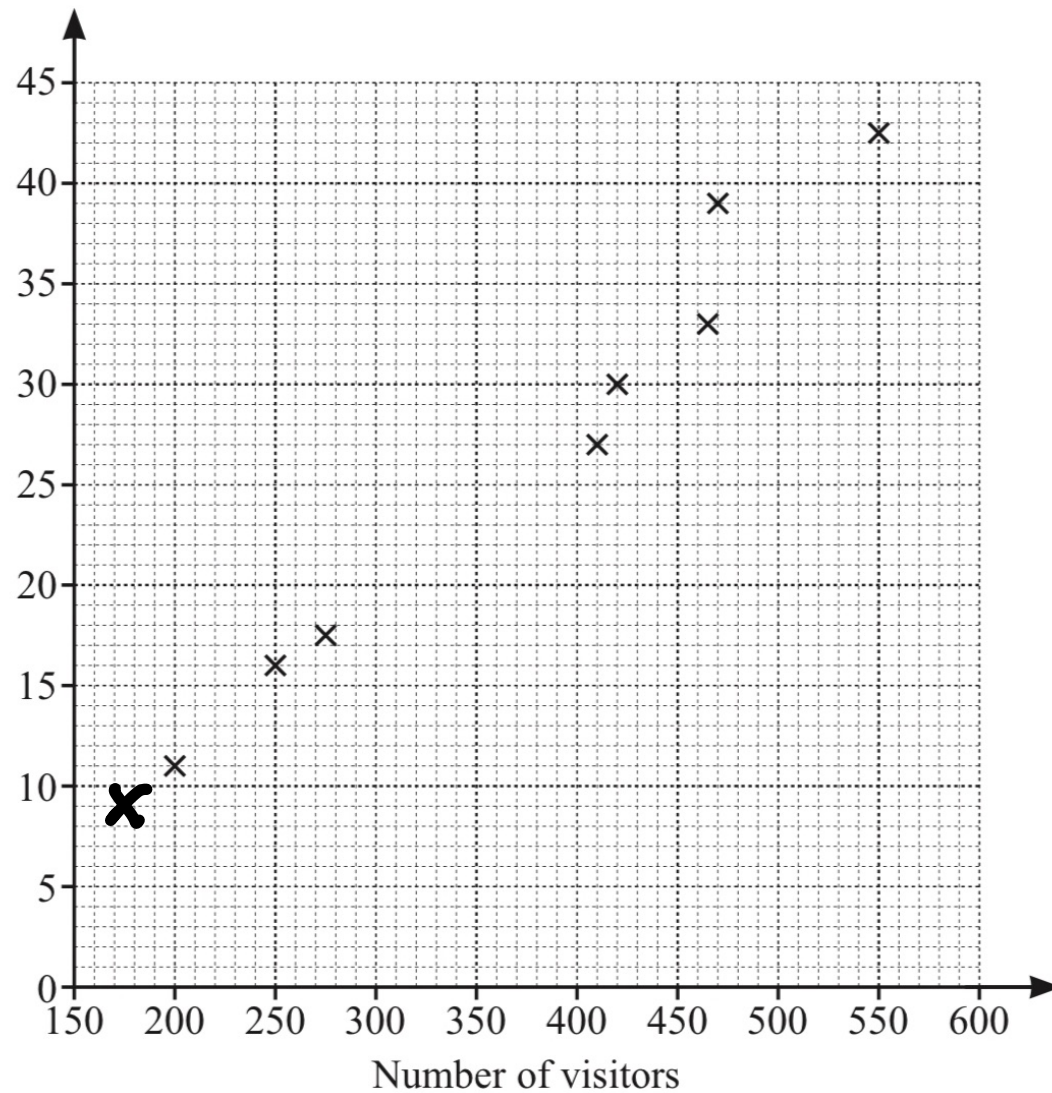
Number of visitors	175
Total amount spent (\$1000)	9

Plot this information on the scatter diagram.

[1]

7

Total amount spent (\$1000)



(c) Draw a line of best fit on the scatter diagram.

[1]

(d) On the tenth day the total amount spent is \$22 000.

Estimate the number of visitors on this day.

..... [1]



11 As the temperature increases, people eat more ice cream.

What type of correlation does this statement describe?

..... [1]

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Pg 102

5 The number of bowls of hot soup sold decreases when the temperature rises.

What type of correlation does this statement describe?

..... [1]

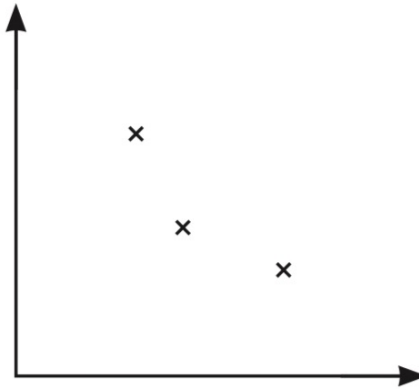
0580/22/F/M/21



5 (a) Henrik draws this scatter diagram.

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Q 102



Put a ring around the **one** correct statement about this scatter diagram.

It shows no correlation.

It is not possible to tell if there is correlation as there are not enough points.

It shows negative correlation.

It shows positive correlation.

(b) Each of the four scatter diagrams shows the same set of data. A line has been drawn on each diagram.

[1]

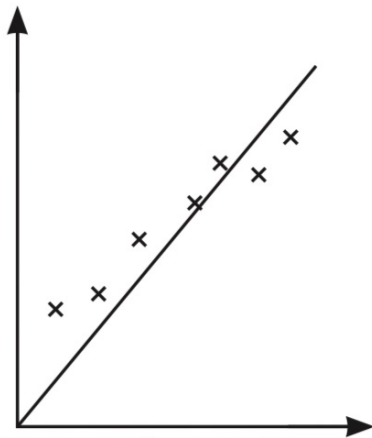


Diagram A

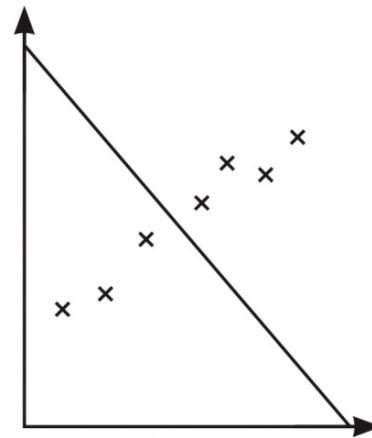


Diagram B

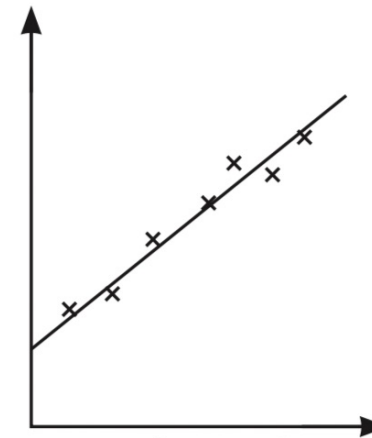


Diagram C

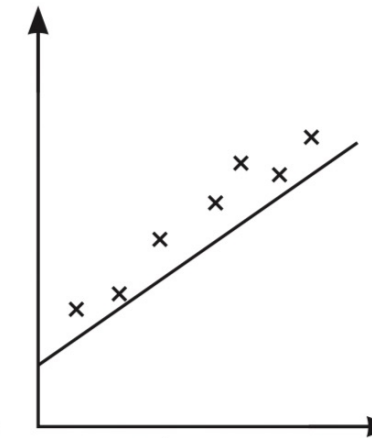


Diagram D

Complete the statement.

The line in Diagram is the most appropriate line of best fit.

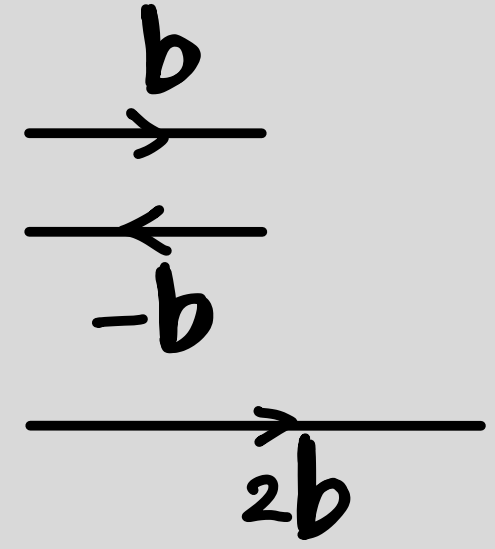
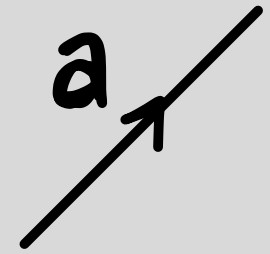
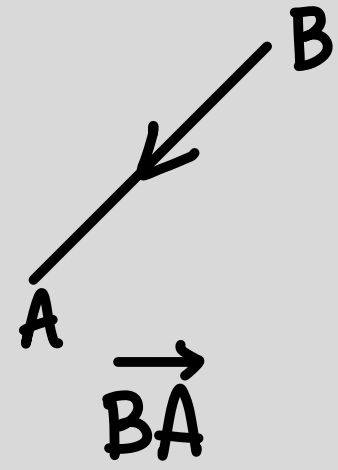
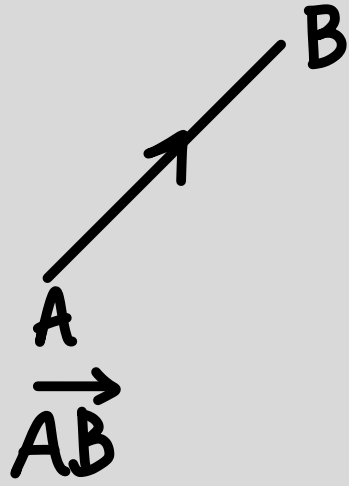
[1]



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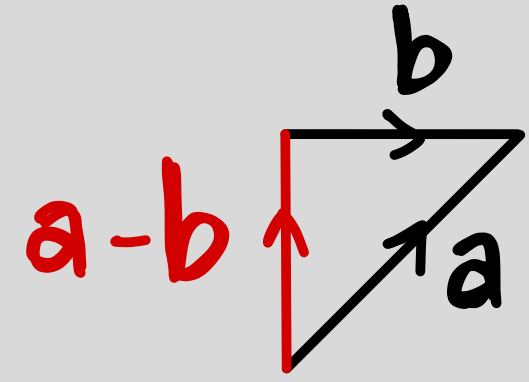
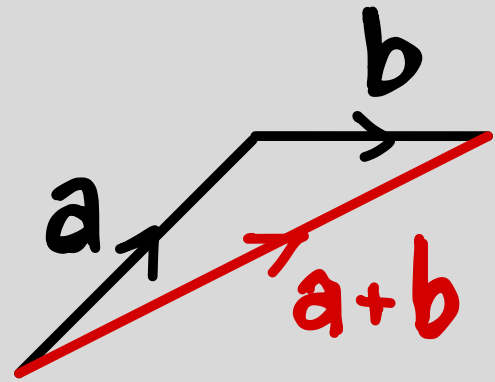
5.4 Vectors

Scalar + Direction



Addition

Subtract





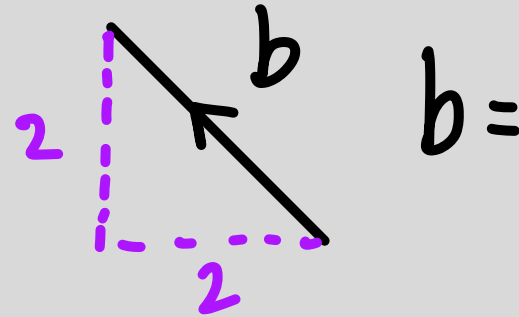
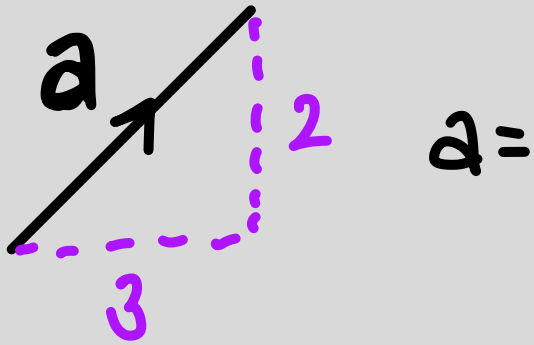
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5.4 Vectors

Column vector

$$a = \begin{pmatrix} x \\ y \end{pmatrix}$$

⊖ ↔ ⊕
⊖ ↓ ↑ ⊕



a) $2a =$

b) $a + b =$

c) $a - 2b =$

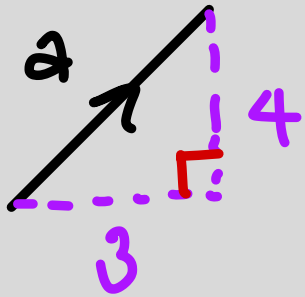


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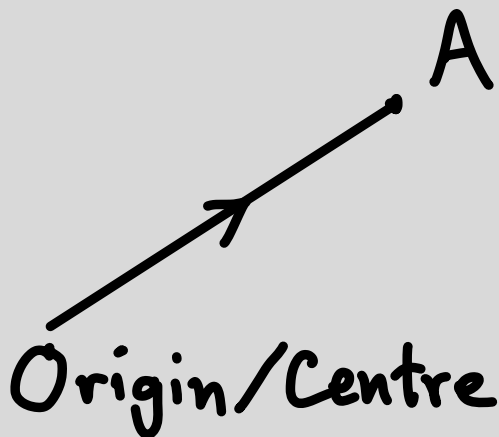
5.4 Vectors

Modulus/Magnitude

$$|\vec{AB}| \text{ or } |a| = \sqrt{x^2 + y^2}$$



Position vector of a point A



Start with the origin/centre.

$$= \vec{OA}$$



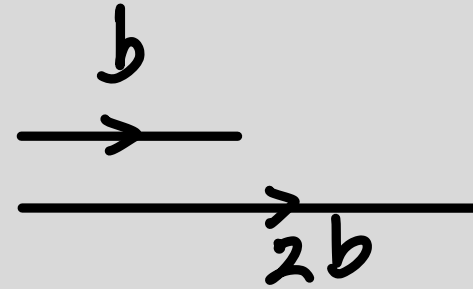
5.4 Vectors

Parallel vectors

$$\mathbf{a} = k\mathbf{b}$$

Ex.

$$\mathbf{a} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$



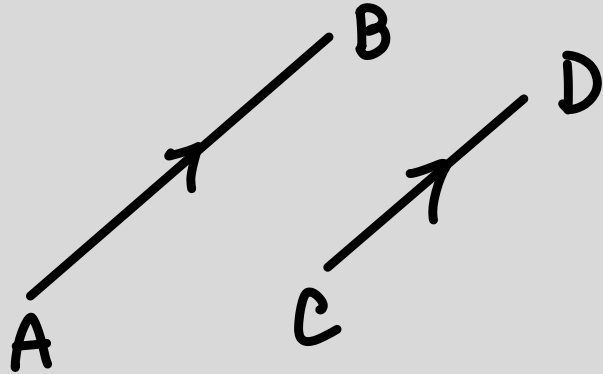
Vector \mathbf{a} is a multiple of \mathbf{b} .
Vector \mathbf{a} and \mathbf{b} are parallel.



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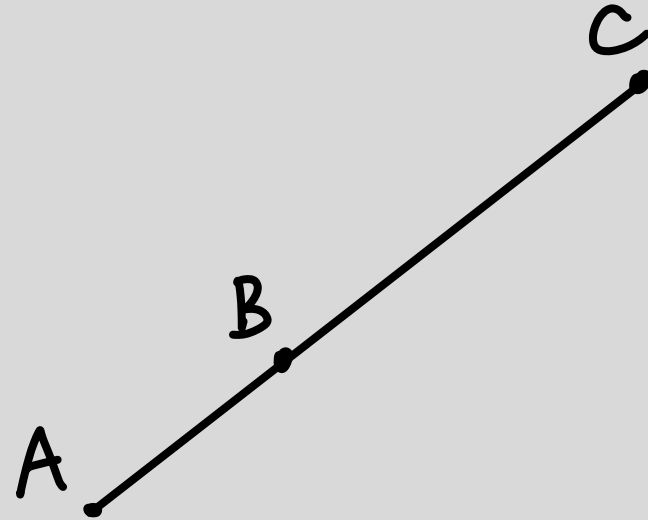
5.4 Vectors

Parallel vectors



$$\vec{AB} = k \vec{CD}$$

- 1) AB and CD are parallel.
- 2) Length $AB = kCD$



$$\vec{AB} = k \vec{BC}$$

B is a common point.

A, B and C lie on the same straight line.

OR A, B and C are collinear.



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5.4 Vectors



6 (a) $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Find

(i) $3\mathbf{q}$,

$\left(\quad \right)$ [1]

(ii) $\mathbf{p} - \mathbf{q}$,

$\left(\quad \right)$ [1]

(iii) $|\mathbf{p}|$.

..... [2]

(b) B is the point $(2, 7)$ and $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$.

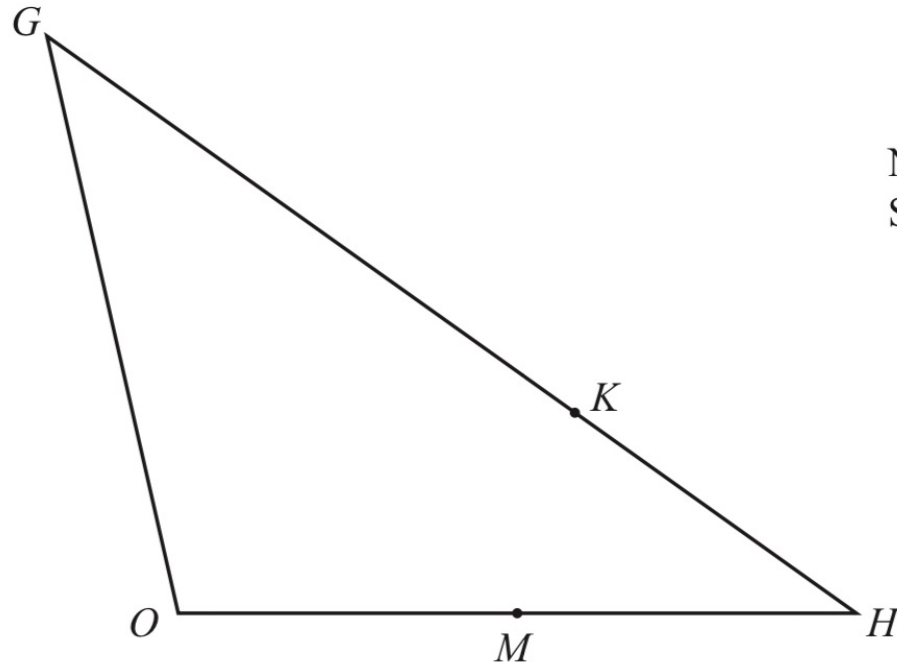
Find the coordinates of A .

(.....,) [2]

6 (c)



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NOT TO
SCALE

In triangle OGH , M is the midpoint of OH and K divides GH in the ratio $5 : 2$.

$\overrightarrow{OG} = \mathbf{g}$ and $\overrightarrow{OH} = \mathbf{h}$.

Find \overrightarrow{MK} in terms of \mathbf{g} and \mathbf{h} .

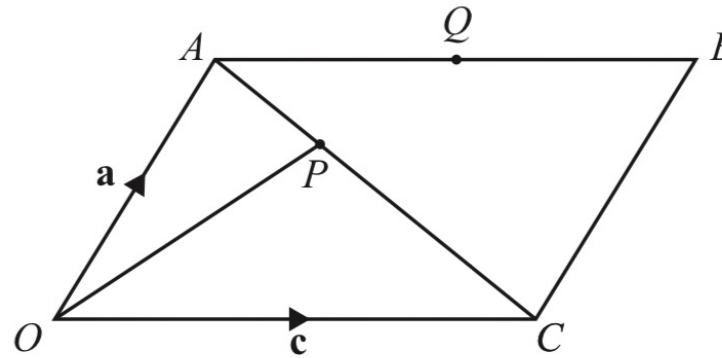
Give your answer in its simplest form.

$\overrightarrow{MK} = \dots\dots\dots$ [4]



9. (b)

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NOT TO
SCALE

$OACB$ is a parallelogram.

P is a point on AC and Q is the midpoint of AB .

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(ii) $\overrightarrow{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$

(i) Find, in terms of \mathbf{a} and/or \mathbf{c}

(a) \overrightarrow{AQ} ,

(a) Show that O, P and Q lie on a straight line.

$\overrightarrow{AQ} = \dots\dots\dots$ [1]

(b) \overrightarrow{OQ} .

(b) Write down the ratio $OP : OQ$.
Give your answer in the form $1 : n$.

[2]

$\overrightarrow{OQ} = \dots\dots\dots$ [1]

$1 : \dots\dots\dots$ [1]

12 (b) A is the point $(7, 2)$ and B is the point $(-5, 8)$.

(iii) AB is one side of the parallelogram $ABCD$ and

- $\overrightarrow{BC} = \begin{pmatrix} -a \\ -b \end{pmatrix}$ where $a > 0$ and $b > 0$
- the gradient of BC is 1
- $|\overrightarrow{BC}| = \sqrt{8}$.

Find the coordinates of D .

(..... ,) [4]



9

- (b) Ahmed finds the magnitude of the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

From this list, put a ring around the correct calculation.

$\sqrt{2^2 + -3^2}$

$2^2 - 3^2$

$\sqrt{2^2 - 3^2}$

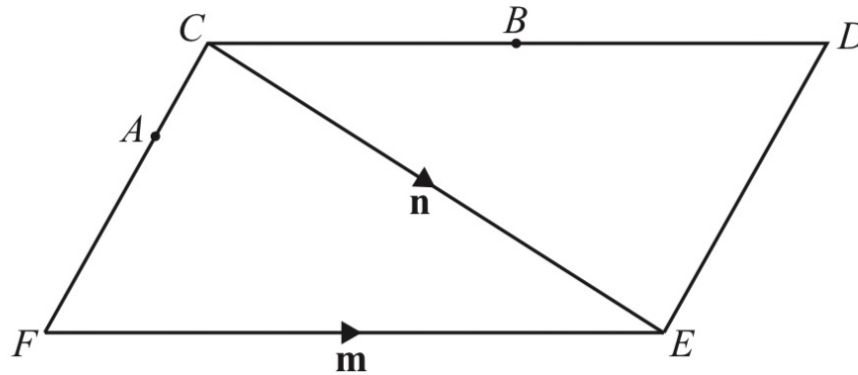
$2^2 + (-3)^2$

$\sqrt{2^2 + (-3)^2}$

[1]



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The diagram shows a parallelogram $CDEF$.
 $\vec{FE} = \mathbf{m}$ and $\vec{CE} = \mathbf{n}$.
 B is the midpoint of CD .
 $FA = 2AC$

Find an expression, in terms of \mathbf{m} and \mathbf{n} , for \vec{AB} .
 Give your answer in its simplest form.

$\vec{AB} = \dots\dots\dots$ [3]

(b) $\vec{GH} = \frac{5}{6}(2\mathbf{p} + \mathbf{q})$ $\vec{JK} = \frac{5}{18}(2\mathbf{p} + \mathbf{q})$

Write down **two** facts about vectors \vec{GH} and \vec{JK} .

.....

..... [2]

21 $\vec{XY} = 3\mathbf{a} + 2\mathbf{b}$ and $\vec{ZY} = 6\mathbf{a} + 4\mathbf{b}$.

Write down two statements about the relationship between the points X , Y and Z .

1

2 [2]

