

Intensive Maths IGCSE -Oct Nov 2023

Day:2



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Topic 

Straight line graphs

Harder Graph

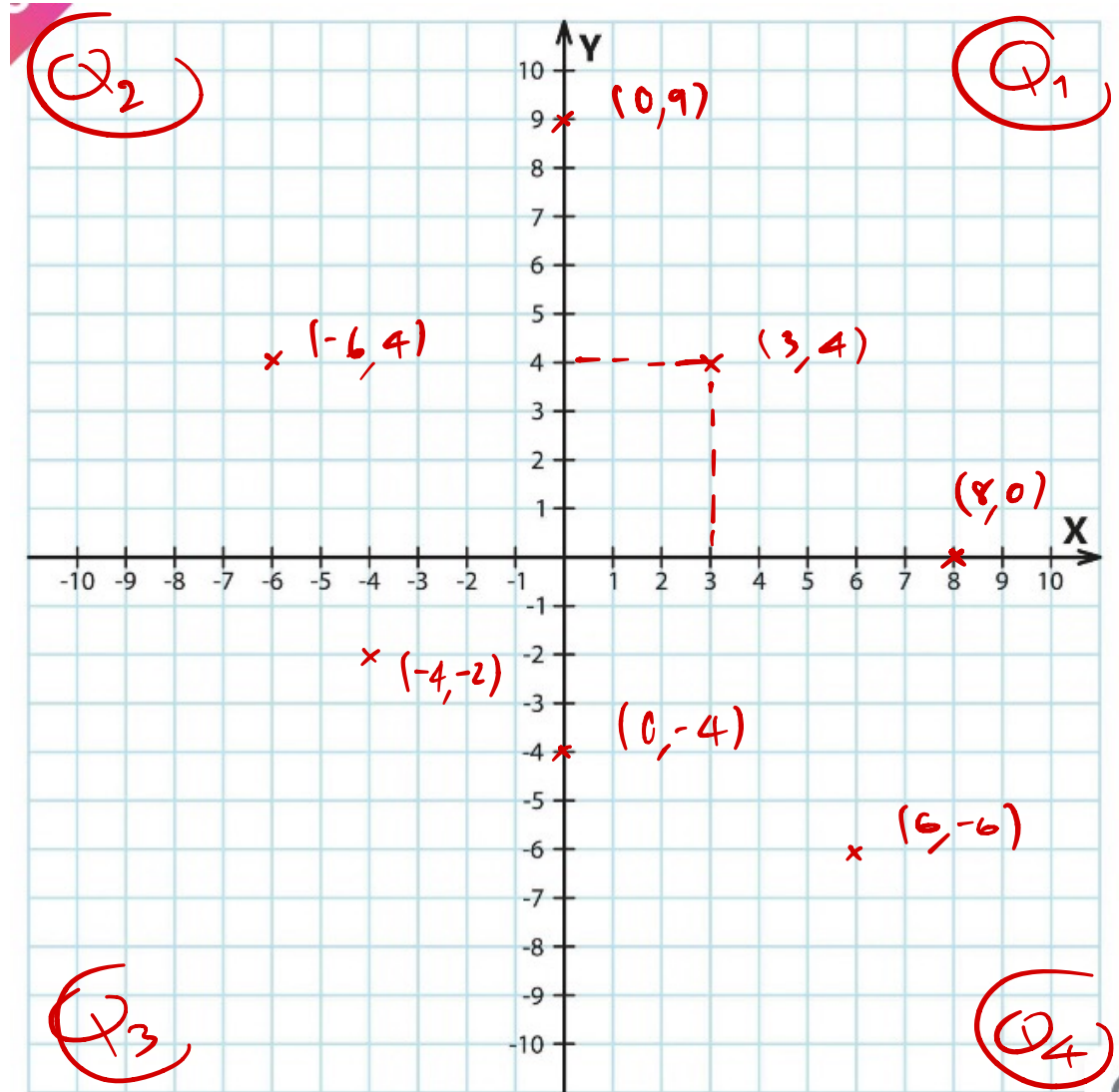
Function and Graph

Differentiation

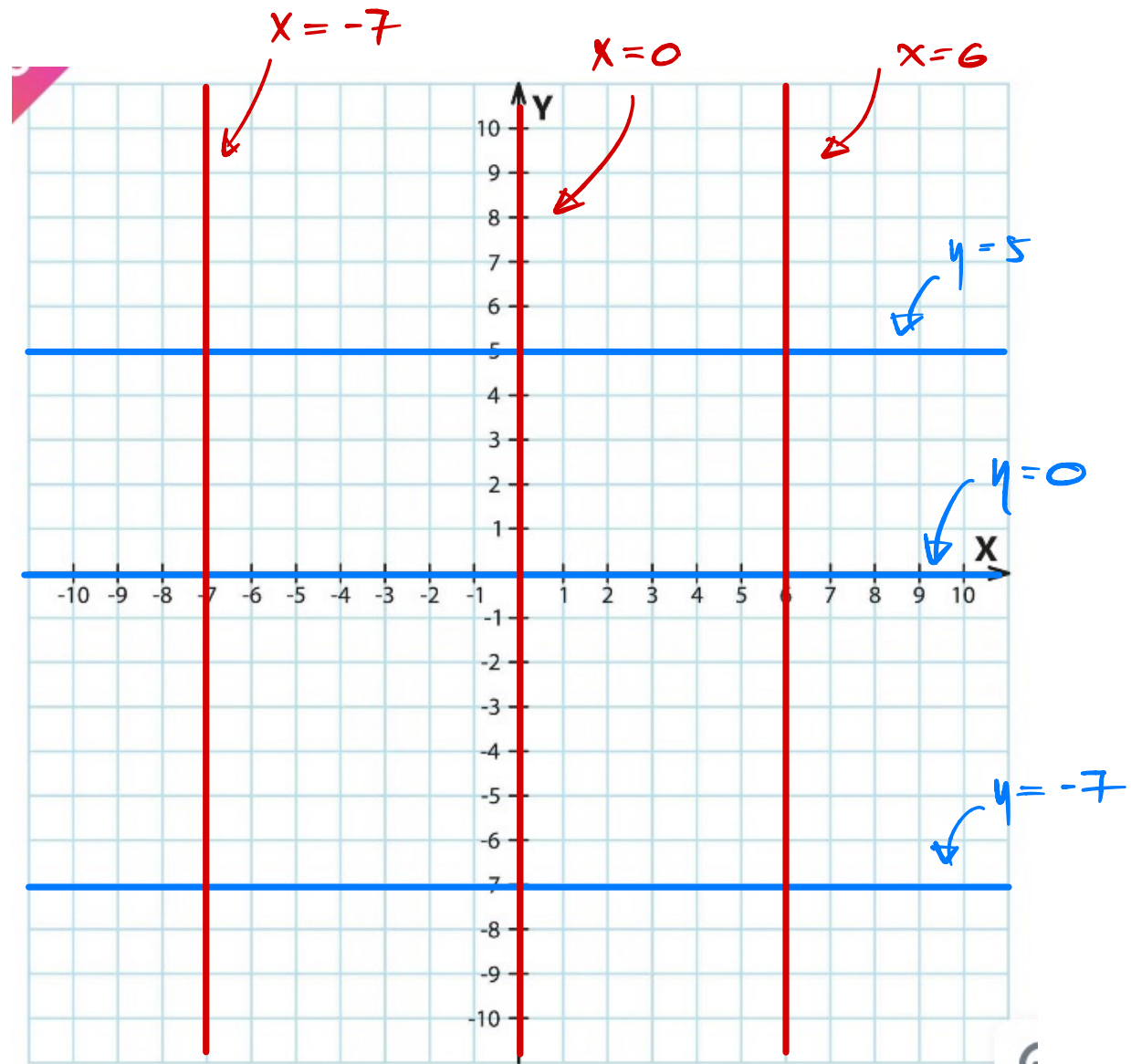
Part - ครูส้ม 

Graphs: Straight Lines.

Coordinate



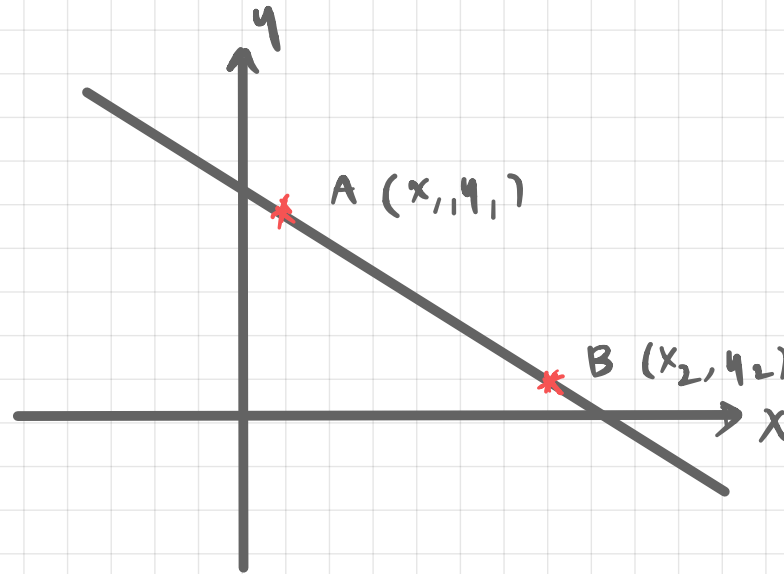
Lines: Basic



Graphs: Straight Lines.

Two points

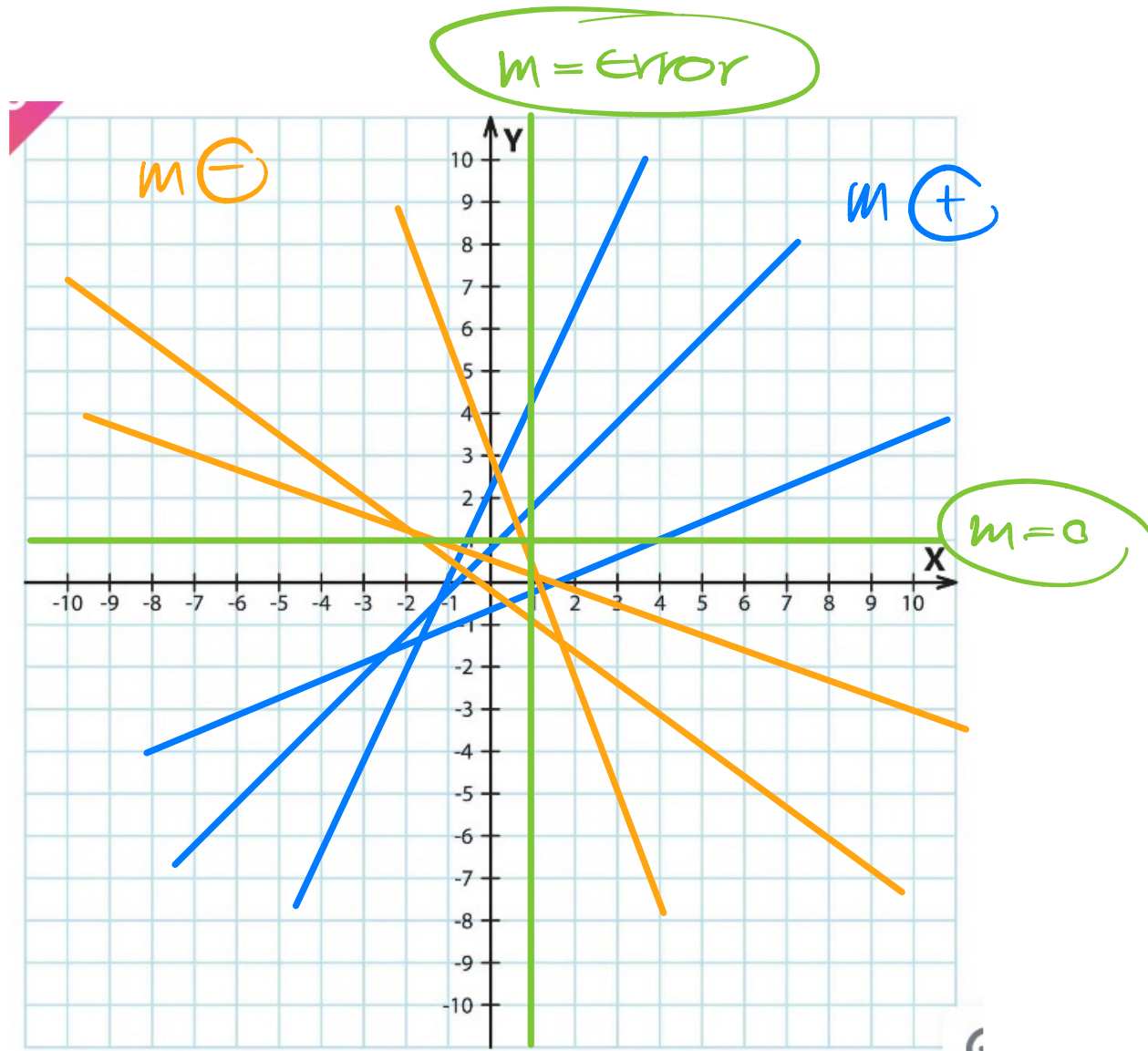
$$A(x_1, y_1) \text{ and } B(x_2, y_2)$$



1) Gradient (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Graphs: Straight Lines.



2) Midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

3) Distance between point (Length)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graphs: Straight Lines.

4) Equation of line

↓

gradient (m)
y-intercept (0, c) } $y = mx + c$

Example: Gradient = $\frac{1}{3}$
y-intercept (0, -7)

→ gradient (m)
point (x₁, y₁) } $y = mx + c$
 $y_1 = m(x_1) + c$

$c = c$

$y = mx + c$

Example: Gradient = $\frac{3}{2}$, passes through (4, 1)

Equation of lines

1) Change between $y=mx+c$ and $ax+by+c=0$

Example:

$$y = \frac{3}{2}x + 5 \longrightarrow ax + by + c = 0$$

Example:

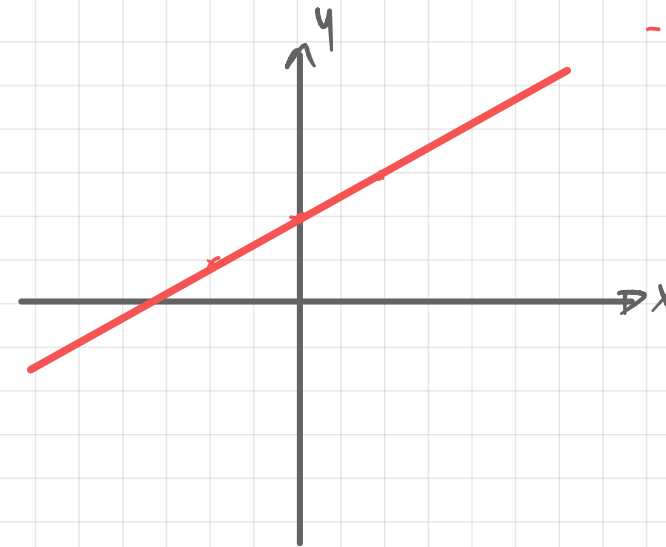
$$5x - 2y + 4 = 0 \longrightarrow y = mx + c$$

2) Sketch graph

Table

x	-2	0	2
y	y_1	y_2	y_3

Intercept

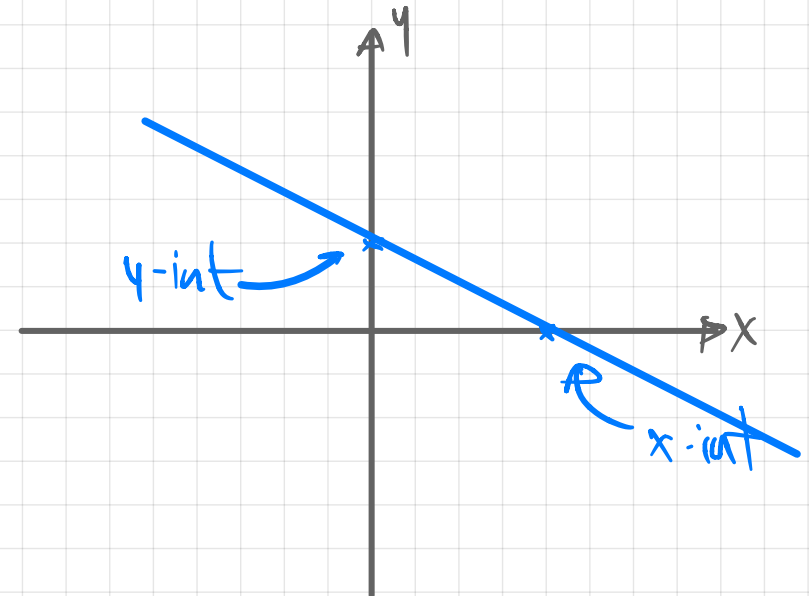


Cross the x-axis

x-intercept $\rightarrow y=0 ; x=?$

Cross the y-axis

y-intercept $\rightarrow x=0 ; y=?$



3) Gradient

$$y = mx + c \quad \longrightarrow \quad y = \frac{3}{2}x + 7$$

$$y = -4x + 1$$

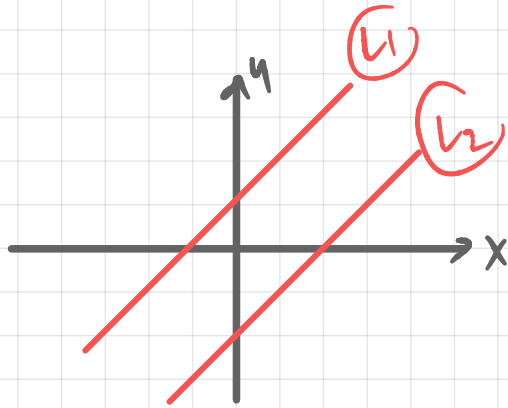
$$y = \frac{5 - 7x}{2}$$

$$ax + by + c = 0 \quad \longrightarrow \quad 3x + 2y - 4 = 0$$



$$y = mx + c$$

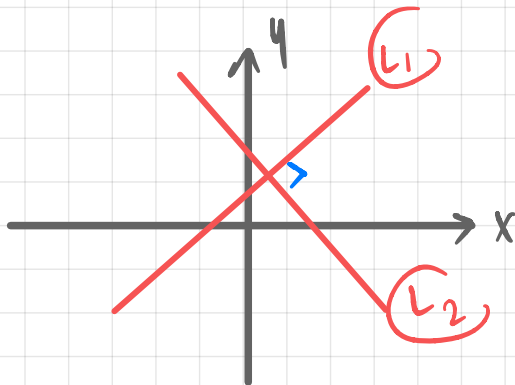
4) Parallel lines



$L_1 \parallel L_2 \rightarrow$

$$m_1 = m_2$$

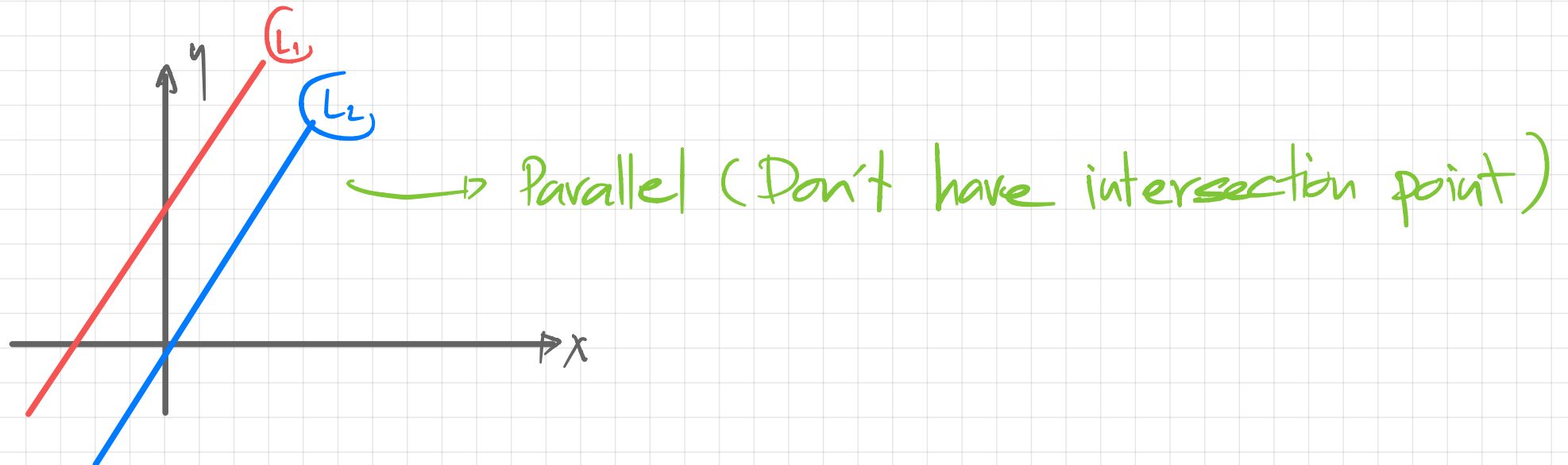
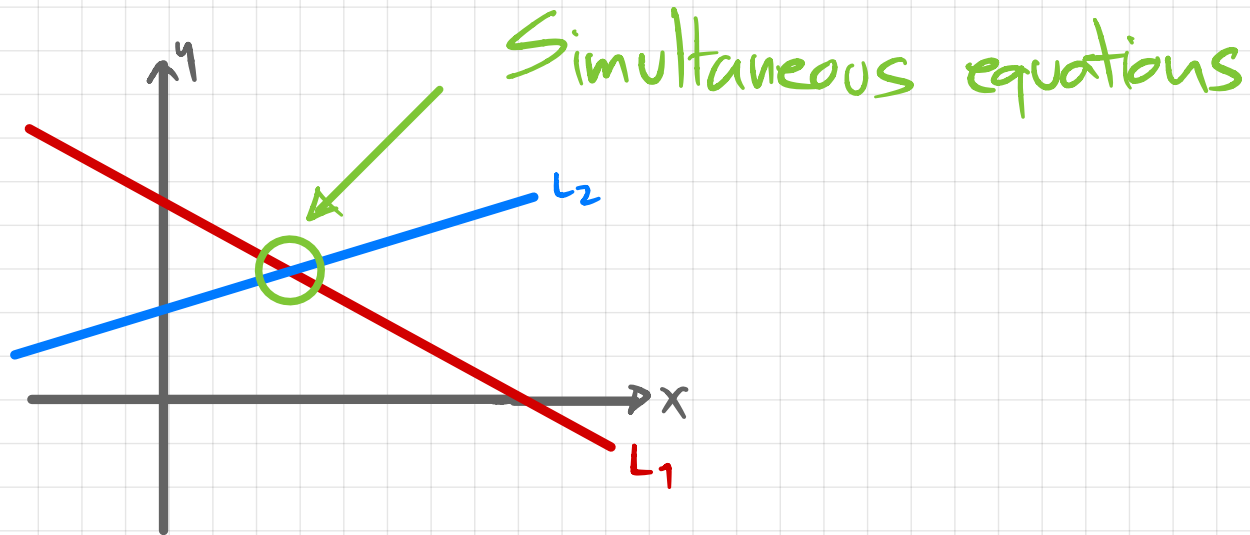
5) Perpendicular lines



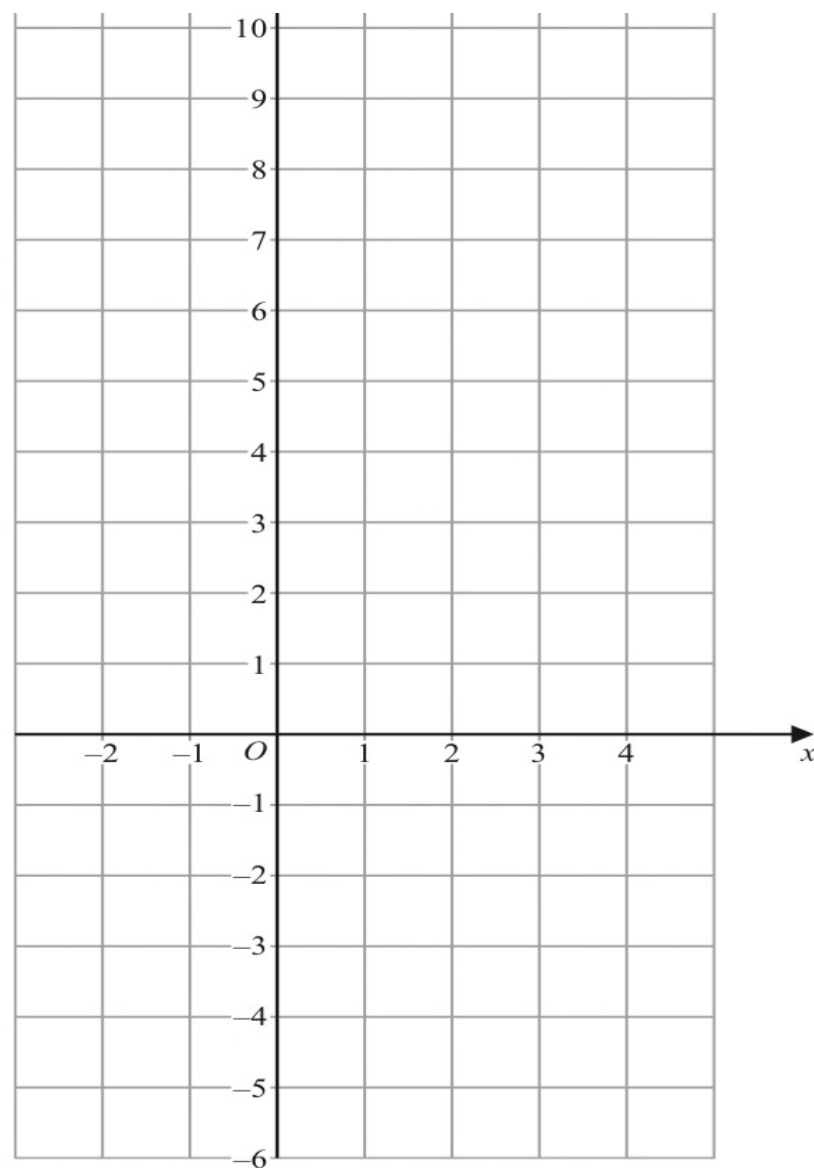
$L_1 \perp L_2 \rightarrow$

$$m_1 \times m_2 = -1$$

6) Intersection point

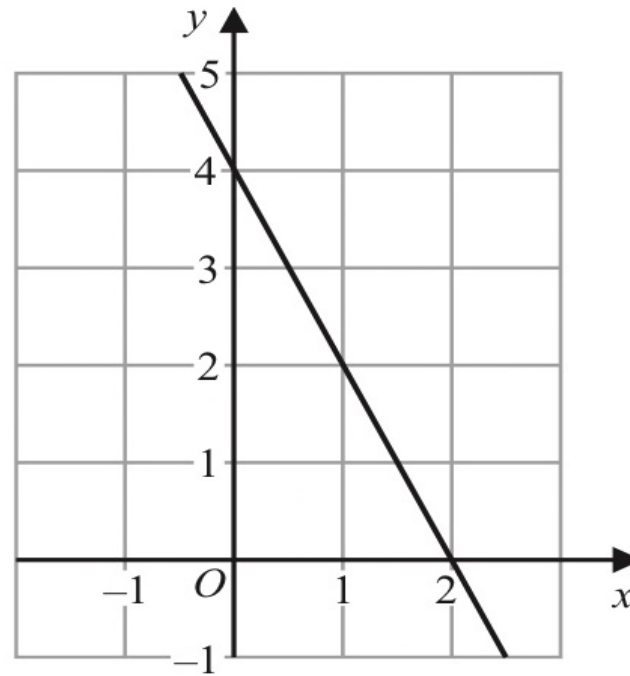


- 5 On the grid, draw the graph of $5x + 2y = 10$ for values of x from -2 to 4



(Total for Question 5 is 3 marks)

9) The diagram shows a straight line drawn on a grid.



(d) Write down an equation of the line.

.....
(2)

13 A is the point with coordinates $(-5, 12)$

B is the point with coordinates $(19, -48)$

Find an equation of the straight line that passes through the points A and B

10 The straight line **L** has equation $2y + 7x = 10$

(a) Find the gradient of **L**

.....
(2)

(b) Find the coordinates of the point where **L** crosses the y -axis.

(.....,)
(1)

(Total for Question 10 is 3 marks)

6 The points A and B are on a coordinate grid.

The coordinates of A are $(6, 4)$

The coordinates of B are $(17, j)$ where j is a constant.

The midpoint of AB has coordinates $(k, 15)$ where k is a constant.

Find the value of j and the value of k

$$j = \dots\dots\dots$$

$$k = \dots\dots\dots$$

(Total for Question 6 is 3 marks)

22 $ABCD$ is a rhombus.

The diagonals, AC and BD , intersect at the point M .

The coordinates of M are $(6, -11)$

The points A and C both lie on the line with equation $2y + 7x = 20$

Find the exact coordinates of the point where the line through B and D intersects the y -axis.

(.....,))

25 $ABCD$ is a trapezium with AB parallel to DC

A is the point with coordinates $(-4, 6)$

B is the point with coordinates $(2, 3)$

D is the point with coordinates $(-1, 8)$

The trapezium has one line of symmetry.

The line of symmetry intersects CD at the point E

Work out the coordinates of the point E

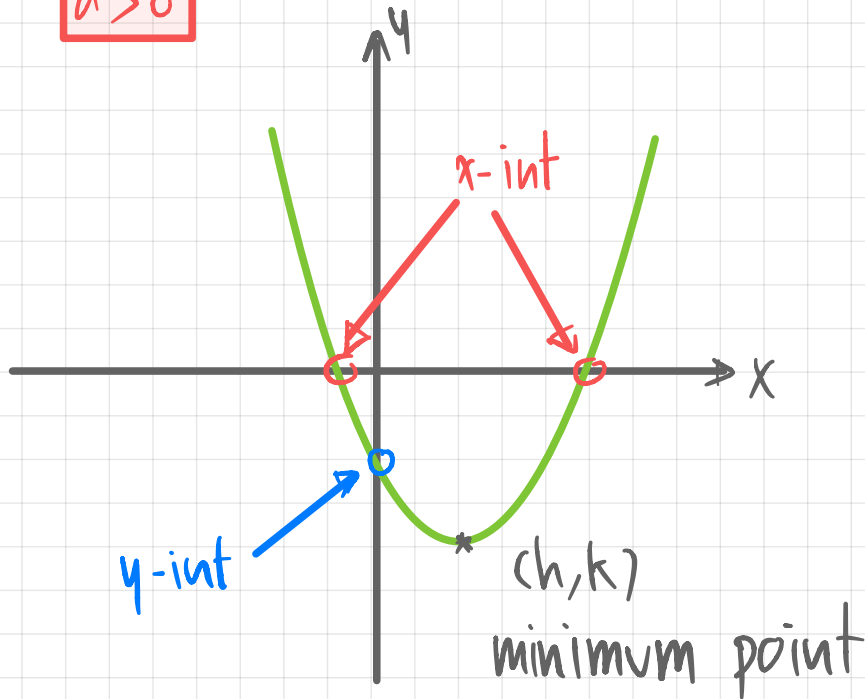
(.....,))

(Total for Question 25 is 6 marks)

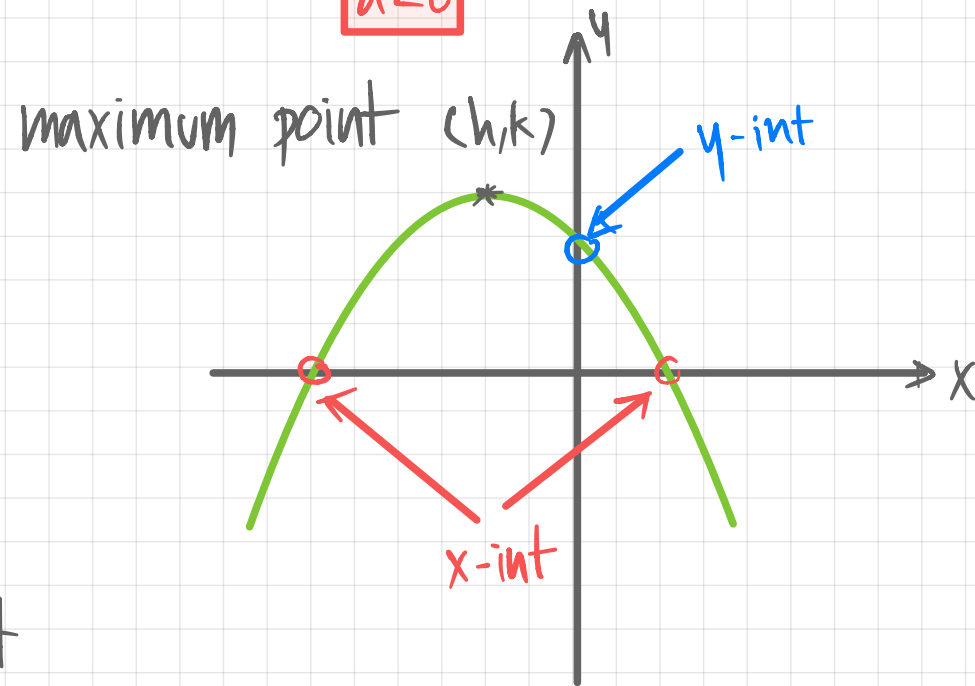
Graphs: Quadratic Graphs.

$$y = a(x-h)^2 + k$$

$$a > 0$$



$$a < 0$$



Symmetry line $x = h$

Minimum value $y = k$ or Maximum value $y = k$

Graphs: Quadratic Graphs.

Example: $f(x) = (x-3)^2 + 1$

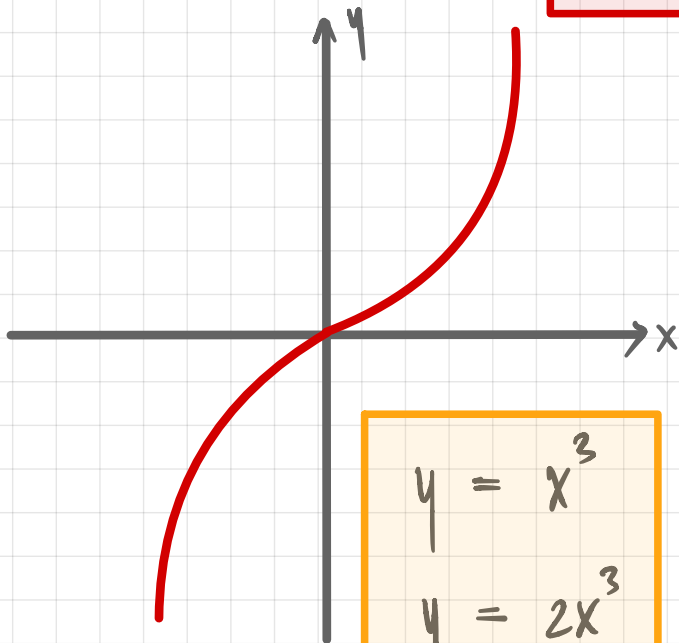
$$y = a(x-h)^2 + k$$

Example: $f(x) = -\frac{1}{2}(x+2)^2 - 3$

Cubic Graphs

$$y = ax^3$$

$$a > 0$$

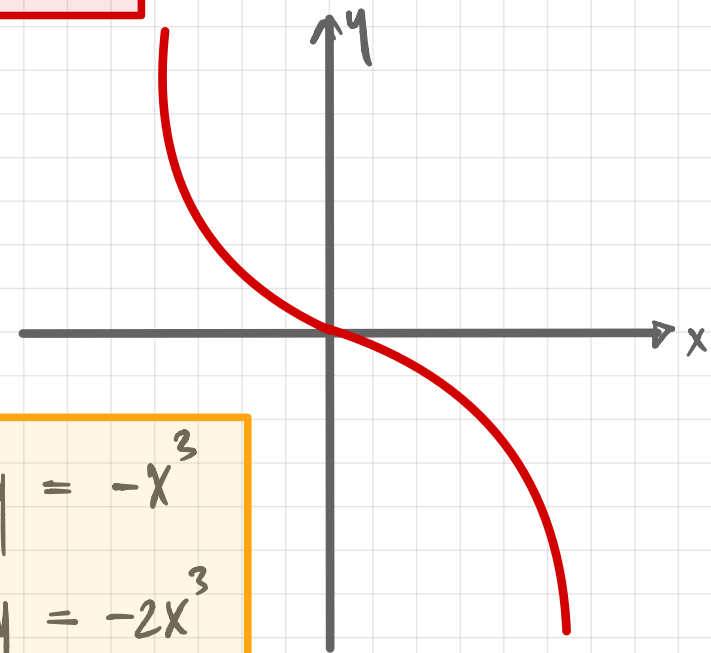


$$y = x^3$$

$$y = 2x^3$$

$$y = \frac{1}{2}x^3$$

$$a < 0$$



$$y = -x^3$$

$$y = -2x^3$$

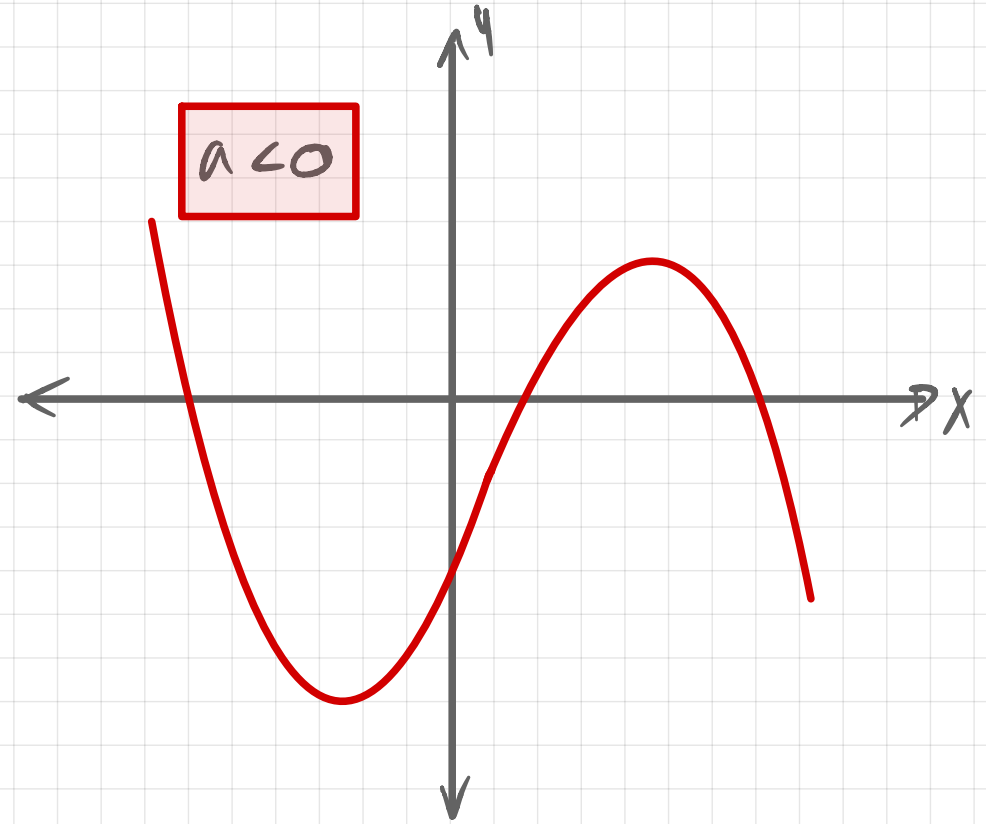
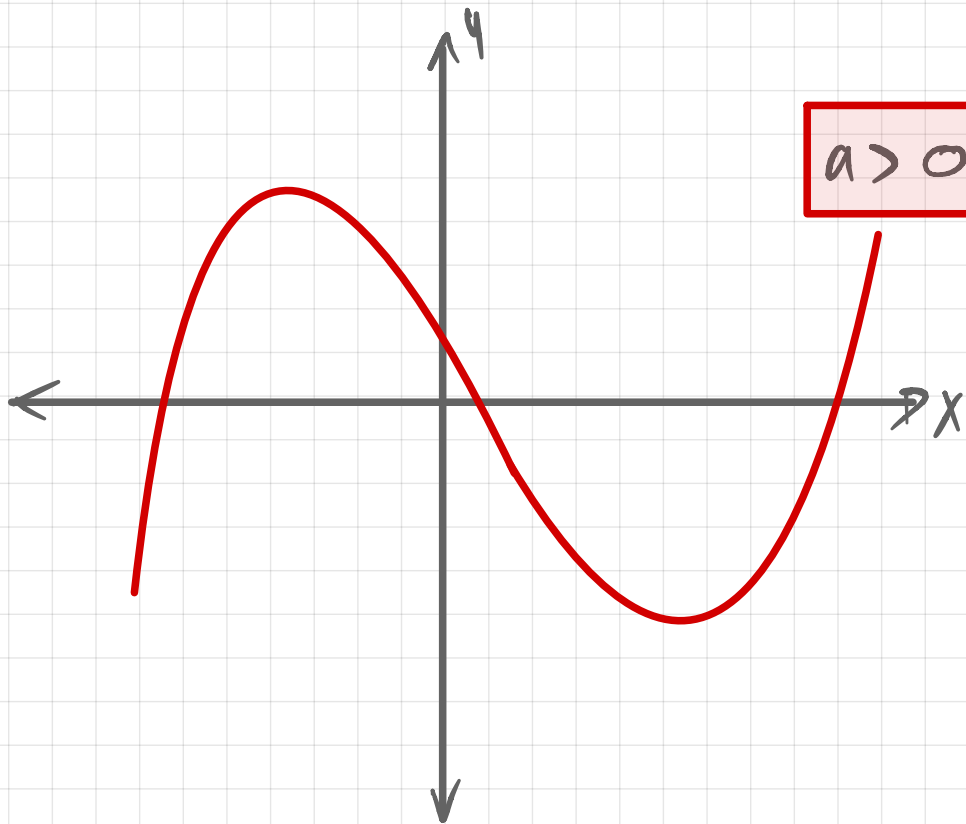
$$y = -\frac{1}{2}x^3$$

Graphs: Harder Graphs.

Cubic Graphs

$$y = ax^3 + bx^2 + cx + d$$

$$y = (\quad)(\quad)(\quad)$$



Graphs: Harder Graphs.

Reciprocal Graphs

$$y = \frac{a}{x}$$

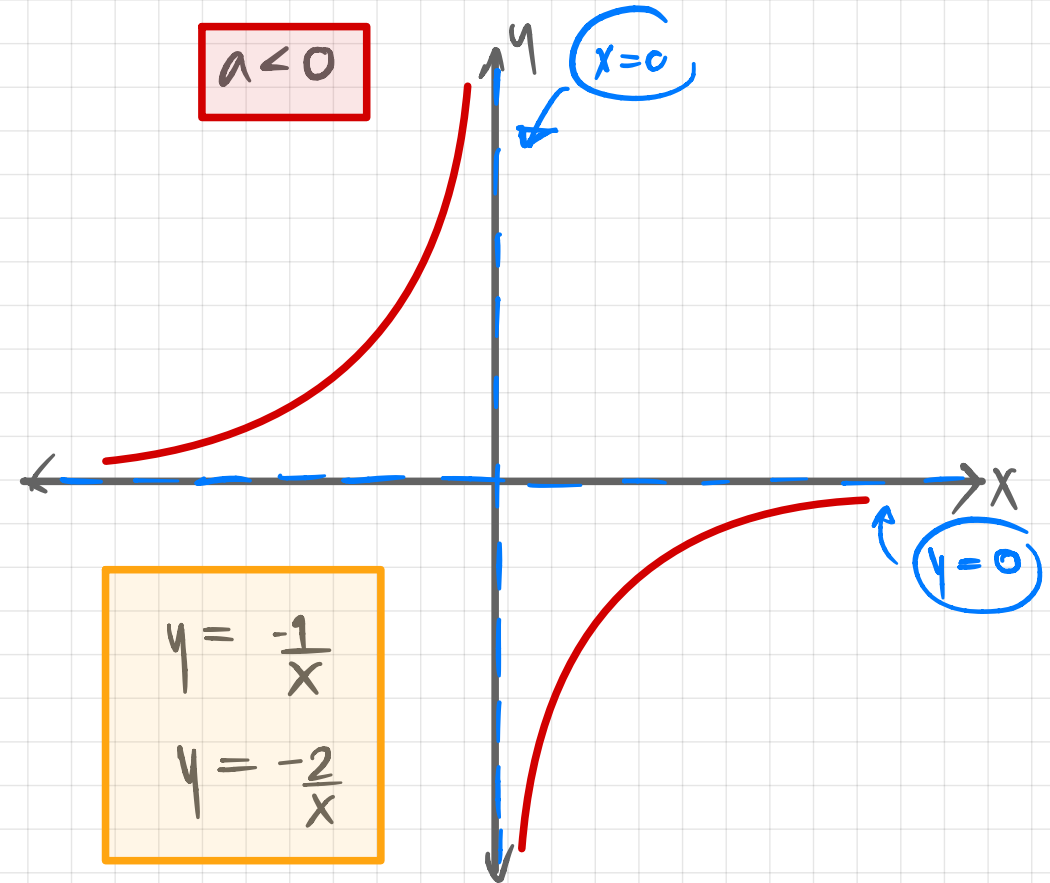
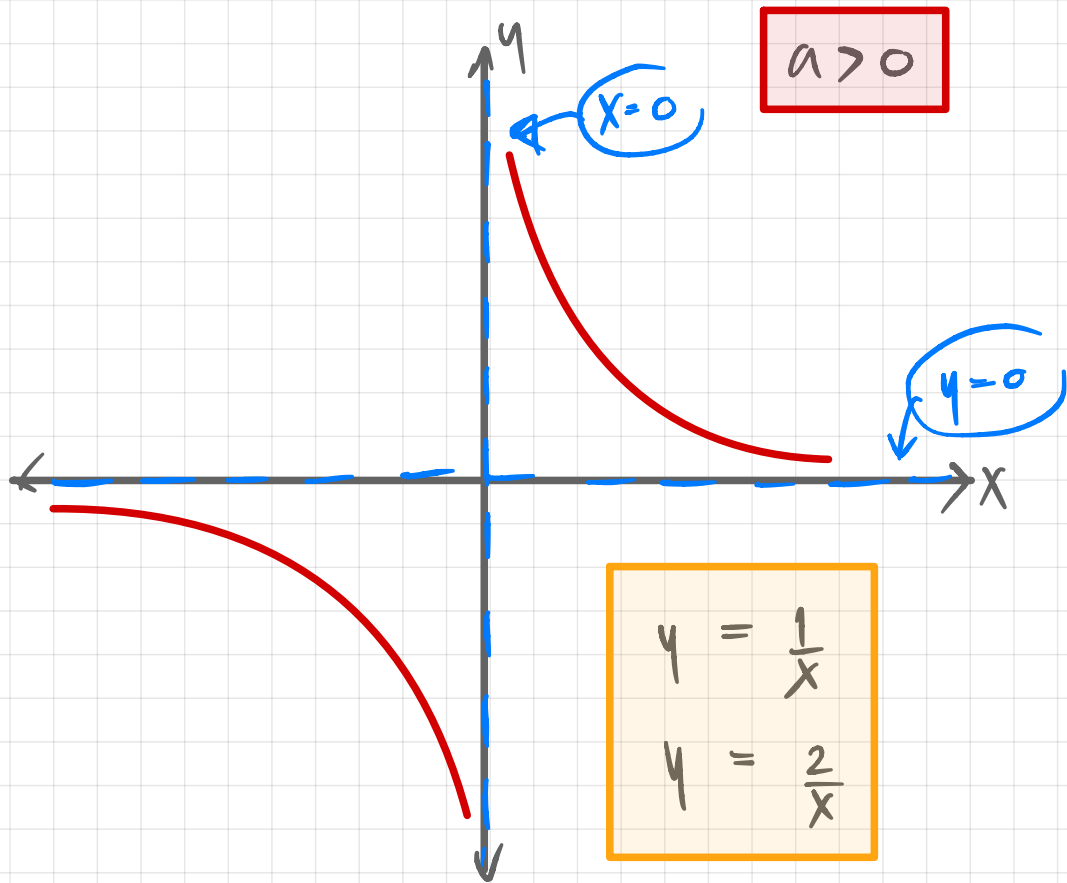


Asymptote $\rightarrow x=0$
 $y=0$



$$xy = a$$

$xy = 5$
 $xy = -3$



Graphs: Harder Graphs.

$$y = \frac{a}{x^2}$$

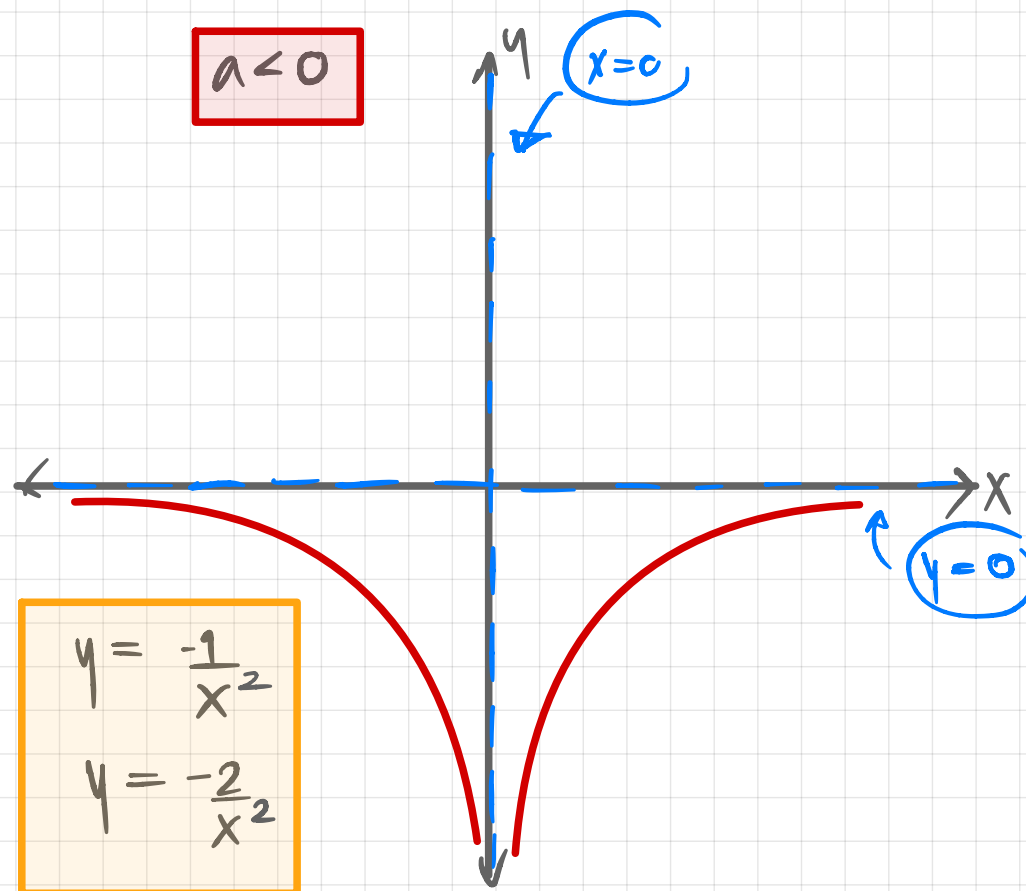
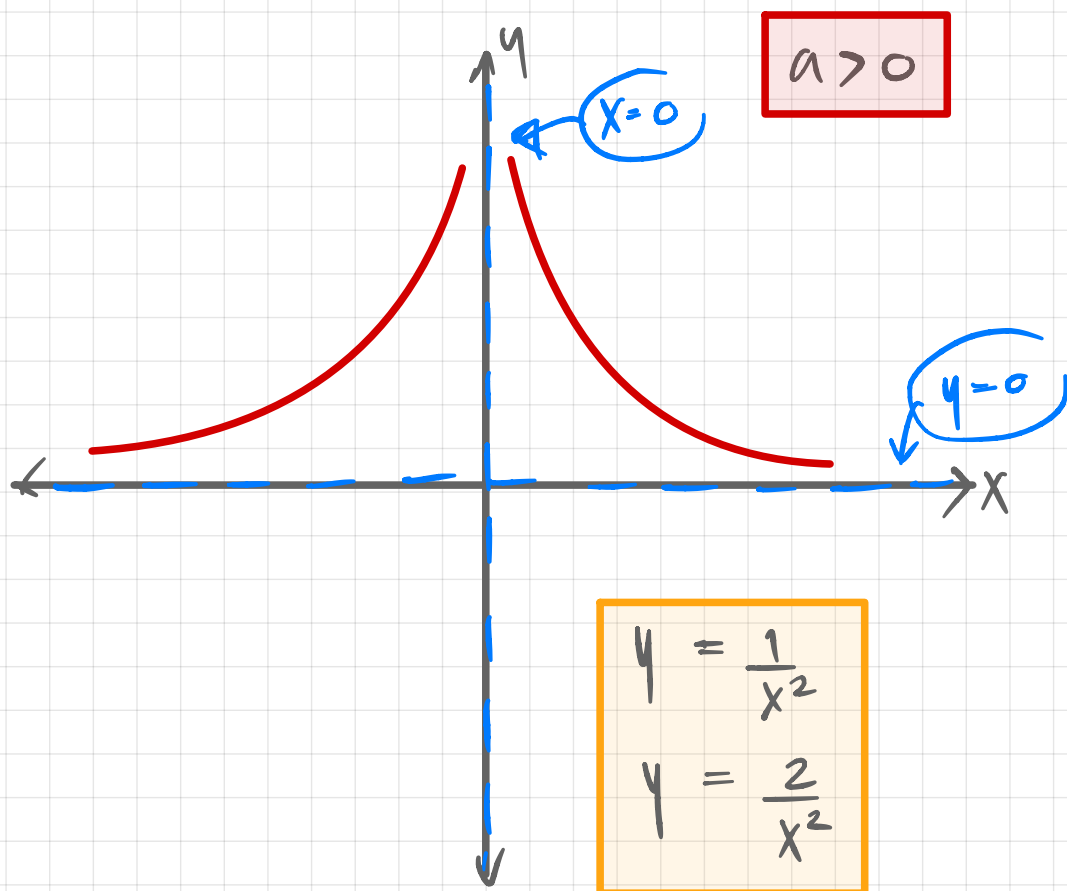


Asymptote $\rightarrow x=0$
 $y=0$



$$yx^2 = a$$

$\rightarrow yx^2 = 3$
 $\rightarrow yx^2 = -5$



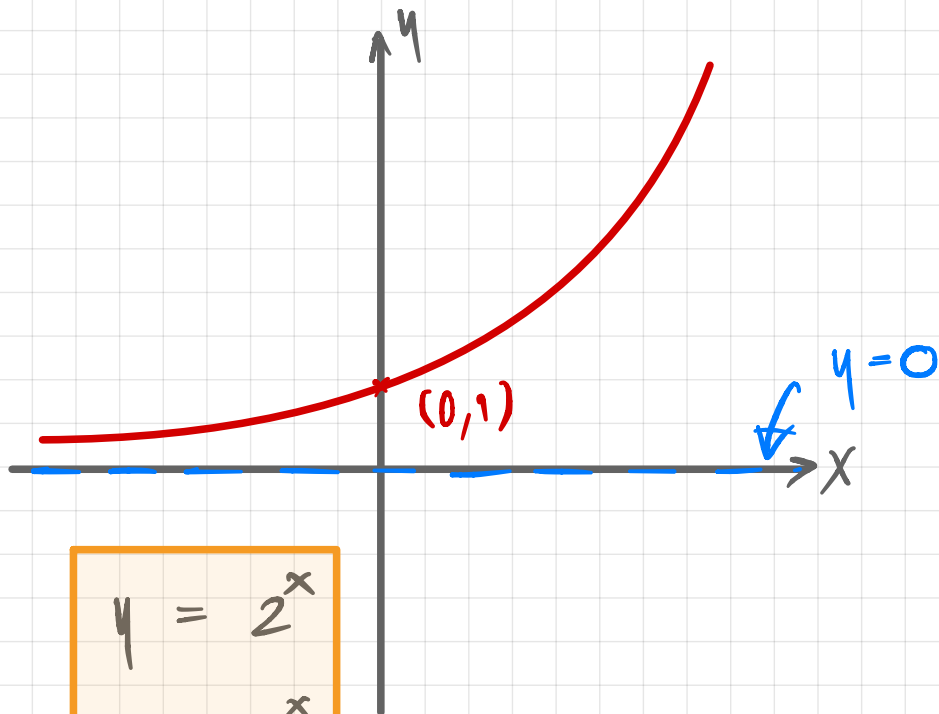
Graphs: Harder Graphs.

Exponential Graphs

$$y = a^x \longrightarrow$$

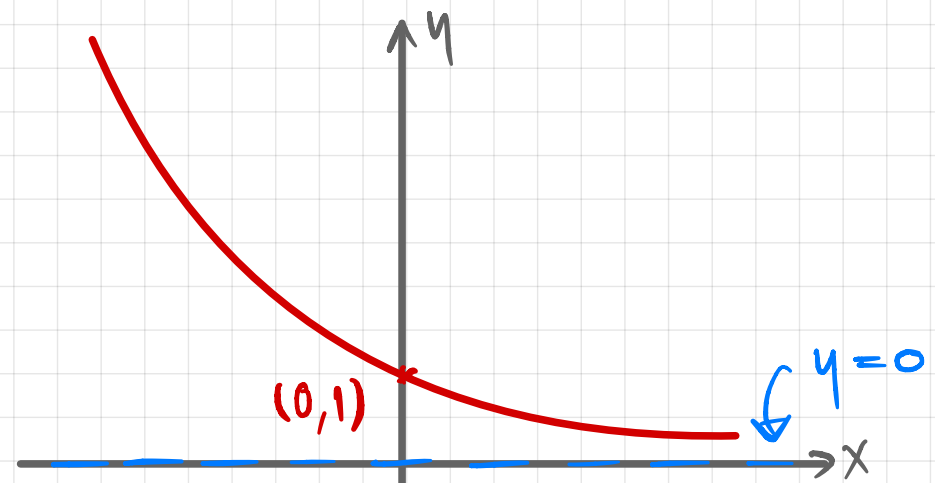
Asymptote $\rightarrow y = 0$

$$a > 1$$



$$y = 2^x$$
$$y = e^x$$

$$0 < a < 1$$



$$y = \left(\frac{1}{2}\right)^x$$
$$y = 3^{-x}$$

$$\left(3^{-1}\right)^x = \left(\frac{1}{3}\right)^x$$

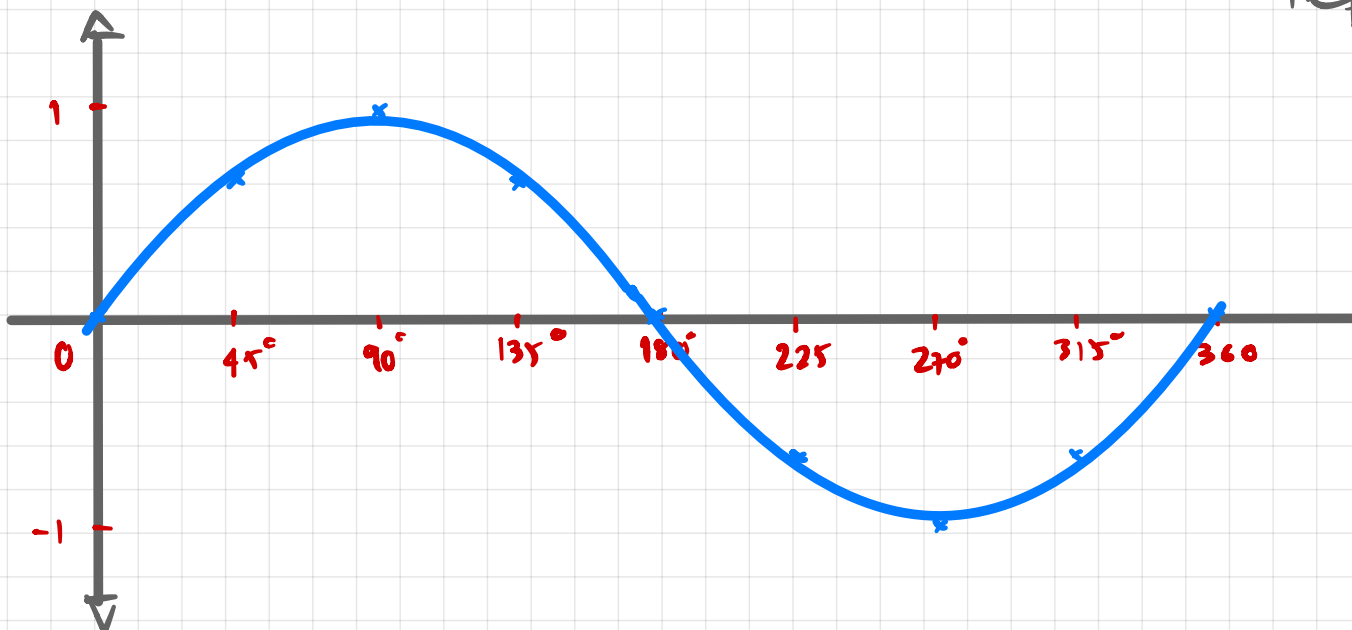
Graphs: Harder Graphs.

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Trigonometry Graphs

$$y = \sin(x) \quad \text{for} \quad 0 \leq x \leq 360$$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	0.7	1	0.7	0	-0.7	-1	-0.7	0



$$\left. \begin{array}{l} \text{Maximum} = 1 \\ \text{Minimum} = -1 \end{array} \right\} \text{Amplitude} = 1$$

Repeats every 360°

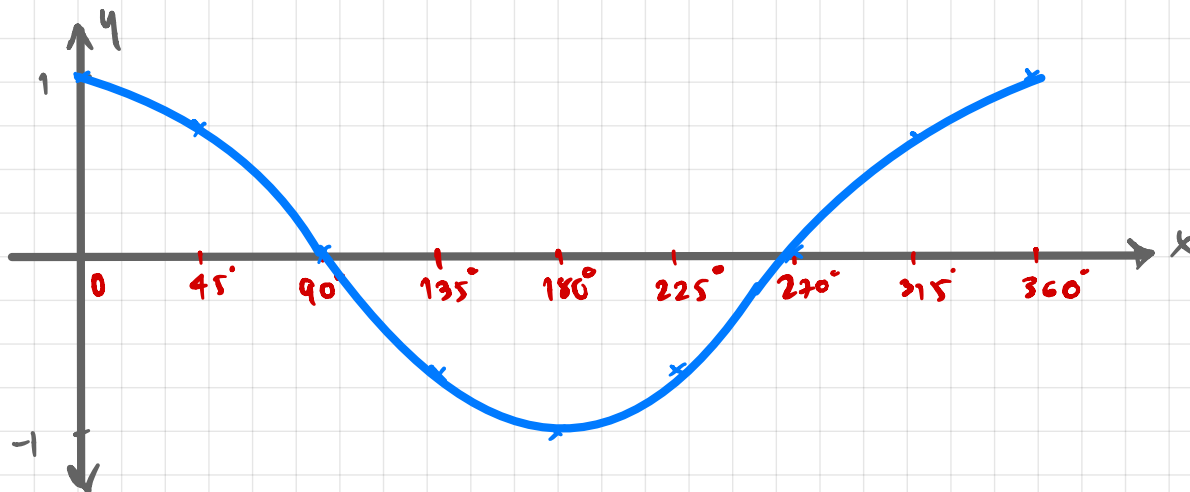
↳ Period = 360°

axis $\rightarrow y = 0$

Graphs: Harder Graphs.

$$y = \cos(x) \quad \text{for} \quad 0 \leq x \leq 360$$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	1	0.7	0	-0.7	-1	-0.7	0	0.7	1



$$\left. \begin{array}{l} \text{Maximum} = 1 \\ \text{Minimum} = -1 \end{array} \right\} \text{Amplitude} = 1$$

Repeats every 360°

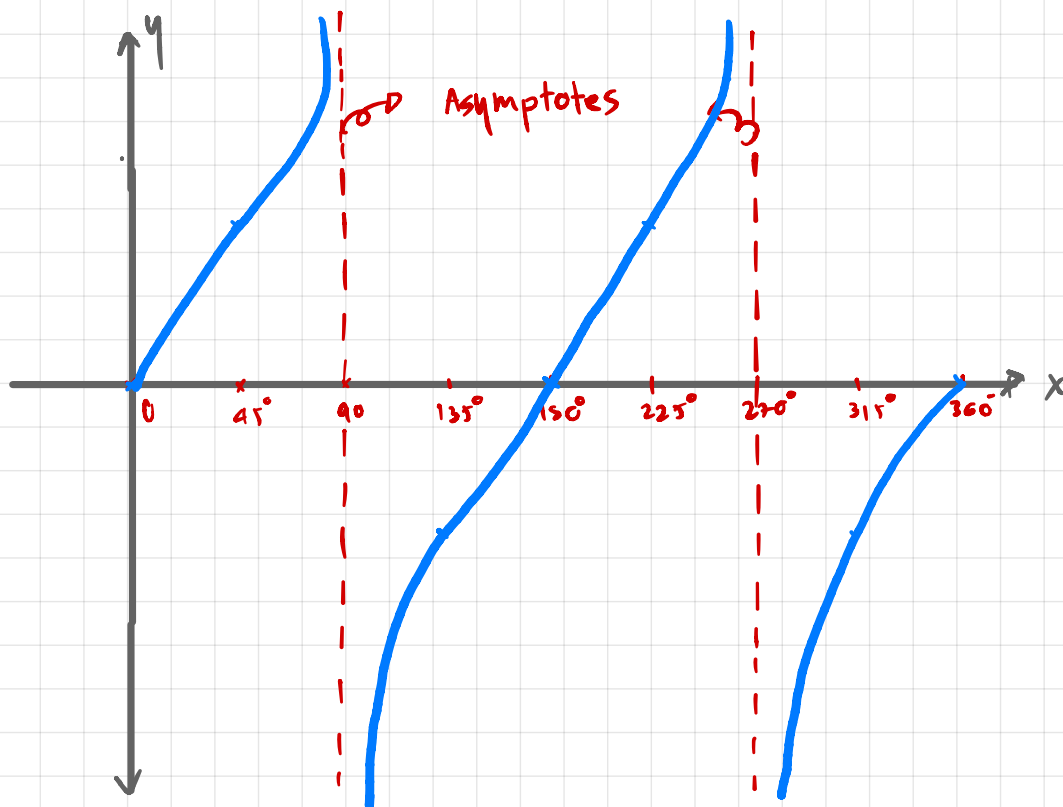
↳ Period = 360°

axis $\rightarrow y = 0$

Graphs: Harder Graphs.

$$y = \tan(x) \quad \text{for} \quad 0 \leq x \leq 360$$

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	1	-	-1	0	1	-	-1	0



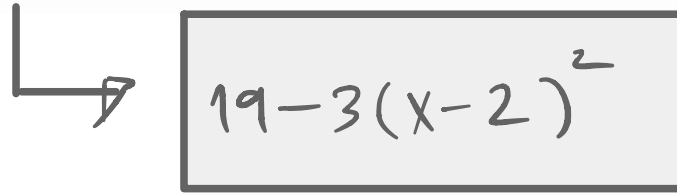
Maximum —

Minimum —

Repeats every = 180°

Asymptotes ; $x = 90$ and $x = 270$

20 (a) Express $7 + 12x - 3x^2$ in the form $a + b(x + c)^2$ where a , b and c are integers.



$19 - 3(x - 2)^2$

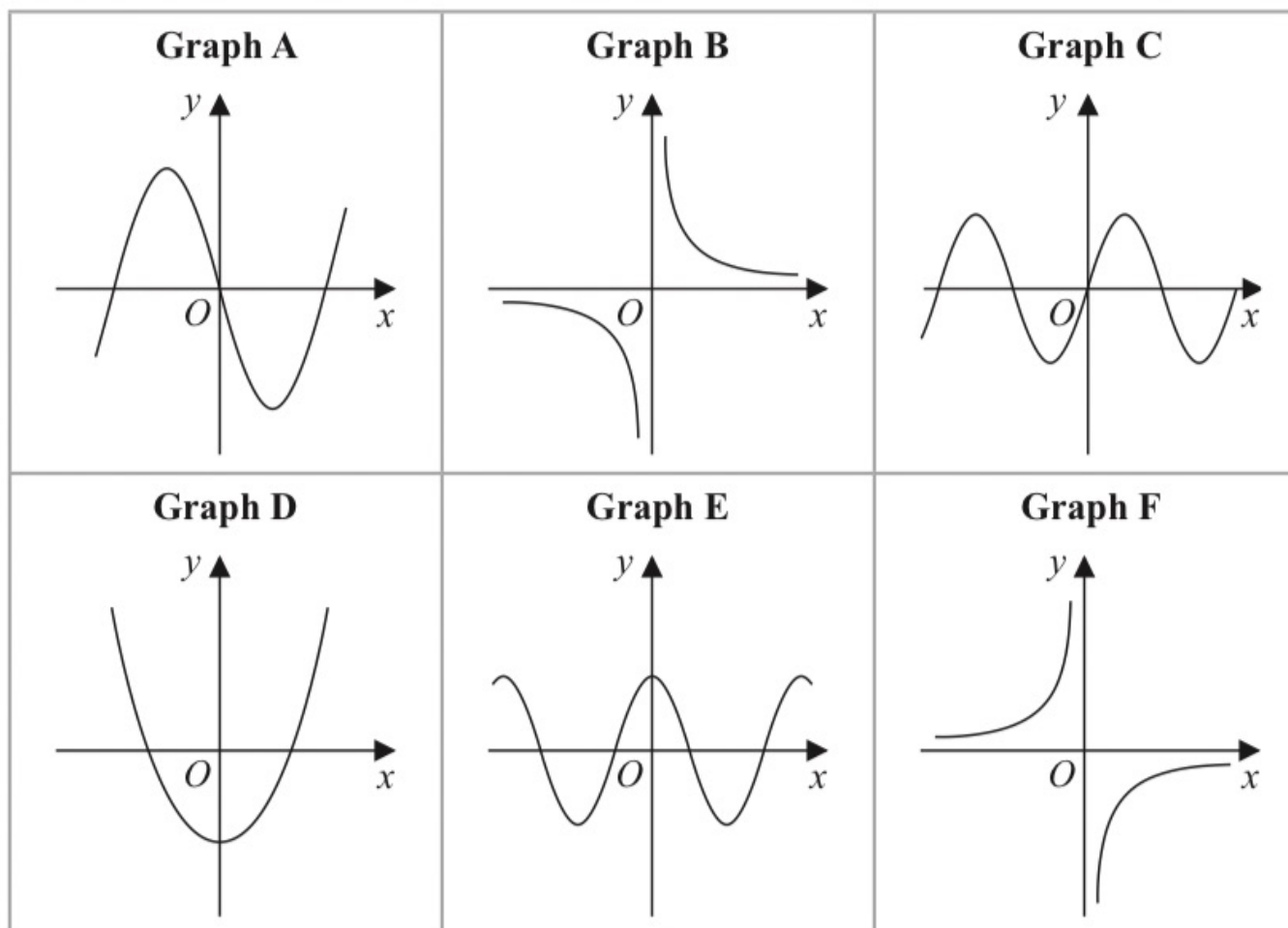
C is the curve with equation $y = 7 + 12x - 3x^2$
The point *A* is the maximum point on **C**

(b) Use your answer to part (a) to write down the coordinates of *A*

(.....,)
(1)

(Total for Question 20 is 4 marks)

18 Here are 6 graphs.

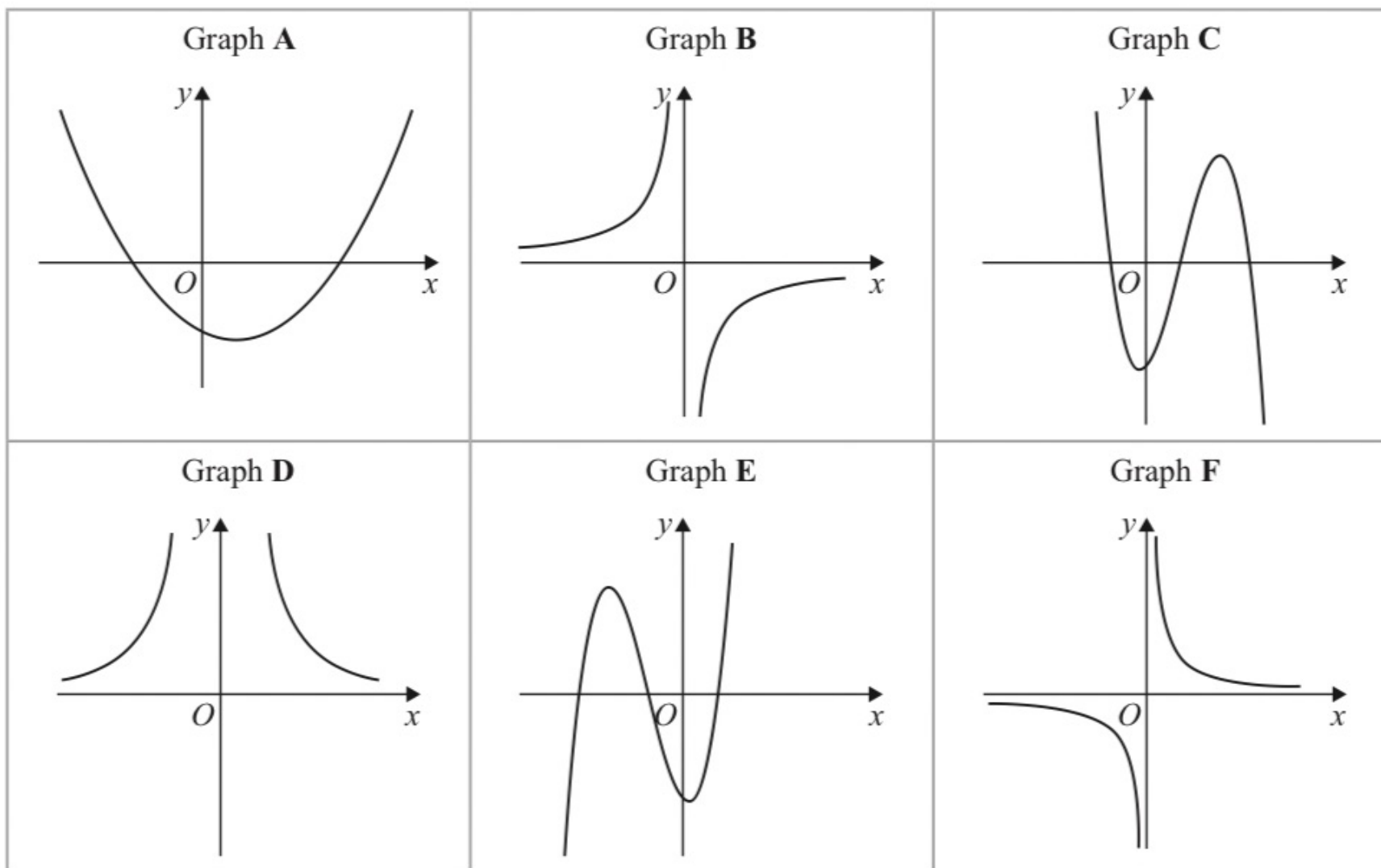


Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = \sin x$
$y = -\frac{3}{x}$
$y = 4x^3 - 5x$

12 Here are six graphs.



Write down the letter of the graph of

(a) $y = \frac{10}{x^2}$

.....
(1)

(b) $y = x - 3 + 3x^2 - x^3$

.....
(1)

(c) $y = -\frac{3}{x}$

.....
(1)

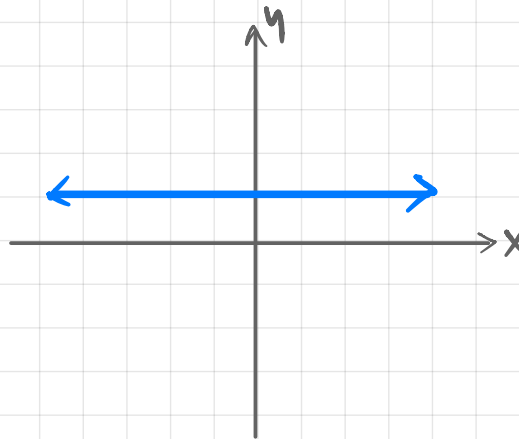
(Total for Question 12 is 3 marks)

Translations

$$f(x+a)$$

$$(x, y) \rightarrow (x-a, y)$$

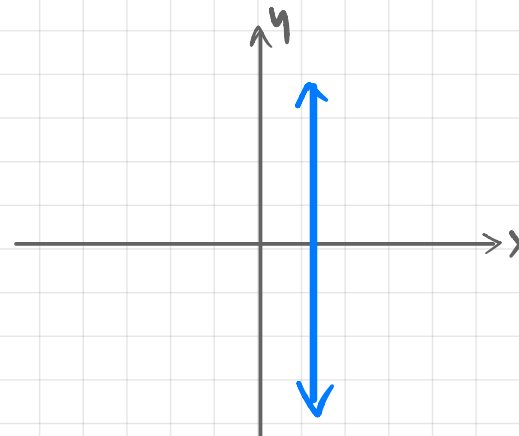
Translations by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$



$$f(x)+a$$

$$(x, y) \rightarrow (x, y+a)$$

Translations by $\begin{pmatrix} 0 \\ a \end{pmatrix}$

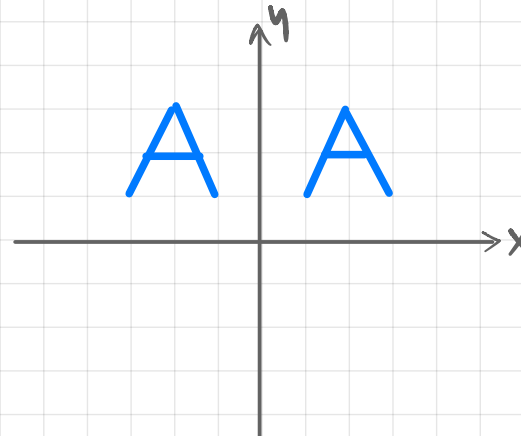


Reflections

$f(-x)$

$$(x, y) \rightarrow (-x, y)$$

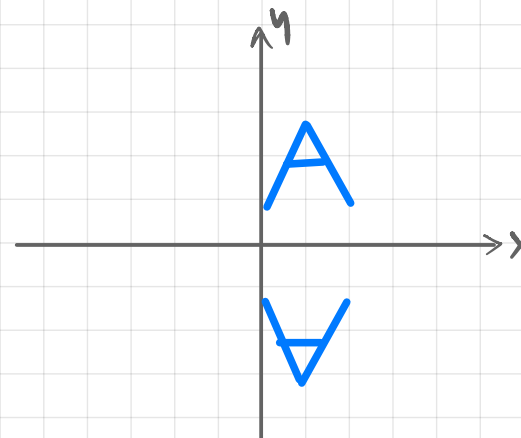
Reflections in the y -axis ($x=0$)



$-f(x)$

$$(x, y) \rightarrow (x, -y)$$

Reflections in the x -axis ($y=0$)

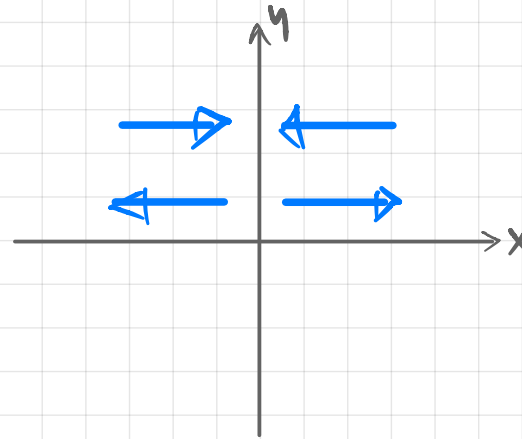


Stretches

$f(ax)$

$$(x, y) \rightarrow \left(\frac{x}{a}, y\right)$$

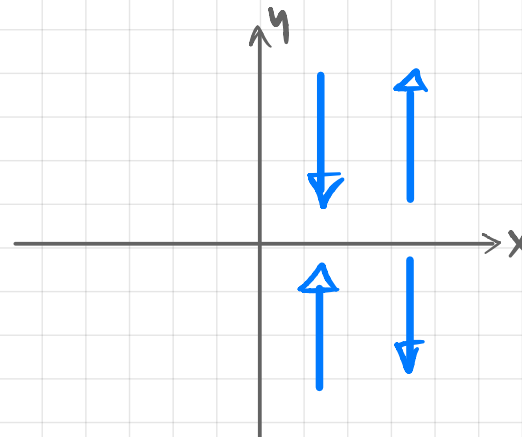
Stretches in the x direction



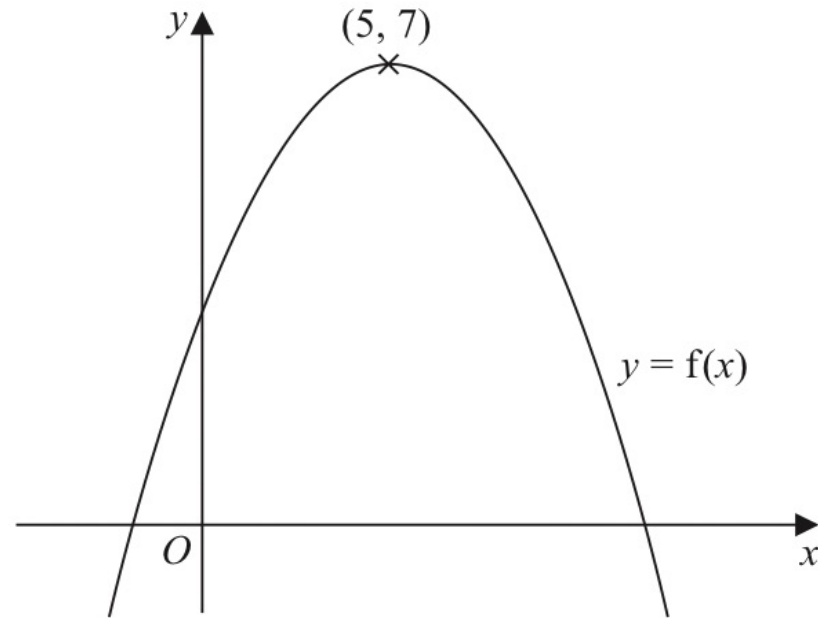
$af(x)$

$$(x, y) \rightarrow (x, ay)$$

Stretches in the y direction



23 The diagram shows a sketch of the curve with equation $y = f(x)$



There is only one maximum point on the curve.
The coordinates of this maximum point are $(5, 7)$

Write down the coordinates of the maximum point on the curve with equation

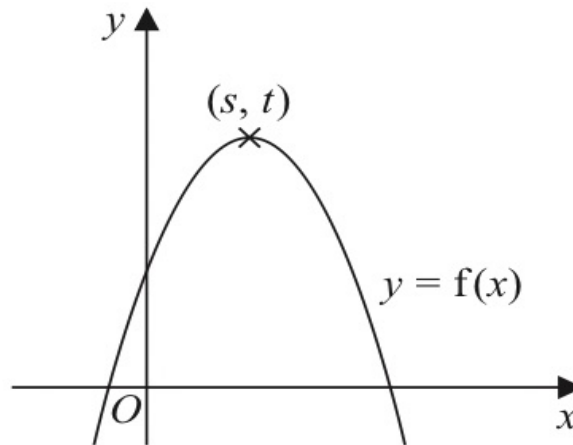
(i) $y = f(x + 9)$

(.....,))

(ii) $y = f(x) + 3$

(.....,))

20 The diagram shows a sketch of part of the curve with equation $y = f(x)$



There is one maximum point on this curve.

The coordinates of this maximum point are (s, t)

Find, in terms of s and t , the coordinates of the maximum point on the curve with equation

(i) $y = f(x - 2)$

(.....,)
(1)

(ii) $y = 3f(x)$

(.....,)
(1)

(Total for Question 20 is 2 marks)

20 Curve **C** has equation $y = f(x)$

The graph of curve **C** has one maximum point.

The coordinates of this maximum point are (3, 5)

(a) Write down the coordinates of the maximum point on the curve with equation

(i) $y = 2f(x)$

(.....,)
(1)

(ii) $y = f(x) - 7$

(.....,)
(1)

(iii) $y = f(-x)$

(.....,)
(1)

20 The curve with equation $y = f(x)$ has one turning point.

The coordinates of this turning point are $(-6, -4)$

(a) Write down the coordinates of the turning point on the curve with equation

(i) $y = f(x) + 5$

(.....,))

(ii) $y = f(3x)$

(.....,))

(2)

21 A curve has equation $y = f(x)$

There is only one turning point on the curve.

The coordinates of this turning point are $(6, 5)$

Write down the coordinates of the turning point on the curve with equation

(a) $y = f(x - 4)$

(..... ,)
(1)

(b) $y = f(3x)$

(..... ,)
(1)

(Total for Question 21 is 2 marks)

Curve **L** has equation $y = x^2 + 7x + 20$

Curve **L** is transformed to curve **S** under the translation $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

(b) Find an equation for **S**

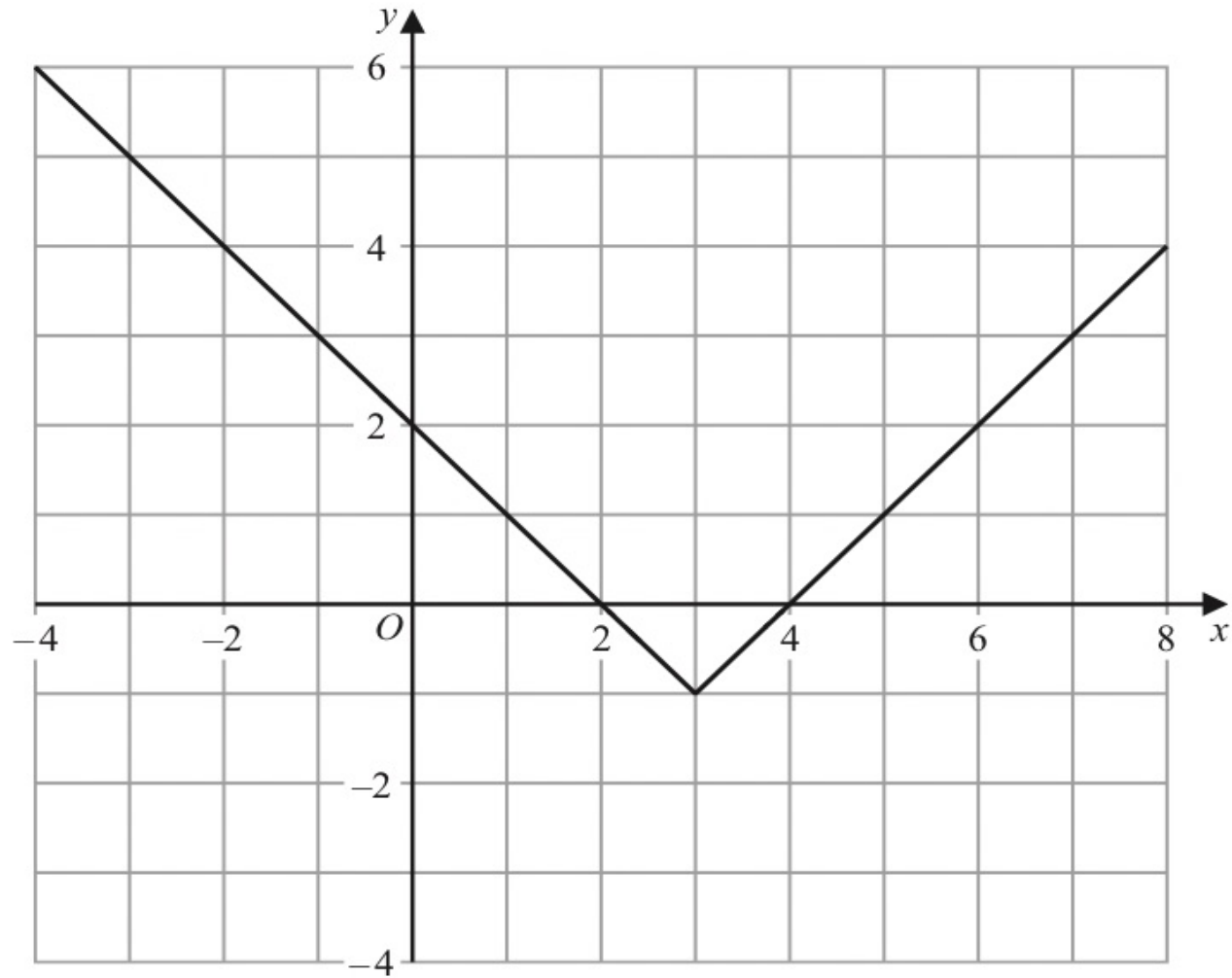
Give your answer in the form $y = ax^2 + bx + c$

$$y = \dots\dots\dots$$

(4)

(Total for Question 20 is 7 marks)

20) The graph of $y = g(x)$ is shown on the grid below.



(b) On the grid, sketch the graph of $y = 2g(x)$ for $-1 \leq x \leq 7$

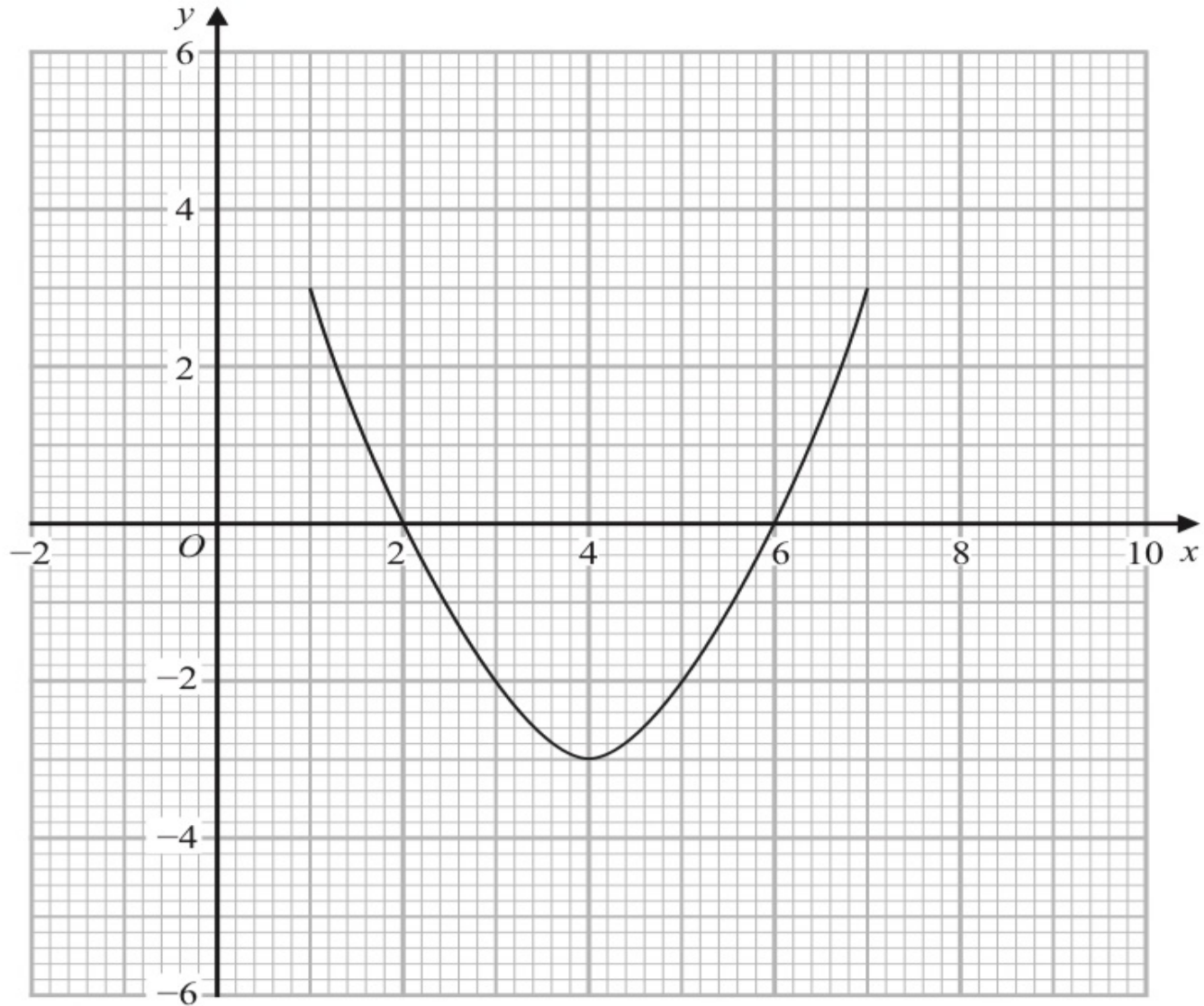
(2)

25 The curve with equation $y = g(x)$ is transformed to the curve with equation $y = -g(x)$ by the single transformation **T**.

(a) Describe fully the transformation **T**.

(1)

25) The diagram shows the graph of $y = f(x)$



(b) On the grid, draw the graph of $y = 2f(x - 1)$

(2)

(Total for Question 25 is 3 marks)

20) The graph of $y = h(x)$ intersects the x -axis at two points.
The coordinates of the two points are $(-1, 0)$ and $(6, 0)$

The graph of $y = h(x + a)$ passes through the point with coordinates $(2, 0)$, where a is a constant.

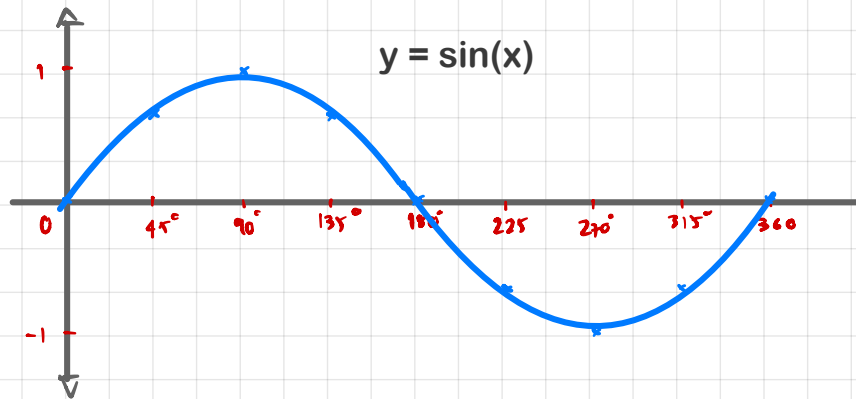
(c) Find the two possible values of a

..... ,

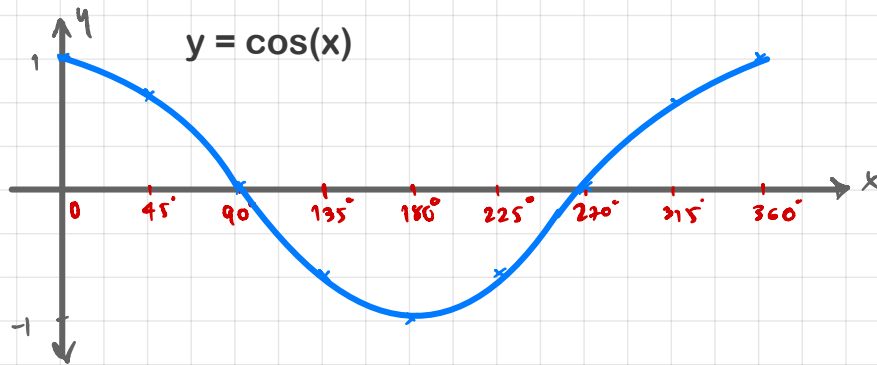
(2)

(Total for Question 20 is 6 marks)

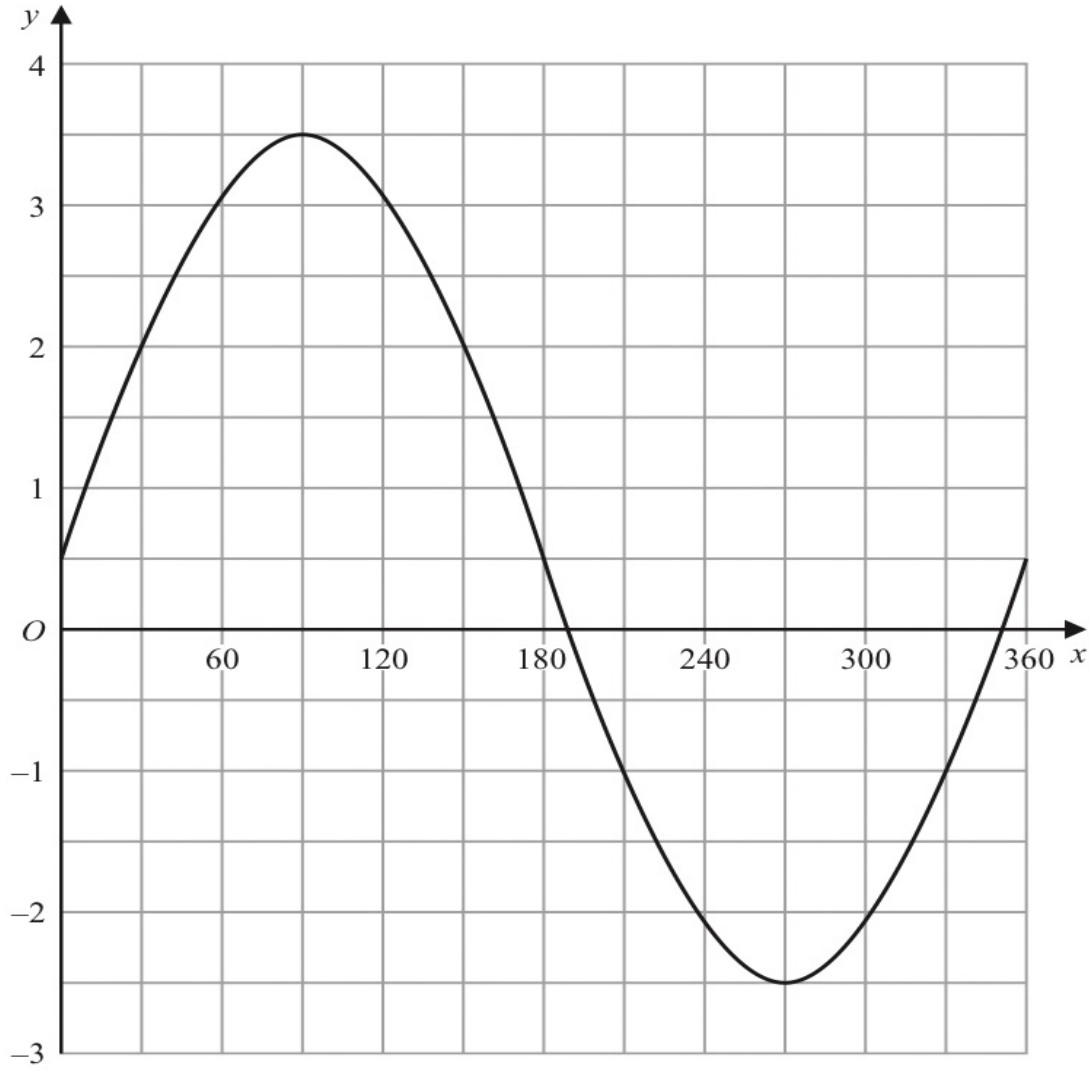
Graph Transformations.



Graph Transformations.



24 The graph of $y = a \sin x^\circ + b$ is drawn on the grid.



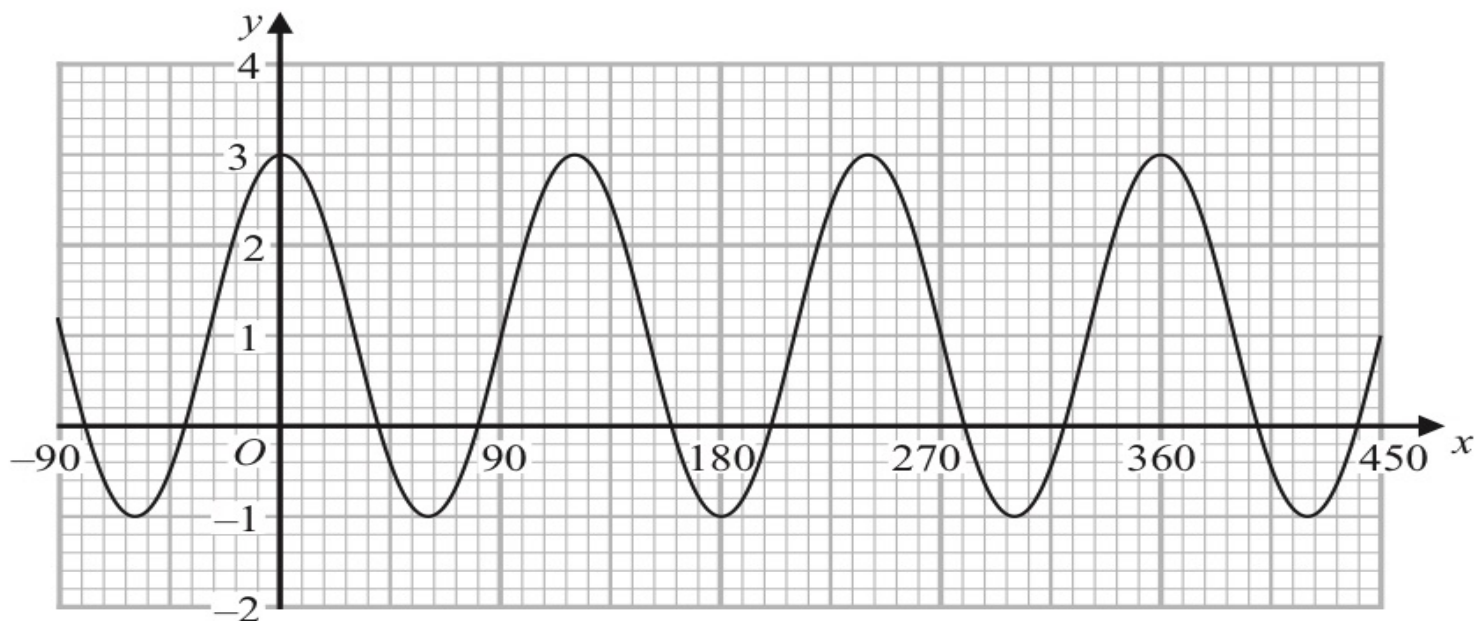
Find the value of a and the value of b

$a = \dots\dots\dots$

$b = \dots\dots\dots$

(Total for Question 24 is 2 marks)

26 Here is a sketch of the curve with equation $y = a \cos bx^\circ + c$ where $-90 \leq x \leq 450$



Find the value of a , the value of b and the value of c

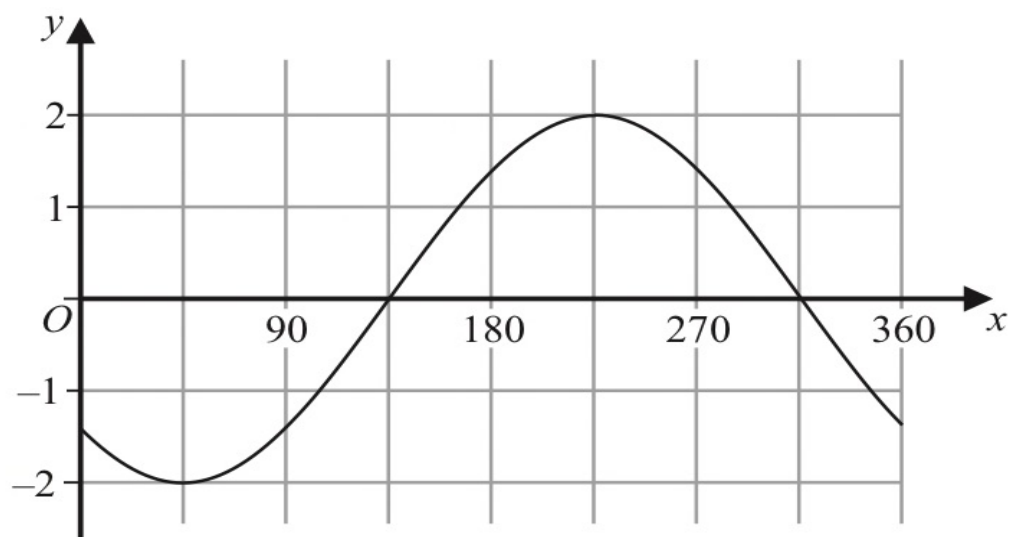
$a =$

$b =$

$c =$

(Total for Question 26 is 3 marks)

20 Here is a sketch of the curve $y = a \cos(x + b)^\circ$ for $0 \leq x \leq 360$



Given that $0 < b < 180$

find the value of a and the value of b

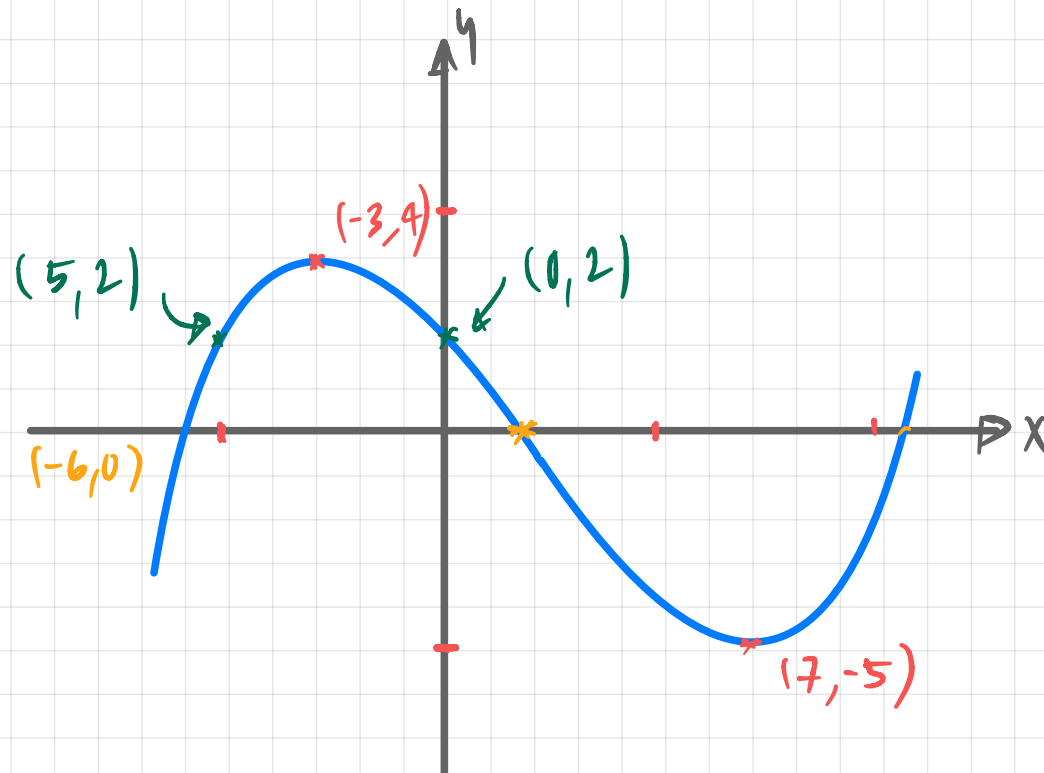
$a = \dots\dots\dots$

$b = \dots\dots\dots$

(Total for Question 20 is 2 marks)

Function

$$f(x) = x^3 + 2x^2 - \dots$$



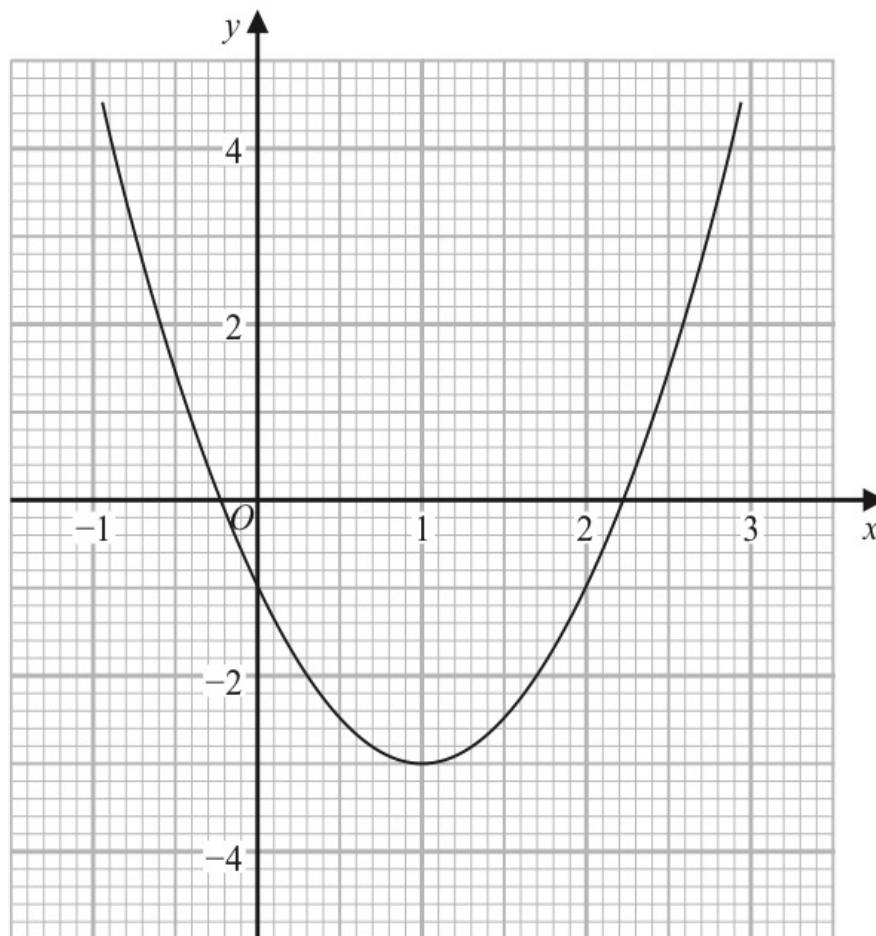
1) Find $f(0)$

2) Find $f(7)$

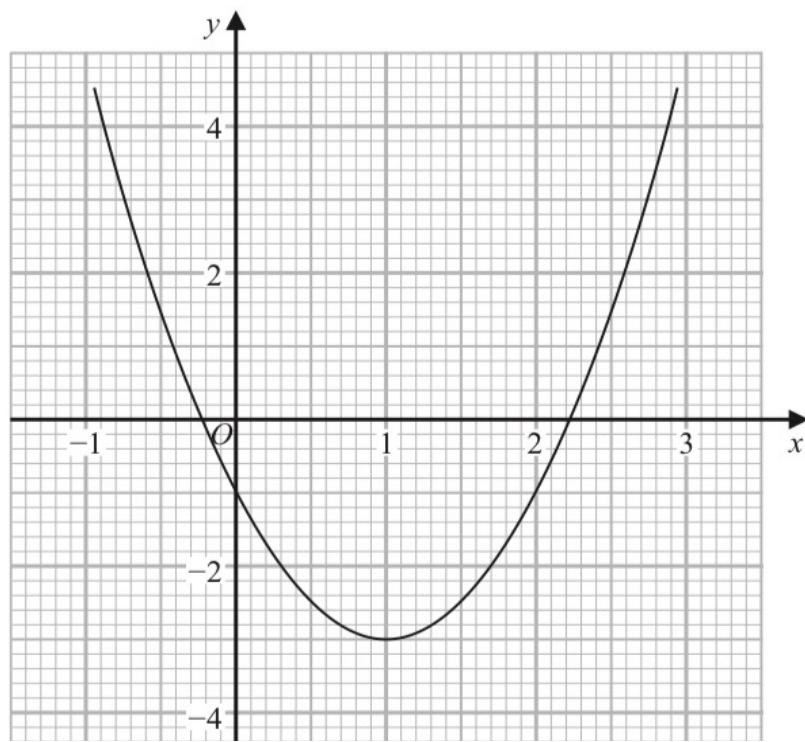
3) Find A when $f(A) = -5$

4) Find B when $f(B) = 2$

21 Part of the graph of $y = 2x^2 - 4x - 1$ is shown on the grid.



- (a) Use the graph to find estimates for the solutions of the equation $2x^2 - 4x - 1 = 0$
Give your solutions correct to one decimal place.



- (b) By drawing a suitable straight line on the grid, find estimates for the solutions of the equation $x^2 - x - 1 = 0$
 Show your working clearly.
 Give your solutions correct to one decimal place.

.....
 (3)

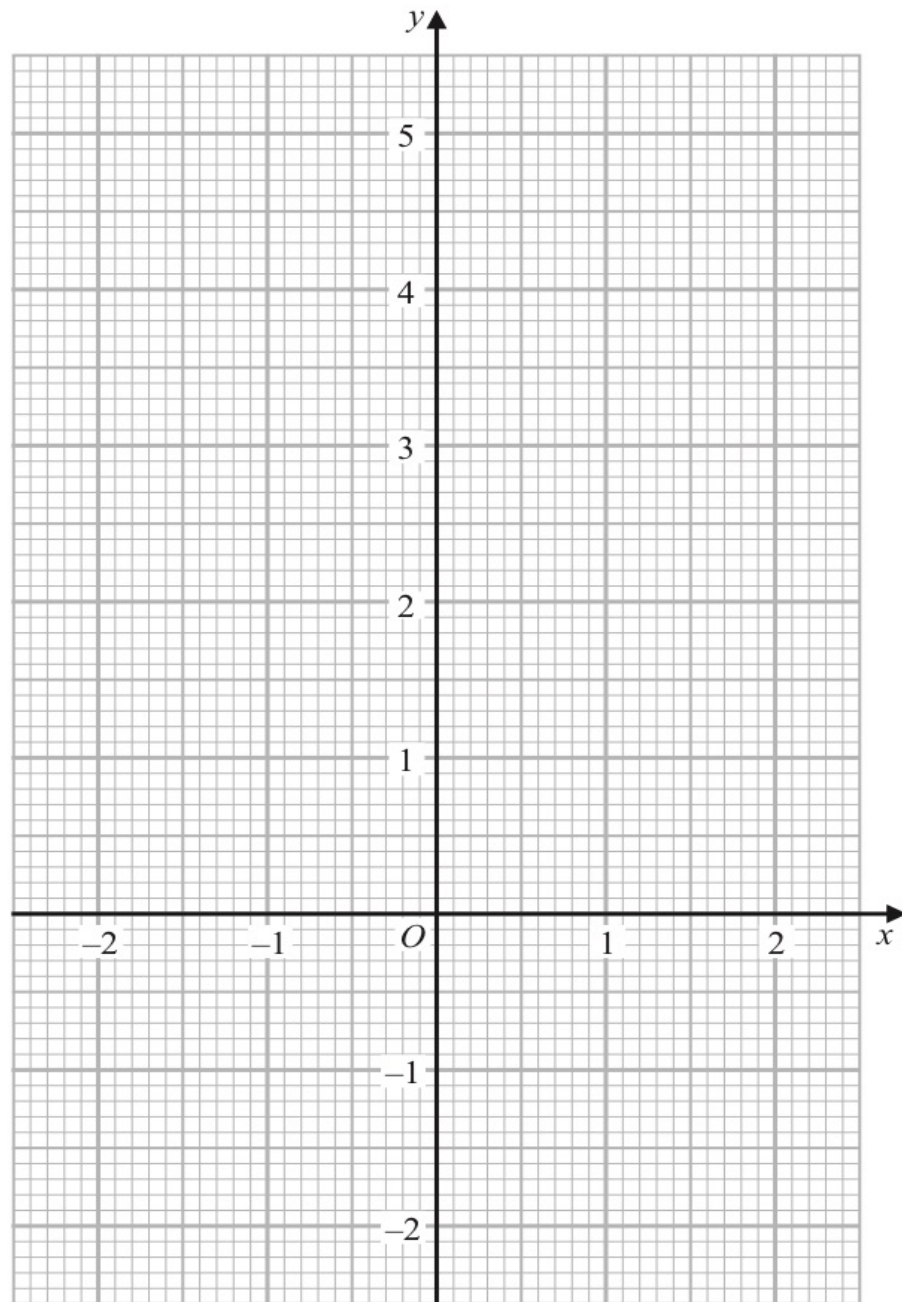
(Total for Question 21 is 5 marks)

15 (a) Complete the table of values for $y = x^3 - 3x + 2$

x	-2	-1	-0.5	0	1	1.5	2
y		4	3.4		0	0.9	

(2)

(b) On the grid, draw the graph of $y = x^3 - 3x + 2$ for values of x from -2 to 2



- (c) By drawing a suitable straight line on the grid, use your graph to find an estimate for the solution of

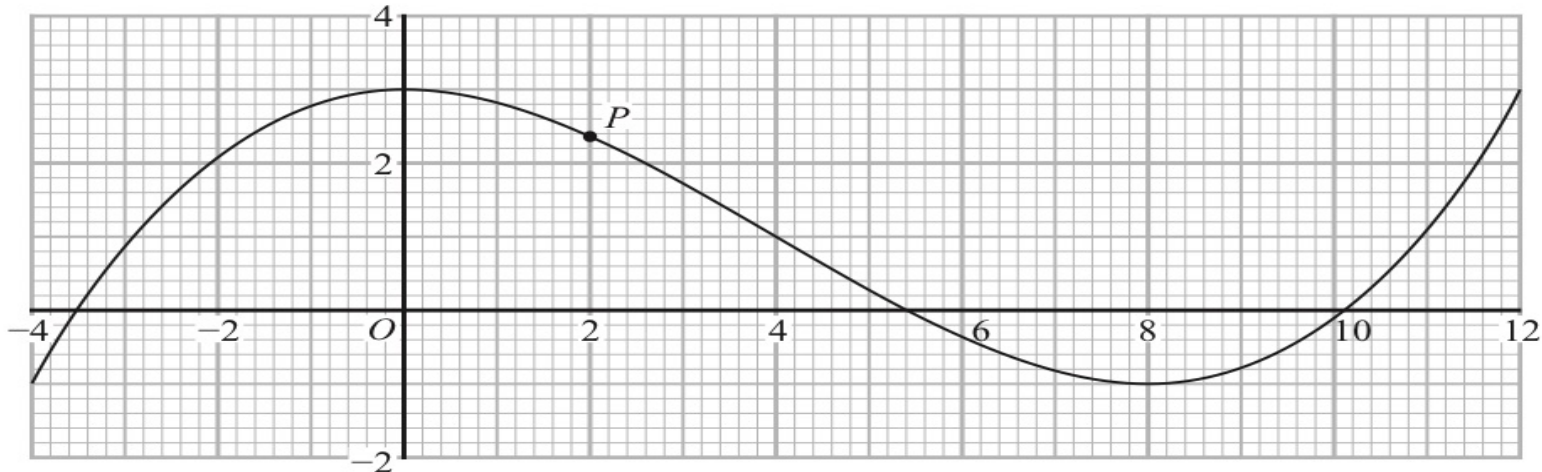
$$2x^3 - 3x + 4 = 0$$

Give your answer correct to one decimal place.

.....
(3)

(Total for Question 15 is 7 marks)

18 The diagram shows the graph of $y = f(x)$ for $-4 \leq x \leq 12$



The point P on the curve has x coordinate 2

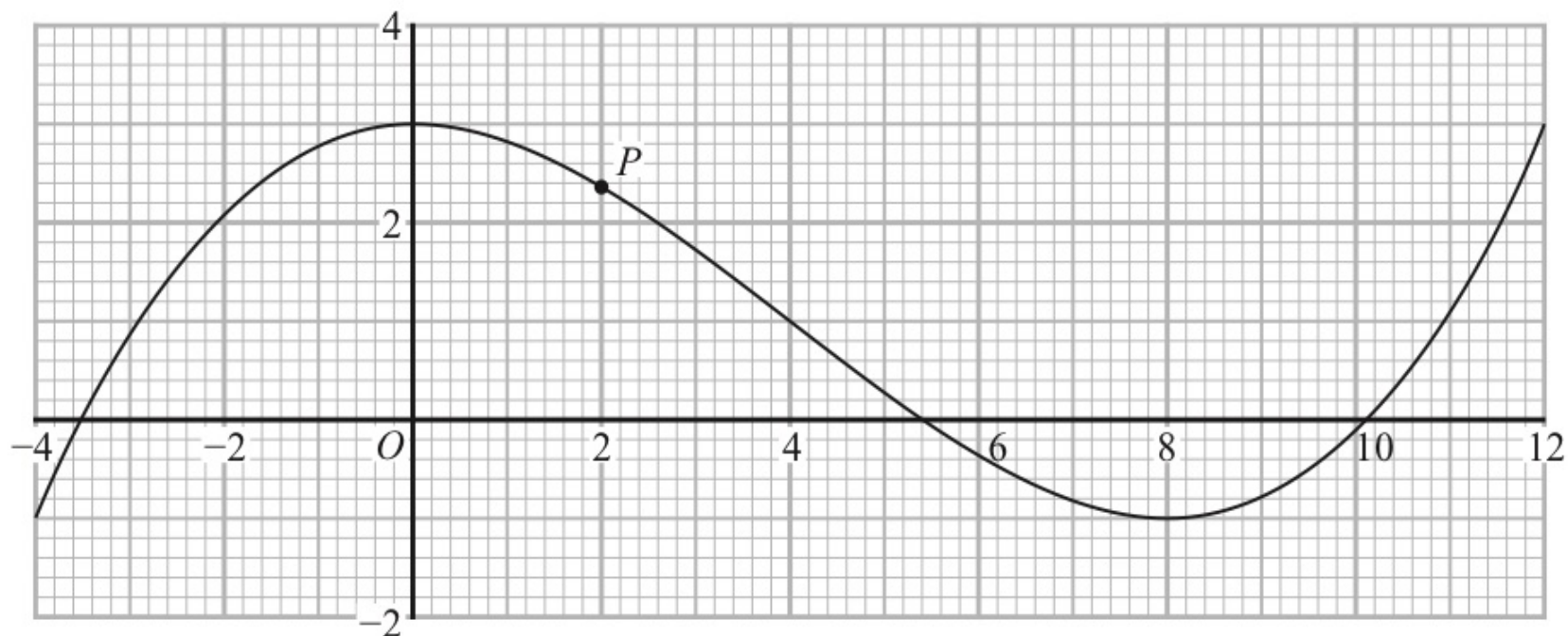
(a) (i) Use the graph to find an estimate for the gradient of the curve at P .

.....
(3)

(ii) Hence find an equation of the tangent to the curve at P .
Give your answer in the form $y = mx + c$

.....
(2)

18 The diagram shows the graph of $y = f(x)$ for $-4 \leq x \leq 12$



The equation $f(x) = k$ has exactly two different solutions for $-4 \leq x \leq 12$

(b) Use the graph to find the two possible values of k .

.....,

(2)

(Total for Question 18 is 7 marks)

Domain and Range of Function

Domain \rightarrow Values of x

Range \rightarrow Values of y

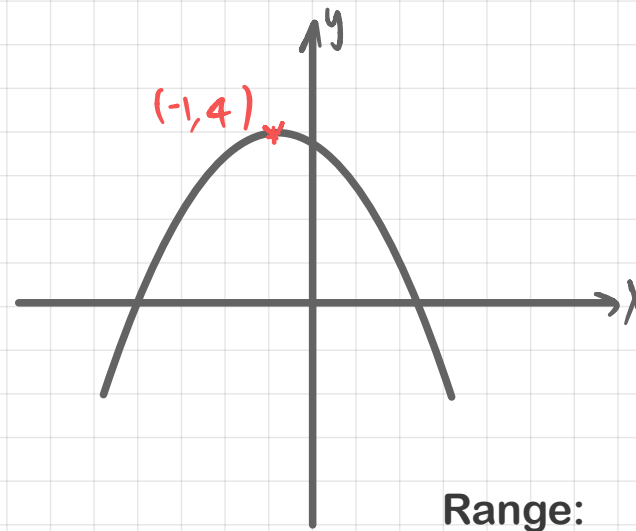
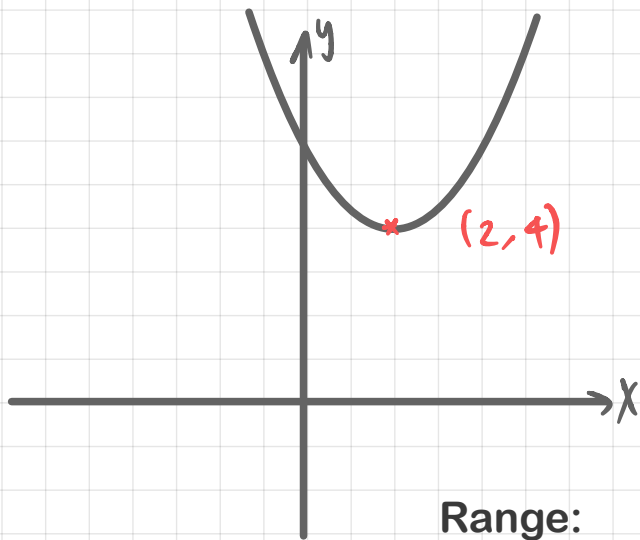
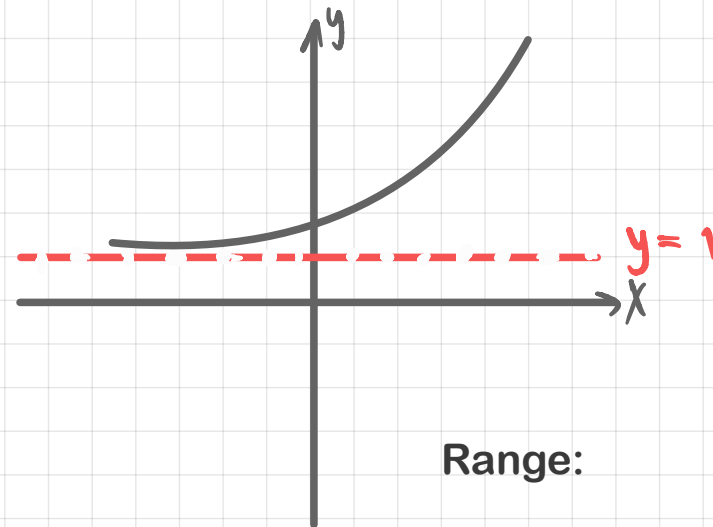
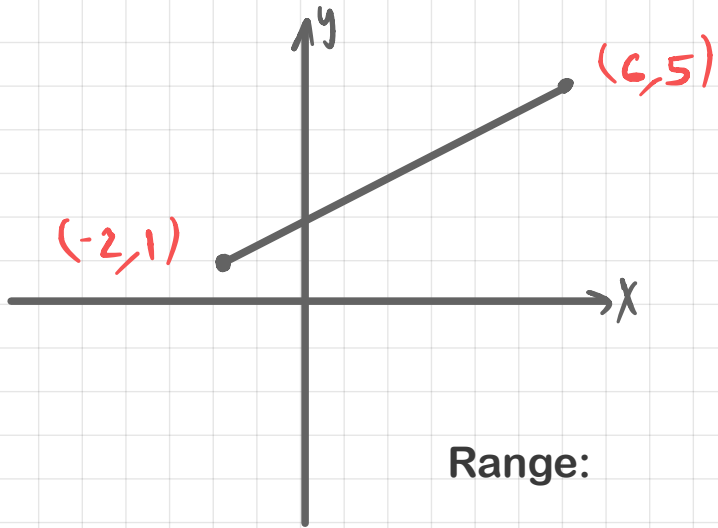
$x \geq 0$ (Domain more than or equal 0)

$x \in \mathbb{R}$ (Domain is element in real number)

$x \in \mathbb{N}$ (Domain is natural number)

$x \neq 2$ (Domain less than 2 or more than 2)

Functions



Which value(s) of x cannot be included in the domain.

Which value(s) of x must be excluded from the domain.

Which value(s) of x for function is undefined

$$f(x) = \frac{\Delta}{\square} \quad \rightsquigarrow \quad \square \neq 0$$

$$f(x) = \sqrt{\square} \quad \rightsquigarrow \quad \square \geq 0$$

Functions

Example: Which value(s) of x for function is undefined.

$$f(x) = \frac{3}{x}$$

$$f(x) = \sqrt{4x+1}$$

$$f(x) = \frac{-4}{4x+1}$$

$$f(x) = 7 + \sqrt{x^2 - 9}$$

$$f(x) = 4 + \frac{5}{x^2 + 2x - 3}$$

Inverse Function

$$f(x) = 3x + 1$$

Range of $f(x)$ \longrightarrow Domain of $f^{-1}(x)$

Domain of $f(x)$ \longrightarrow Range of $f^{-1}(x)$

Function of Inverse Function

$$f(x) = 3x + 1 \longrightarrow f^{-1}(x) = \frac{x - 1}{3}$$



17 The function f is such that $f(x) = (x - 4)^2$ for all values of x .

(a) Find $f(1)$

.....
(1)

(b) State the range of the function f .

.....
(1)

The function g is such that $g(x) = \frac{4}{x+3}$ $x \neq -3$

(c) Work out $fg(2)$

.....
(2)

(Total for Question 17 is 4 marks)

16 The function f is such that

$$f(x) = \frac{2}{3x - 5} \quad \text{where } x \neq \frac{5}{3}$$

(a) Find $f\left(\frac{1}{3}\right)$

.....
(1)

(b) Find $f^{-1}(x)$

$f^{-1}(x) =$
(2)

17 The functions g and h are such that

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \quad x \geq 0$$

(a) What value of x must be excluded from any domain of g ?

.....
(1)

(b) Solve $gh(x) = 1$

$$g(x) = \frac{11}{2x - 5}$$

$$h(x) = x^2 + 4 \quad x \geq 0$$

.....
(3)

(Total for Question 17 is 4 marks)

14 The function f is defined as

$$f:x \mapsto \frac{2x}{x-6} \quad x \neq 6$$

(b) Express the inverse function f^{-1} in the form $f^{-1}:x \mapsto \dots$

$$f^{-1}:x \mapsto \dots\dots\dots$$

(3)

19 $f(x) = x^2 - 4$

$$g(x) = 2x + 1$$

Solve $fg(x) > 0$

Show clear algebraic working.

18 The function f is such that $f(x) = \frac{k}{x}$ where $x \neq 0$ and k is an integer.

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) = \dots$

$$f^{-1}(x) = \dots\dots\dots$$

(1)

The function g is such that $g(x) = 2 - 3x^4$ where $x \neq 0$

The function h is such that $h(x) = \frac{3x}{2-x}$ where $x \neq 2$

(b) (i) Find $g(-2)$

$$\dots\dots\dots$$

(1)

- (ii) Express the composite function hg in the form $hg(x) = \dots$
Give your answer in its simplest form.

$$g(x) = 2 - 3x^4 \text{ where } x \neq 0$$
$$h(x) = \frac{3x}{2-x} \text{ where } x \neq 2$$

$$hg(x) = \dots\dots\dots$$

(2)

(Total for Question 18 is 4 marks)

Review:Indices

$$\longrightarrow ax^n$$

$$a = ax^0 \longrightarrow$$

$$ax = ax^1 \longrightarrow$$

$$(x^m)(x^n) = x^{m+n} \longrightarrow$$

$$\frac{x^m}{x^n} = x^{m-n} \longrightarrow$$

$$\frac{1}{x^m} = 1x^{-m} \longrightarrow$$

$$\sqrt{x} = x^{\frac{1}{2}} \longrightarrow$$

$$\sqrt[3]{x} = x^{\frac{1}{3}} \longrightarrow$$

Differentiation.

Differentiation:

$$\begin{array}{l} y = x \dots \xrightarrow{D} \frac{dy}{dx} = \dots \\ f(x) = x \dots \xrightarrow{D} f'(x) = \dots \end{array}$$

$$ax^n \xrightarrow{D} a(n)x^{n-1}$$

$$3x^5 \xrightarrow{D}$$

$$-4x^{\frac{1}{2}} \xrightarrow{D}$$

$$2x^{-3} \xrightarrow{D}$$

Differentiation.

$$ax \xrightarrow{D} a$$

Prove! $ax = ax^1 \xrightarrow{D} a(1)x^{1-1}$
 $ax^0 = a$

$$5x \xrightarrow{D}$$

$$-100x \xrightarrow{D}$$

$$\frac{2}{3}x \xrightarrow{D}$$

Differentiation.

$$a \xrightarrow{D} 0$$

$$ax^n \xrightarrow{D} a(n)x^{n-1}$$

Prove!

$$a = ax^0 \xrightarrow{D} a(0)x^{0-1} = 0$$

$$5 \xrightarrow{D} \rightarrow$$

$$-100 \xrightarrow{D} \rightarrow$$

$$\frac{2}{3} \xrightarrow{D} \rightarrow$$

Differentiation.

Top Maths IGCSE

Example: Differentiation

$$f(x) = (3x - 5)^2$$

Differentiation.

Example: Differentiation

$$f(x) = 4\sqrt{x} + \frac{5}{2x}$$

Variables

$$y = 7x^2 + 3x \quad \xrightarrow{D}$$

$$A = x^3 + 5 \quad \xrightarrow{D}$$

$$x = 4y - 1 \quad \xrightarrow{D}$$

$$A = \pi r^2 \quad \xrightarrow{D}$$

$$s = 3t^2 + 5t - 2 \quad \xrightarrow{D}$$

Second order derivative

$$y = x \dots \xrightarrow{D} \frac{dy}{dx} = \dots \xrightarrow{D} \frac{d^2y}{dx^2} = \dots$$

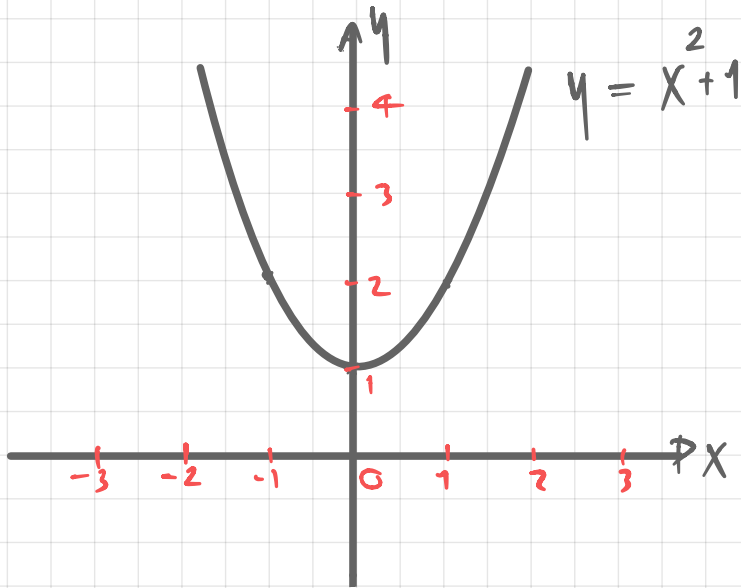
$$f(x) = x \dots \xrightarrow{D} f'(x) = \dots \xrightarrow{D} f''(x) = \dots$$

$$s = t \dots \xrightarrow{D} \frac{ds}{dt} = \dots \xrightarrow{D} \frac{d^2s}{dt^2} = \dots$$

Differentiation.

Top Maths IGCSE

Differentiation to find a Gradient.



1) Differentiating the equation of curve.

2) Find the gradient of the curve at any point by substituting the value for x into 1)...

Find the gradient of curve at $x=1$

Example:

Find the gradient of the graph $y = x^3 - 12x + 1$ at

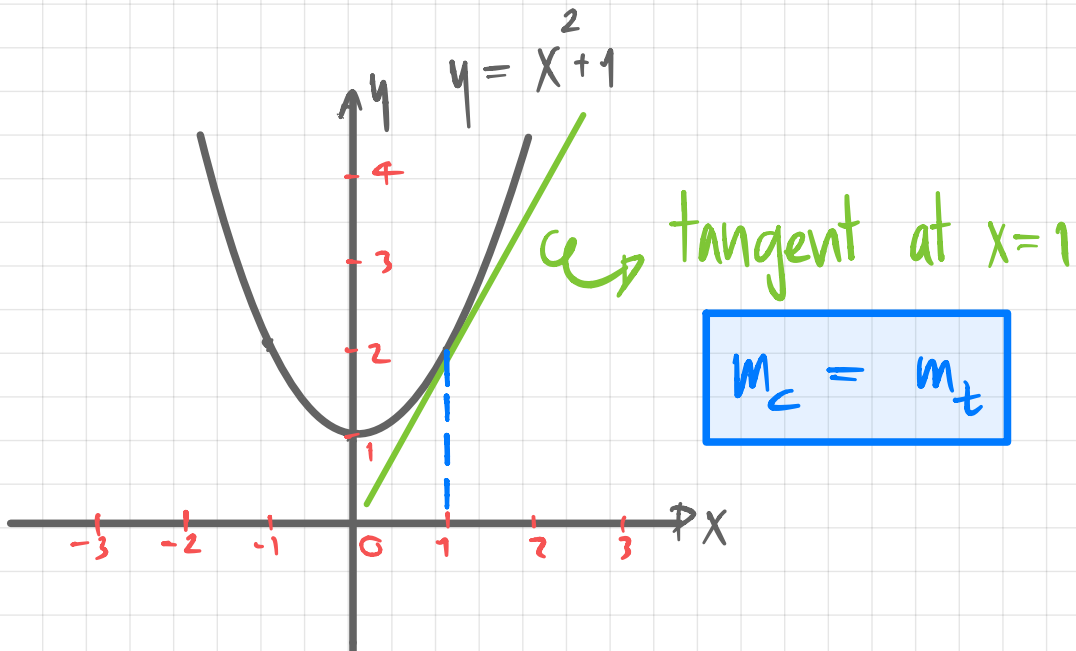
1) $x = -1$

2) $x = 2$

3) $x = \frac{5}{2}$

Differentiation.

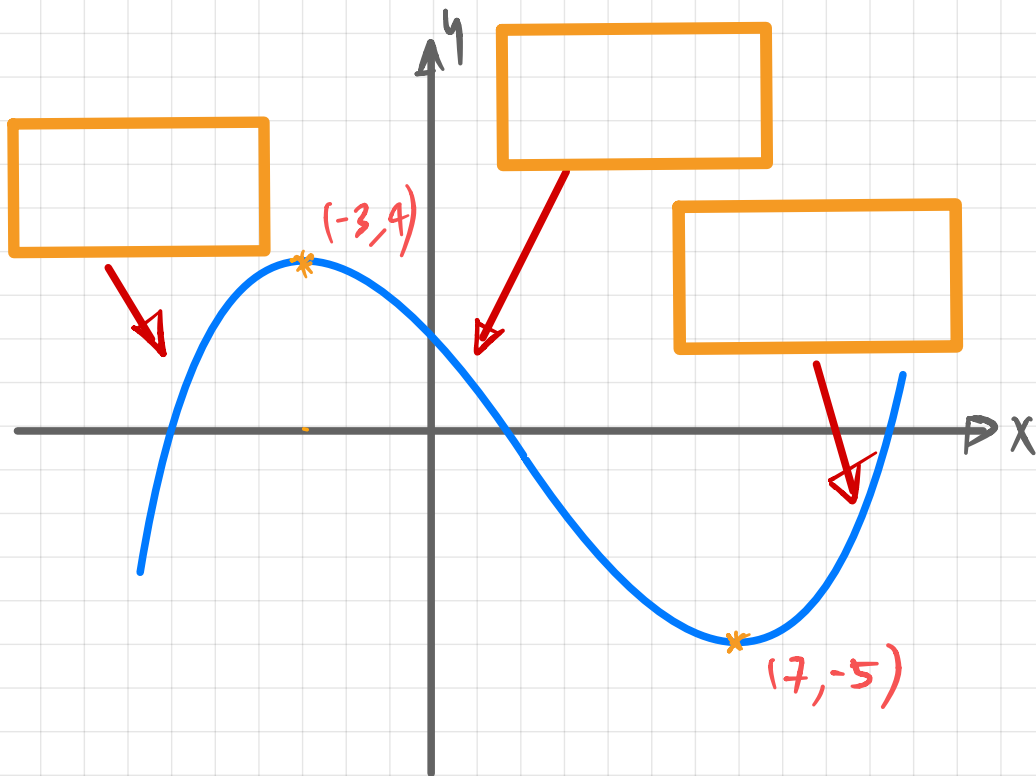
Gradient of the curve = Gradient of the tangent.



Find equation of tangent to curve

$y = x^2 + 1$ at point $(1, 2)$

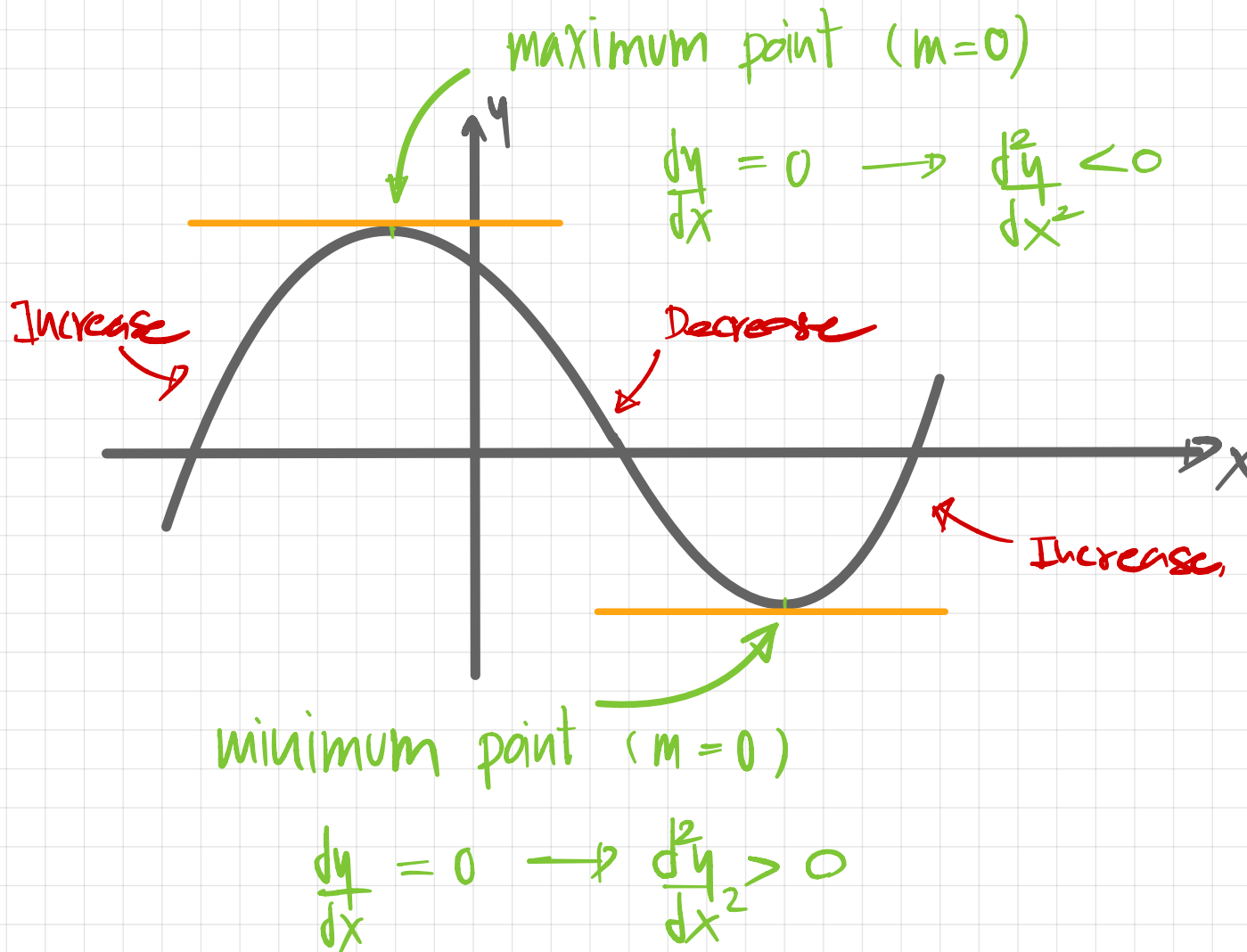
Increase function and Decrease function



1) Gradient of graph is positive
Increasing function $\rightarrow \frac{dy}{dx} > 0$

2) Gradient of graph is negative
Decreasing function $\rightarrow \frac{dy}{dx} < 0$

Differentiation for Stationary points (Turning points)



Example Find the stationary point of curve $y = -x^2 - 4x + 1$ and determine.

1) Differentiate

2) Differentiate=0 ; Solve for x

3) Substitute x to Original equation to find y for coordinates

4) Second order derivative for Max or Min

11 The curve **C** has equation $y = 4x^3 + x^2 - 20x$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \dots\dots\dots$$

(2)

- (b) Find the x coordinates of the points on C where the gradient is 4
Show clear algebraic working.

.....
(4)

(Total for Question 11 is 6 marks)

21 The curve **T** has equation $y = x^3 - 2x^2 - 9x + 15$

(a) Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \dots\dots\dots$$

(2)

- (b) Find the range of values of x for which \mathbf{T} has a positive gradient.
Give your values correct to 3 significant figures.
Show your working clearly.

.....
(4)

(Total for Question 21 is 6 marks)

- 22 The diagram shows a sketch of part of the curve with equation $y = x^2 - \frac{p}{x}$ where p is a positive constant.

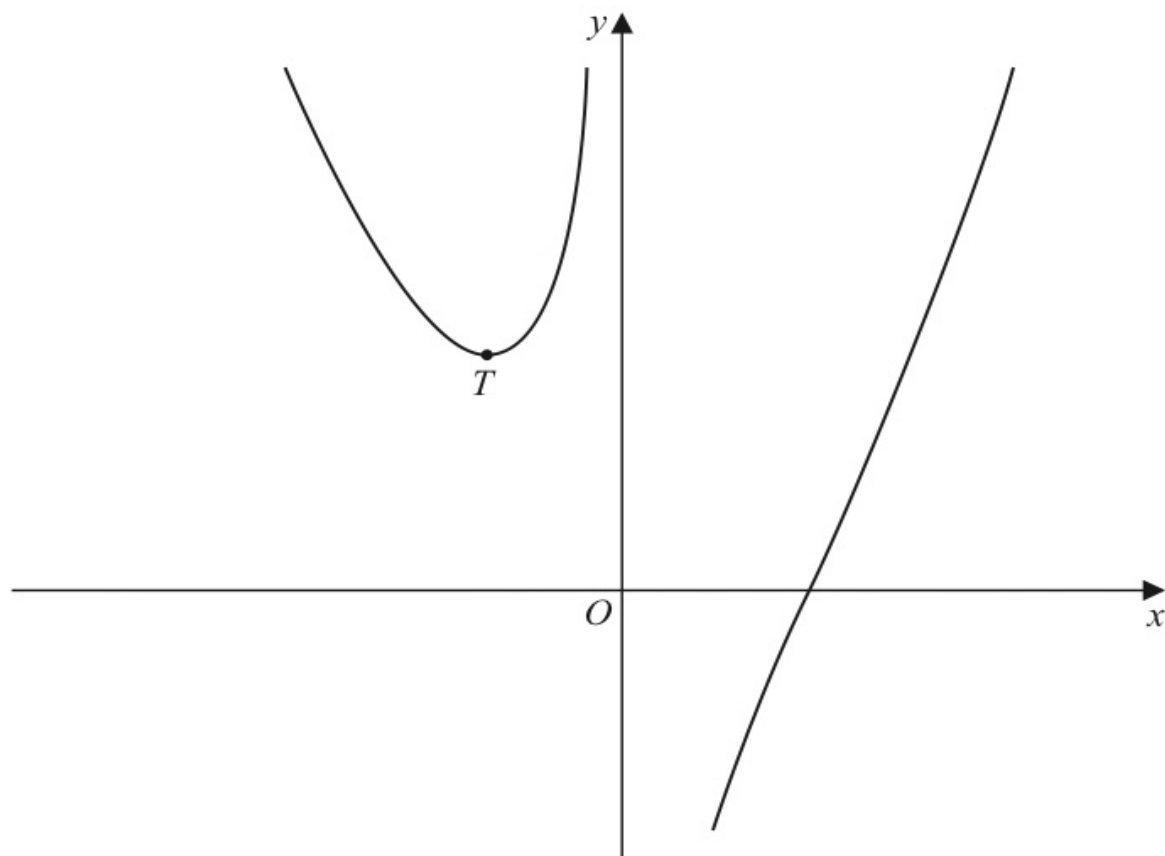


Diagram **NOT**
accurately drawn

For all values of p , the curve has exactly one turning point and this turning point is a minimum shown as the point T in the sketch.

For the curve where the x coordinate of T is -3

- (a) find the value of p

$$p = \dots\dots\dots$$

The line with equation $y = k$ is a tangent to the curve with equation $y = x^2 - \frac{16}{x}$

(b) Find the value of k

$$k = \dots\dots\dots$$

(3)

(Total for Question 22 is 7 marks)

- 23** G is the point on the curve with equation $y = 8x^2 - 14x - 6$ where the gradient is 10
The straight line Q passes through the point G and is perpendicular to the tangent at G

Find an equation for Q

Give your answer in the form $ax + by + c = 0$ where a , b and c are integers.

24 The curve **C** has equation $y = ax^3 + bx^2 - 12x + 6$ where a and b are constants.

The point A with coordinates $(2, -6)$ lies on **C**

The gradient of the curve at A is 16

Find the y coordinate of the point on the curve whose x coordinate is 3

Show clear algebraic working.

$y = \dots\dots\dots$

(Total for Question 24 is 6 marks)

Differentiation.

Rate of change
(Motion of a particle in a straight line)

Distance
(Displacement)



Velocity
(Speed)



Acceleration(+)

Deceleration(-)

$$s = t_{000}$$

$$x = t_{000}$$

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

24 A particle P moves along a straight line that passes through the fixed point O

The displacement, x metres, of P from O at time t seconds, where $t \geq 0$, is given by

$$x = 4t^3 - 27t + 8$$

The direction of motion of P reverses when P is at the point A on the line.

The acceleration of P at the instant when P is at A is $a \text{ m/s}^2$

Find the value of a

$a = \dots\dots\dots$

(Total for Question 24 is 5 marks)

- 17 A particle P moves along a straight line.
The fixed point O lies on this line.

The displacement of P from O at time t seconds, $t \geq 1$, is s metres where

$$s = 4t^2 + \frac{125}{t}$$

The velocity of P at time t seconds, $t \geq 1$, is v m/s

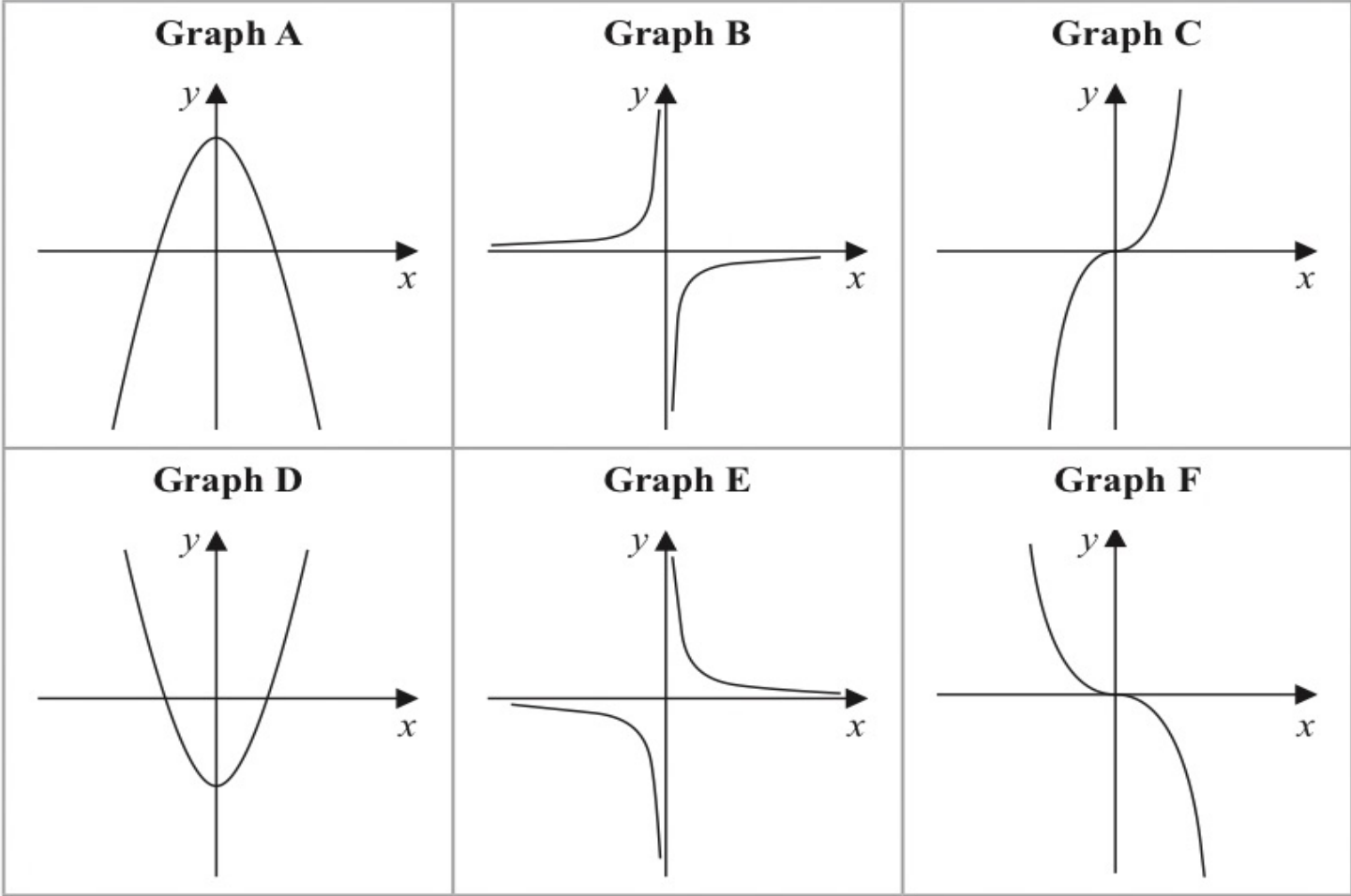
Work out the distance of P from O at the instant when $v = 0$

..... m

(Total for Question 17 is 5 marks)

Extra

11 Here are six graphs.



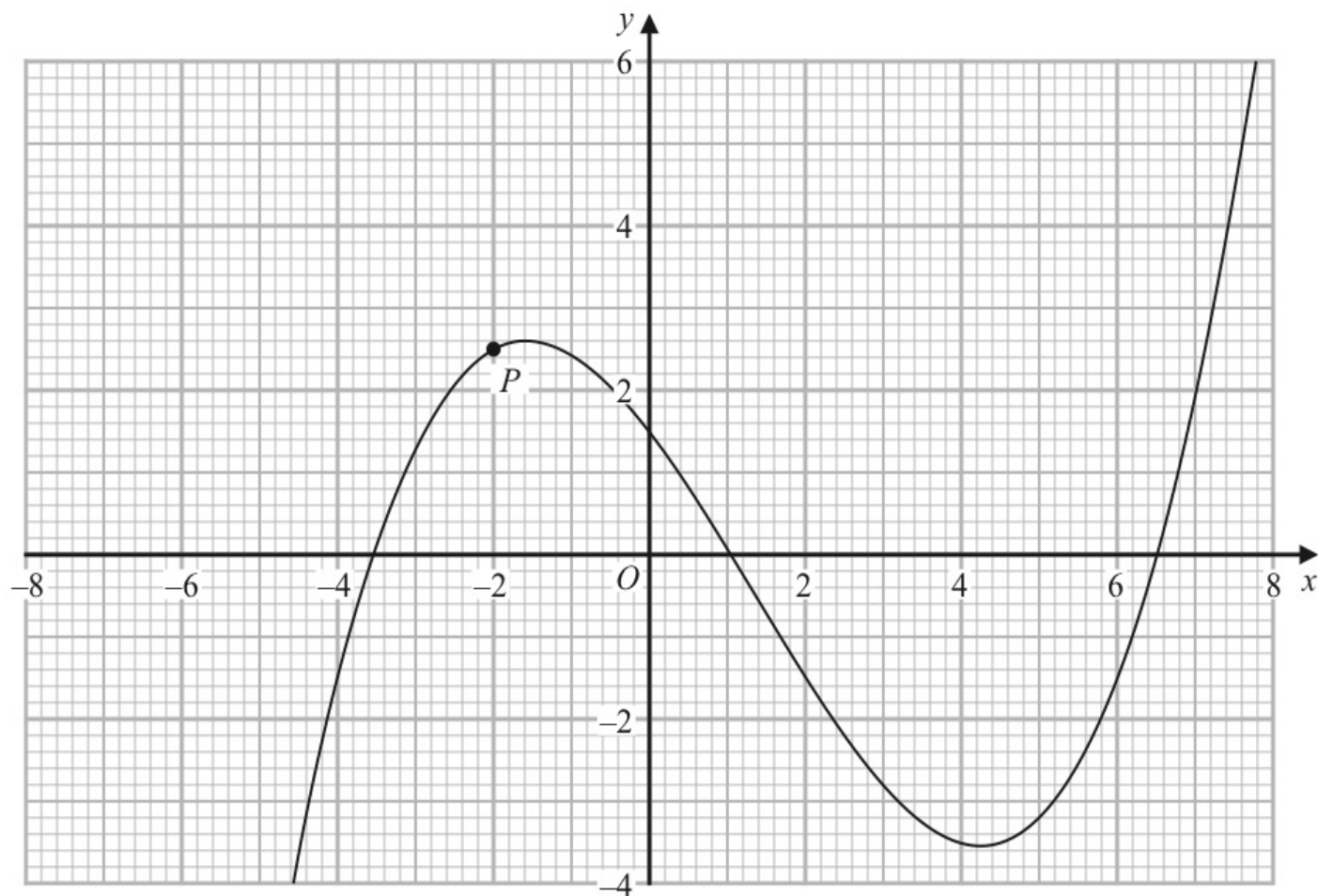
Complete the table below with the letter of the graph that could represent each given equation.

Write your answers on the dotted lines.

Equation	Graph
$y = -\frac{2}{x}$
$y = 5 - x^2$
$y = -2x^3$

(Total for Question 11 is 3 marks)

21 The diagram shows the graph of $y = f(x)$



The point P has x coordinate -2

Use the graph to find an estimate for the gradient of the curve at P

.....
(Total for Question 21 is 3 marks)

23 $ABCD$ is a kite.

$$AB = AD \text{ and } CB = CD$$

The point B has coordinates $(k, 1)$ where k is a negative constant.

The point D has coordinates $(8, 7)$

The straight line L passes through the points B and D

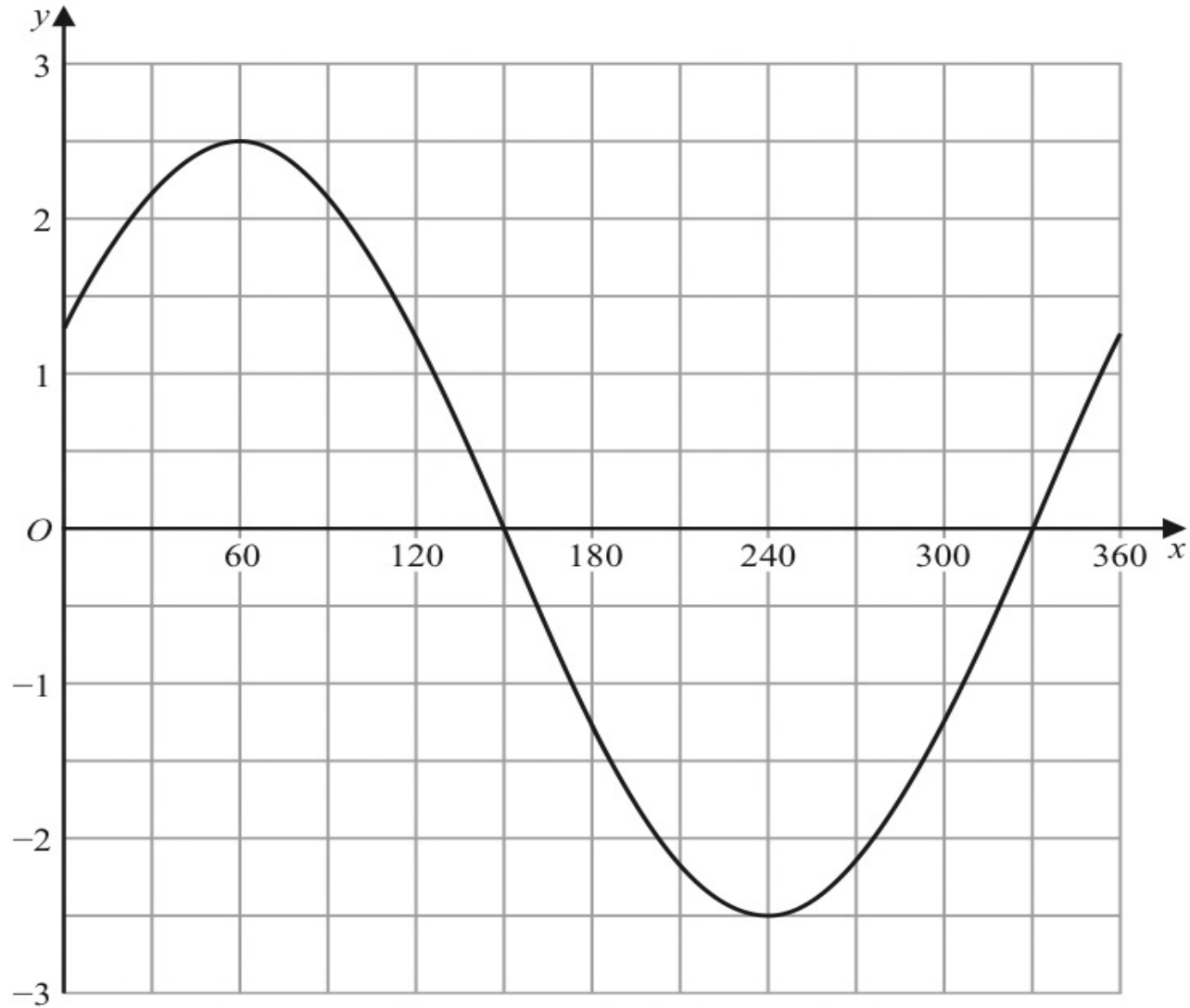
The straight line L is parallel to the line with equation $5y - 3x = 6$

Find an equation of AC

Give your answer in the form $px + qy = r$ where p , q and r are integers.

Show your working clearly.

22 The graph of $y = a \cos(x + b)^\circ$ for $0 \leq x \leq 360$ is drawn on the grid.



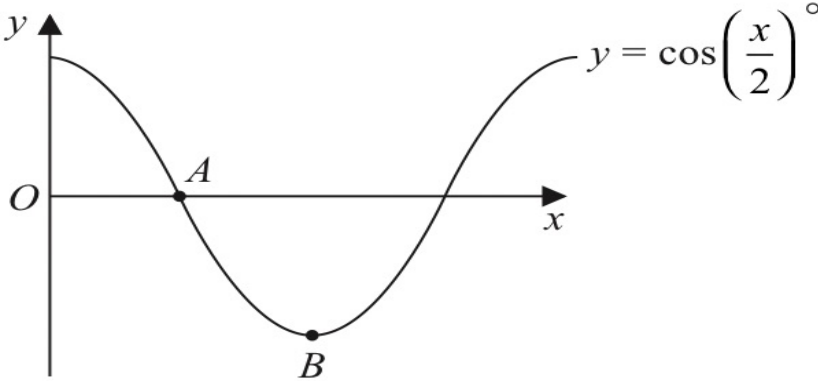
(a) Find the value of a and the value of b .

$a =$

$b =$

(2)

23 The diagram shows a sketch of the graph of $y = \cos\left(\frac{x}{2}\right)^\circ$



(i) Find the coordinates of the point *A*

(..... ,)
(1)

(ii) Find the coordinates of the point *B*

(..... ,)
(1)

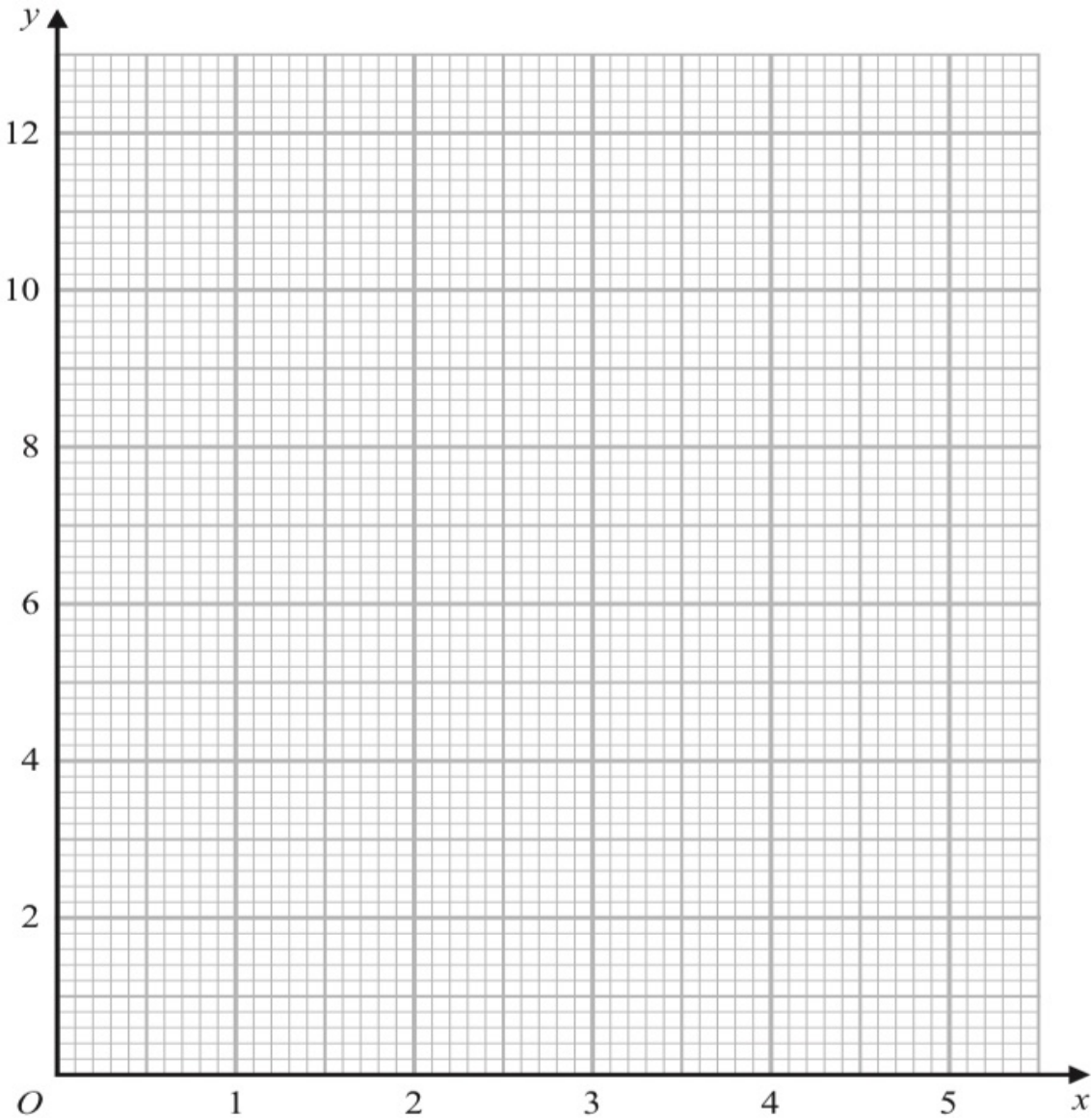
(Total for Question 23 is 2 marks)

14 (a) Complete the table of values for $y = \frac{2}{x}\left(5 - \frac{1}{x}\right)$

x	0.5	1	2	3	4	5
y		8		3.1	2.4	1.9

(1)

(b) On the grid, draw the graph of $y = \frac{2}{x}\left(5 - \frac{1}{x}\right)$ for $0.5 \leq x \leq 5$



(2)

(Total for Question 14 is 3 marks)

22 ABC is a triangle in which angle $ABC = 90^\circ$

p and q are integers such that

the coordinates of A are $(p, 10)$

the coordinates of B are $(-1, -5)$

the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$p = \dots\dots\dots$

$q = \dots\dots\dots$

(Total for Question 22 is 5 marks)