MATHS ONLINE

By: Kru Tar

9/A*
TOP MATHS

คอร์สตะลุยโจทย์

Intensive Maths IGCSE

BOOSTER

OCT/NOV 2023

Kru Tar

Day 2

TOPMaths
A* Level

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By: Kru Tar

6.3 Averages and range

1) Range = Max - Min

2) Mean = $\frac{\text{Sum}}{N} = \frac{\Sigma x}{N} = \frac{\Sigma fx}{N}$

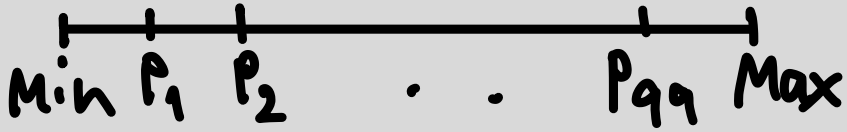
3) Median = Middle (Ordered data) $\frac{N}{2}$ and next

4) Mode = Most common or popular

5) Lower quartile
6) Upper quartile

7) Interquartile range = UQ - LQ

8) The n^{th} percentile



6.3 Averages and range

Ex. a) 3, 4, 6, 2, 8, 8, 5

1) Mean

2) Median

3) Mode

4) Range

5) LQ

6) UQ

7) IQR

8) The 60th percentile

6.3 Averages and range

Ex. b) 13, 26, 22, 30, 32, 48, 29, 27, 26, 32

1) Mean

2) Median

3) Mode

4) Range

5) LQ

6) UQ

7) IQR

8) The 60th percentile

Combining means

Set 1 size $= n_1$, mean $= \bar{x}_1$, sum $= n_1\bar{x}_1$

Set 2 size $= n_2$, mean $= \bar{x}_2$, sum $= n_2\bar{x}_2$

$$\text{Combining means} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

Ex. The mean of a group of 25 observations is 6.4. The mean of a second group of 30 observations is 7.2. Calculate the mean of all 55 observations.

6.3 Averages and range

TOPMaths
A* Level

30.

The students in Class A and in Class B take the same examination.

There are 28 students in Class A and 32 students in Class B.
The mean score for all the students in both classes is 72.6
The mean score for the students in Class A is 75

(a) Work out the mean score for the students in Class B.

.....

(4)

TOPMaths
A* Level

30.

The lowest score in Class A is 39
The range of scores for Class A is 57
The lowest score in Class B is 33
The range of scores for Class B is 60

(b) Find the range of scores for all the students in both classes.

.....

(3)

31.

Twenty students took a Science test and a Maths test.

Both tests were marked out of 50

The table gives information about their results.

	Median	Interquartile range
Science	27	18
Maths	24.5	11

Use this information to compare the Science test results with the Maths test results.
Write down **two** comparisons.

1

2

(Total for Question 11 is 2 marks)

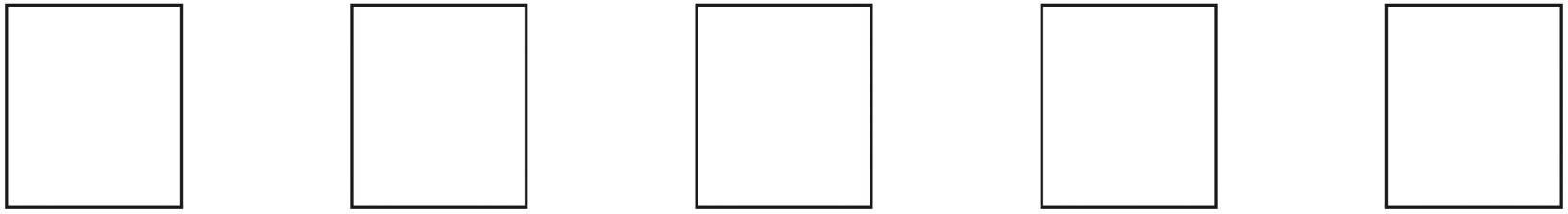
TOPMaths
A* Level
32.

Alexa has five cards.
Each card has a number on it.

The table gives information about the numbers on the five cards.

Total	Median	Mode	Range
45	8	5	10

Using the information in the table, complete each card by writing its number on it.



(Total for Question 4 is 3 marks)

TOPMaths A* Level

33.

Sandeep sat 11 tests in January 2020
Each test was marked out of 60

Here are his test results.

45 41 35 44 38 47 47 39 37 43 42

(a) Find the interquartile range of these test results.
Show your working clearly.

.....

(3)

Sandeep also sat some tests in May 2020
Each test was marked out of 60

The median of the May 2020 test results is 42
The interquartile range of the May 2020 test results is 12

(b) In which month, January or May, were Sandeep's test results more consistent?
Give a reason for your answer.

.....

.....

(1)

TOPMaths
A* Level

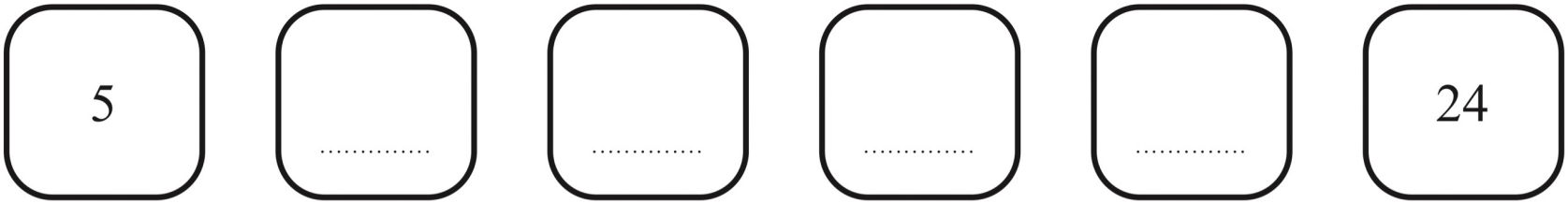
34.

Jenny has six cards.

Each card has a whole number written on it so that

the smallest number is 5
the largest number is 24
the median of the six numbers is 14
the mode of the six numbers is 8

Jenny arranges her cards so that the numbers are in order of size.



(a) For the remaining four cards, write on each dotted line a number that could be on the card.

(3)

TOPMaths
A* Level
34.

A basketball team plays 6 games.
After playing 5 games, the team has a mean score of 21 points per game.
After playing 6 games, the team has a mean score of 23 points per game.

(b) Work out the number of points the team scored in its 6th game.

.....

(3)

35.

Here are the numbers of aces that Rutger served in each of 11 tennis matches.

1 1 2 4 6 8 8 9 11 12 15

(a) Find the interquartile range of the numbers of aces.
Show your working clearly.

.....

(2)

Kim also plays in 11 tennis matches.

For Kim

the median number of aces is 11
the interquartile range of the numbers of aces is 5

(b) State, giving a reason, whether Rutger or Kim

(i) served more aces on average,

.....

.....

(1)

(ii) was more consistent with the number of aces served.

.....

.....

(1)

6.4 Mean of frequency table

Ex.

x	frequency
15	9
15.5	17
16	29
16.5	34
17	12

1) Mean

2) Median

3) Mode

4) Range

5) LQ

6) UQ

7) IQR

8) The 60th percentile

6.4 Mean of frequency table

Class interval

Ex.

x	frequency
30-31	2
32-33	25
34-36	30
37-39	13

1) Mean

2) Median

3) Mode

4) Range

5) LQ

6) UQ

7) IQR

8) The 60th percentile

6.4 Mean of frequency table

TOPMaths
A* Level

36.

The table gives information about the number of days that 100 cars were in an airport car park.

Number of days (d)	Frequency
$0 < d \leqslant 4$	16
$4 < d \leqslant 8$	18
$8 < d \leqslant 12$	19
$12 < d \leqslant 16$	27
$16 < d \leqslant 20$	20

(a) Write down the modal class.

_____

(1)

(b) Work out an estimate for the mean number of days.

_____ days

(4)

TOPMaths A* Level

37.

The table gives information about the times, in hours, some students spent doing sport one week.

Time (T hours)	Frequency
$0 < T \leqslant 2$	5
$2 < T \leqslant 4$	9
$4 < T \leqslant 6$	24
$6 < T \leqslant 8$	40
$8 < T \leqslant 10$	7

Calculate an estimate for the mean time these students spent doing sport.
Give your answer in hours, correct to 1 decimal place.

..... hours

(Total for Question 2 is 4 marks)

TOPMaths
A* Level
38.

The frequency table gives information about the numbers of mice in some nests.

Number of mice	Frequency
5	4
6	13
7	16
8	x
9	6

The mean number of mice in a nest is 7

Work out the value of x.

$x =$

(Total for Question 13 is 4 marks)

TOPMaths A* Level

39.

The table gives information about the number of minutes that Abby spent walking each day in September.

Number of minutes (M)	Frequency
$0 < M \leqslant 30$	5
$30 < M \leqslant 60$	6
$60 < M \leqslant 90$	8
$90 < M \leqslant 120$	9
$120 < M \leqslant 150$	2

Work out an estimate for the total number of minutes that Abby spent walking in September.

..... minutes

(Total for Question 2 is 3 marks)

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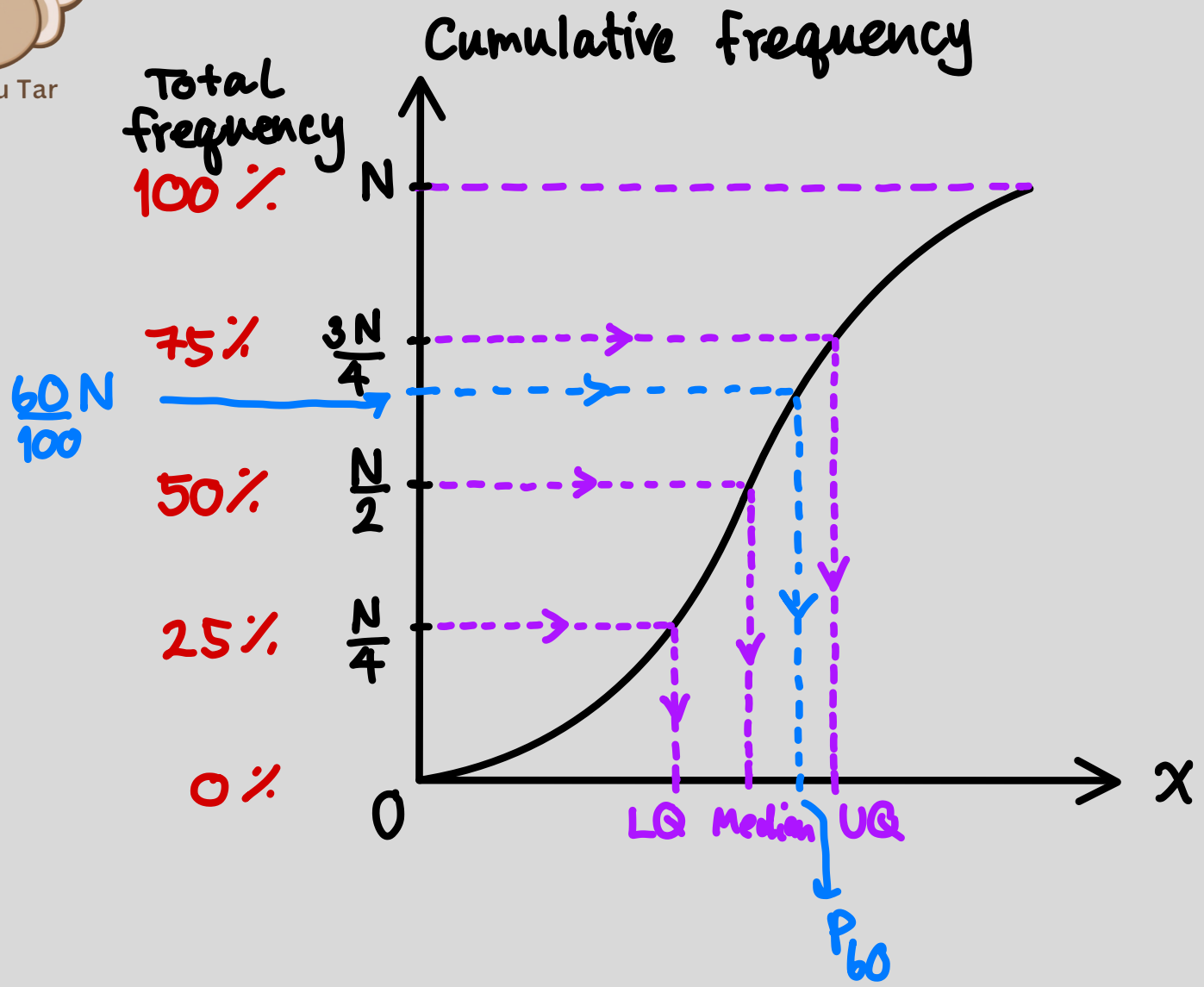
6.5 Quartiles, Cumulative frequency



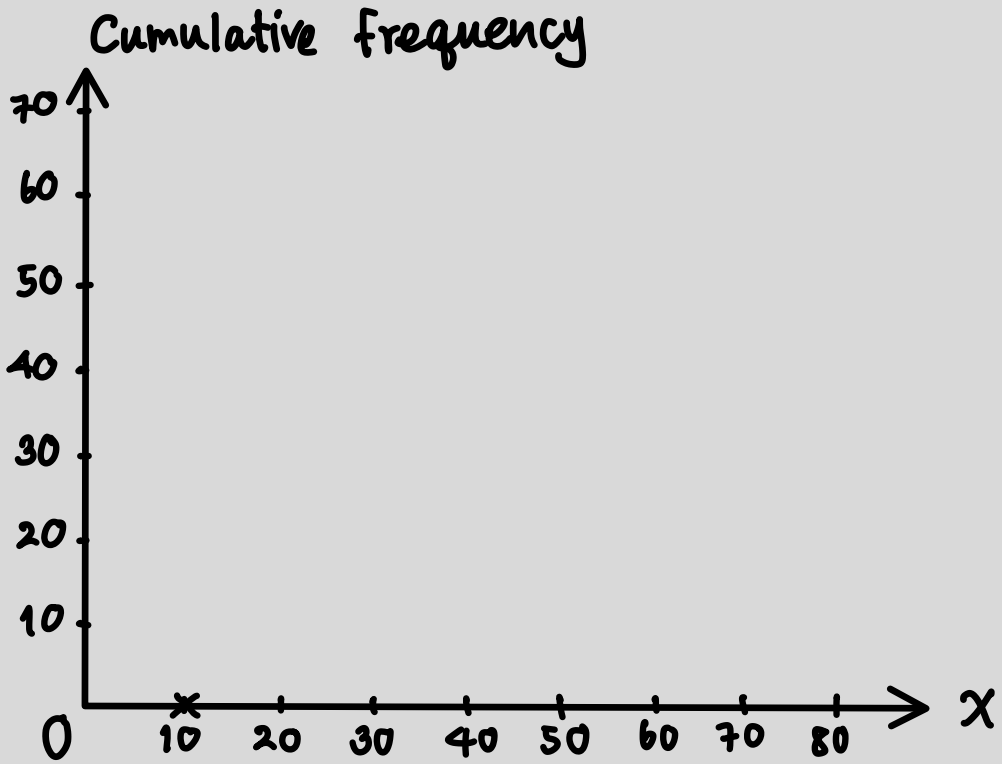
MATHS ONLINE

By: Kru Tar

6.5 Quartiles, Cumulative frequency

x	frequency
10 - 20	2
20 - 35	25
35 - 60	30
60 - 80	13

x	Cumulative frequency
$x \leq 20$	
$x \leq 35$	
$x \leq 60$	
$x \leq 80$	



1) Median

2) LQ

3) UQ

4) IQR

5) The 60th percentile

TOPMaths
A* Level

6.5 Quartiles, Cumulative frequency

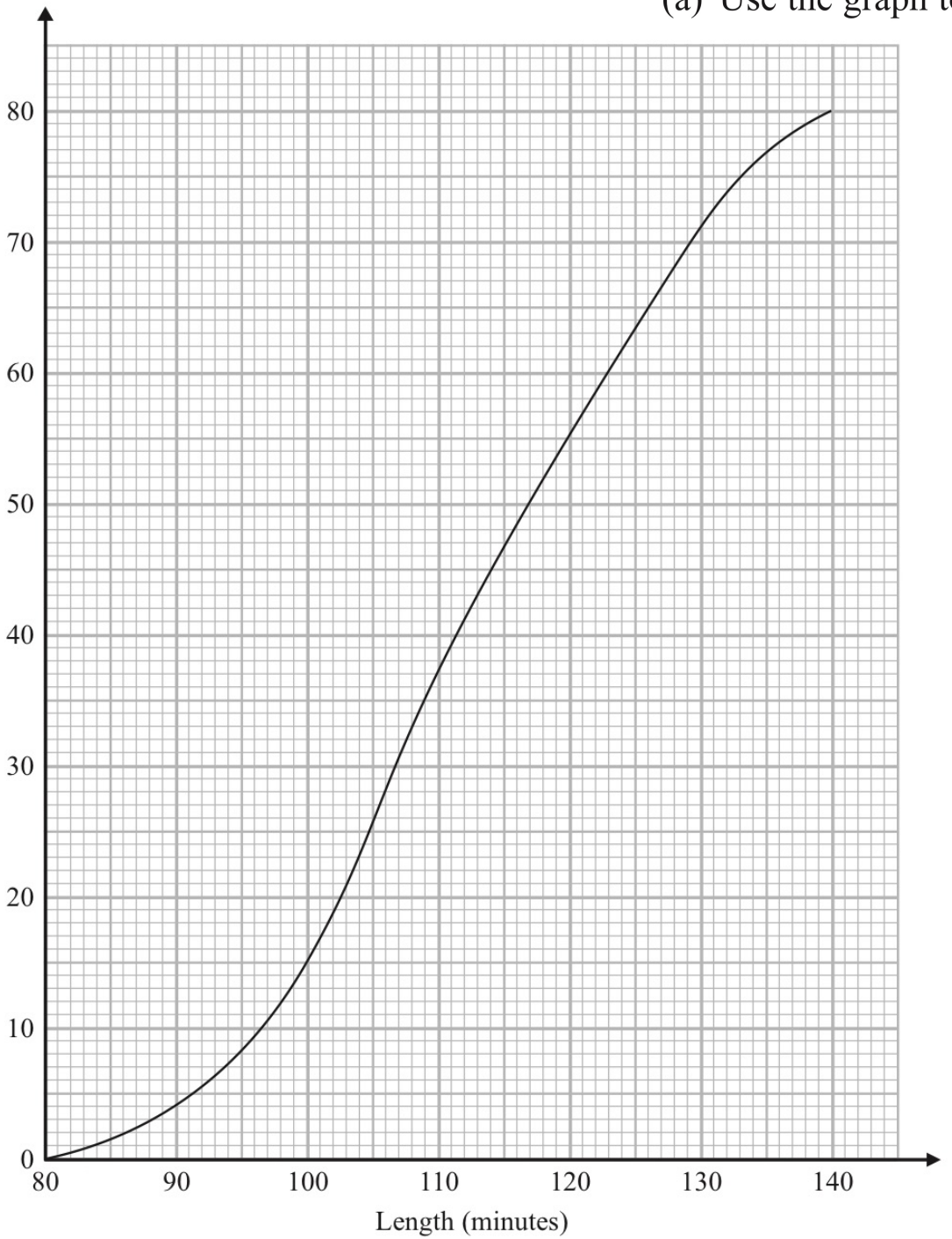
TOPMaths
A* Level

40.

The cumulative frequency graph shows information about the length, in minutes, of each of 80 films.

100

(a) Use the graph to find an estimate for the interquartile range.



..... minutes

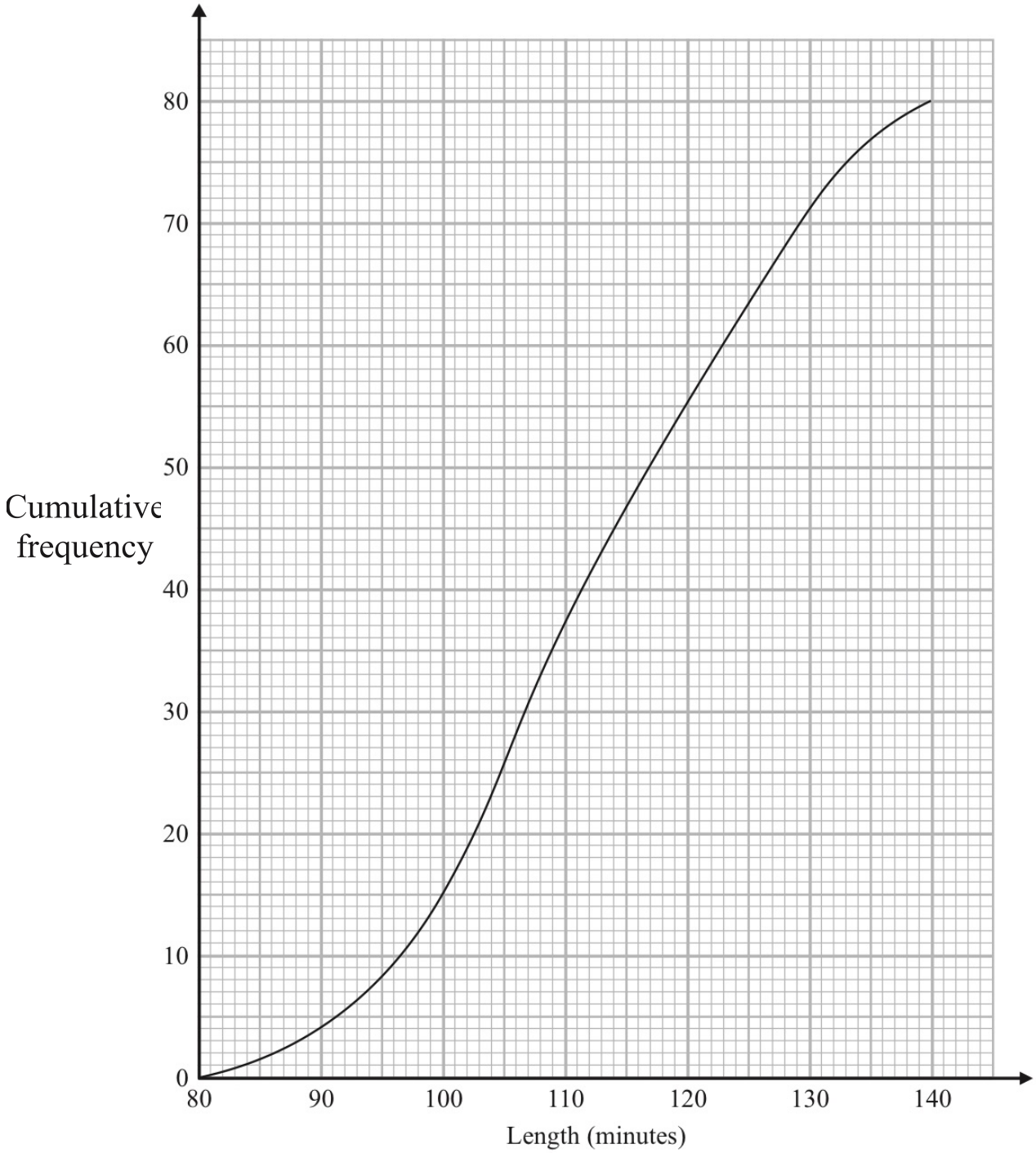
(2)

40.

Clare says,

"More than 35% of these films are over 120 minutes long."

(b) Is Clare correct?
Give a reason for your answer.



.....

.....

(3)

(Total for Question 10 is 5 marks)

TOPMaths A* Level

41.

The table shows information about the amount of money spent on holiday by each of 120 families.

Money spent (£m)	Frequency
$0 < m \leqslant 100$	10
$100 < m \leqslant 200$	36
$200 < m \leqslant 300$	34
$300 < m \leqslant 400$	20
$400 < m \leqslant 500$	15
$500 < m \leqslant 600$	5

Money spent (£m)	Cumulative frequency
$0 < m \leqslant 100$	
$0 < m \leqslant 200$	
$0 < m \leqslant 300$	
$0 < m \leqslant 400$	
$0 < m \leqslant 500$	
$0 < m \leqslant 600$	

(b) Complete the cumulative frequency table for the information in the table.

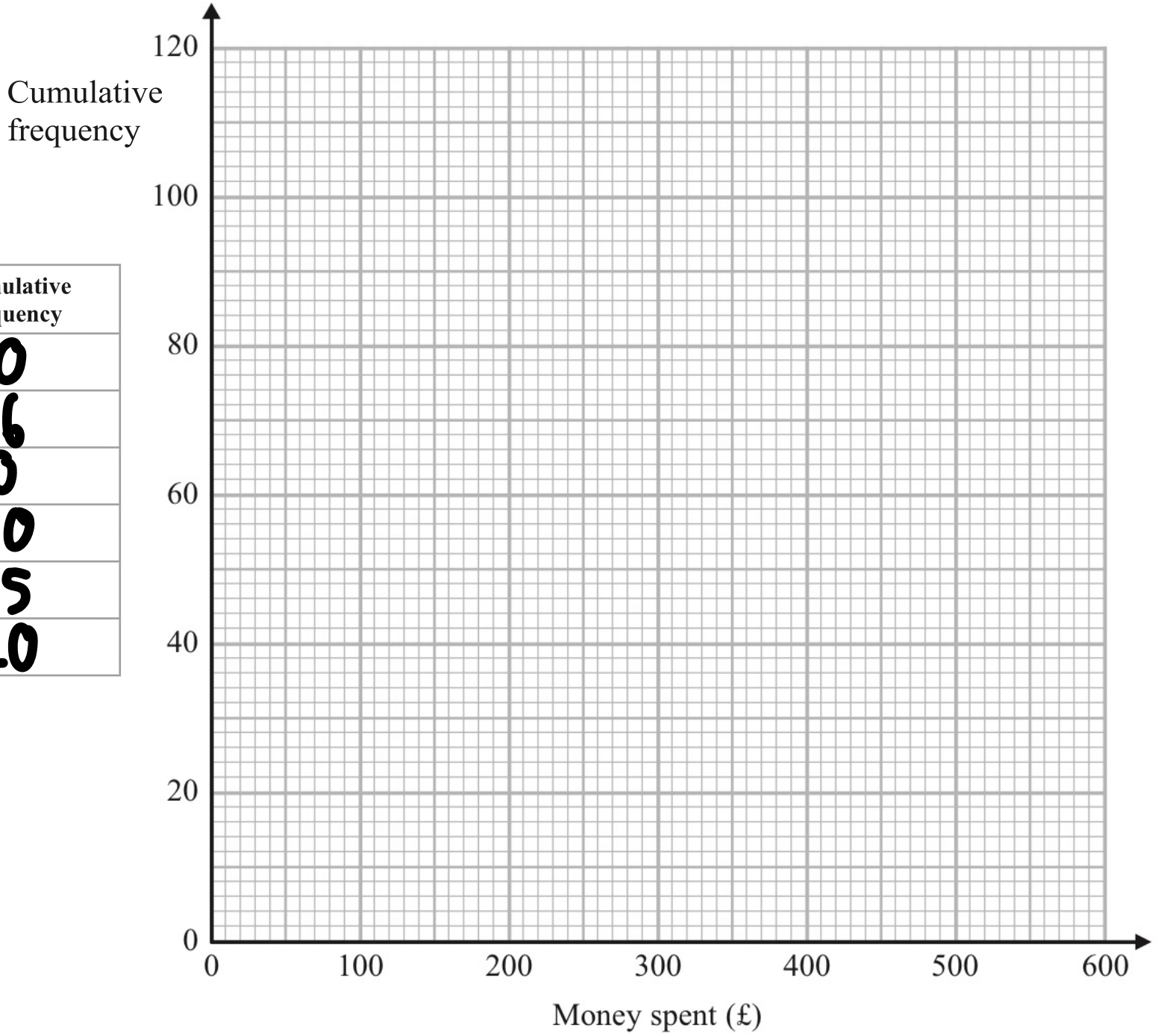
(1)

41.

(c) On the grid, draw a cumulative frequency graph for your table.

(2)

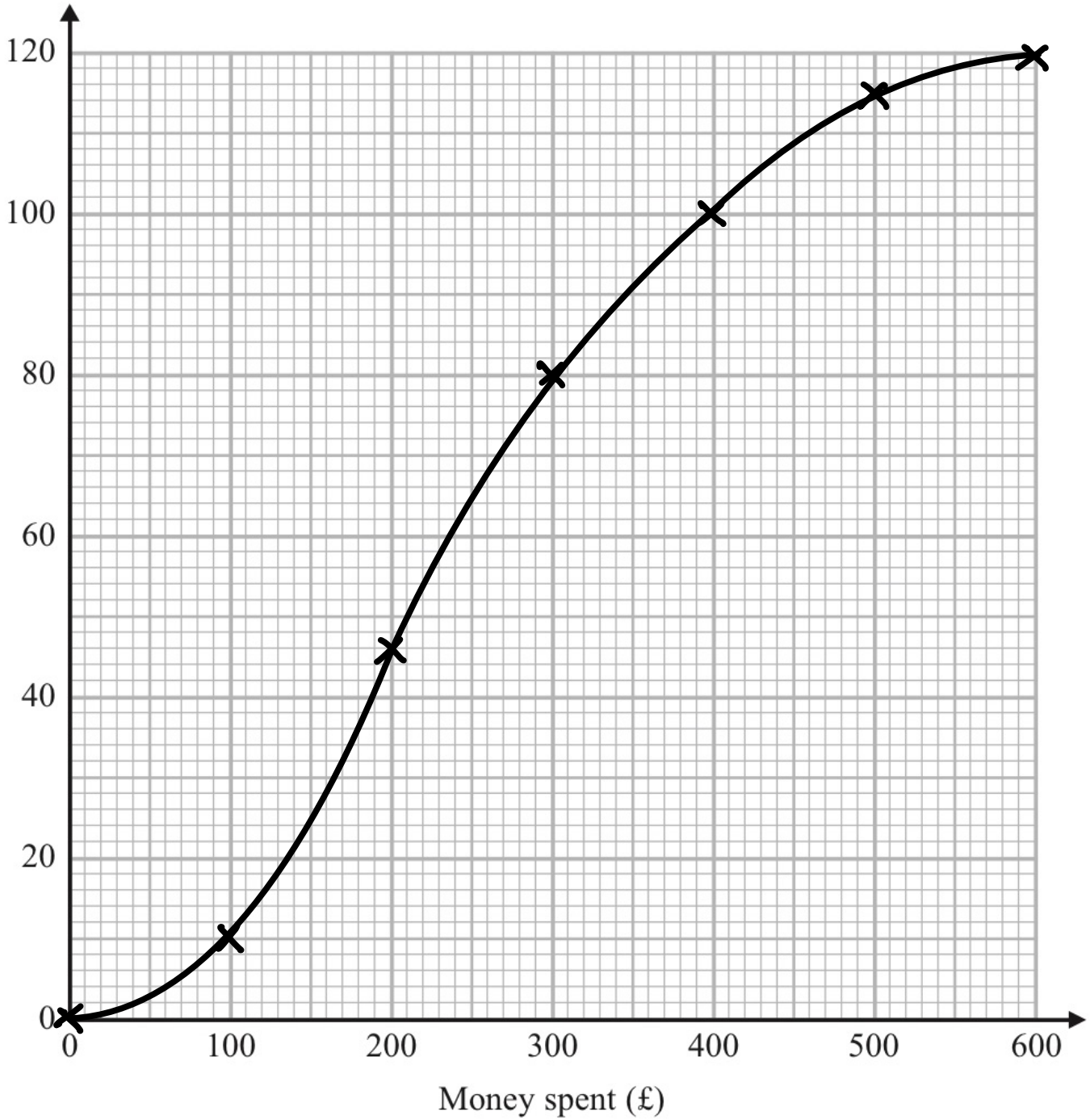
Money spent (£m)	Cumulative frequency
$0 < m \leqslant 100$	10
$0 < m \leqslant 200$	46
$0 < m \leqslant 300$	80
$0 < m \leqslant 400$	100
$0 < m \leqslant 500$	115
$0 < m \leqslant 600$	120



TOPMaths
A* Level

41.

Cumulative frequency



(d) Use your graph to find an estimate for the interquartile range.

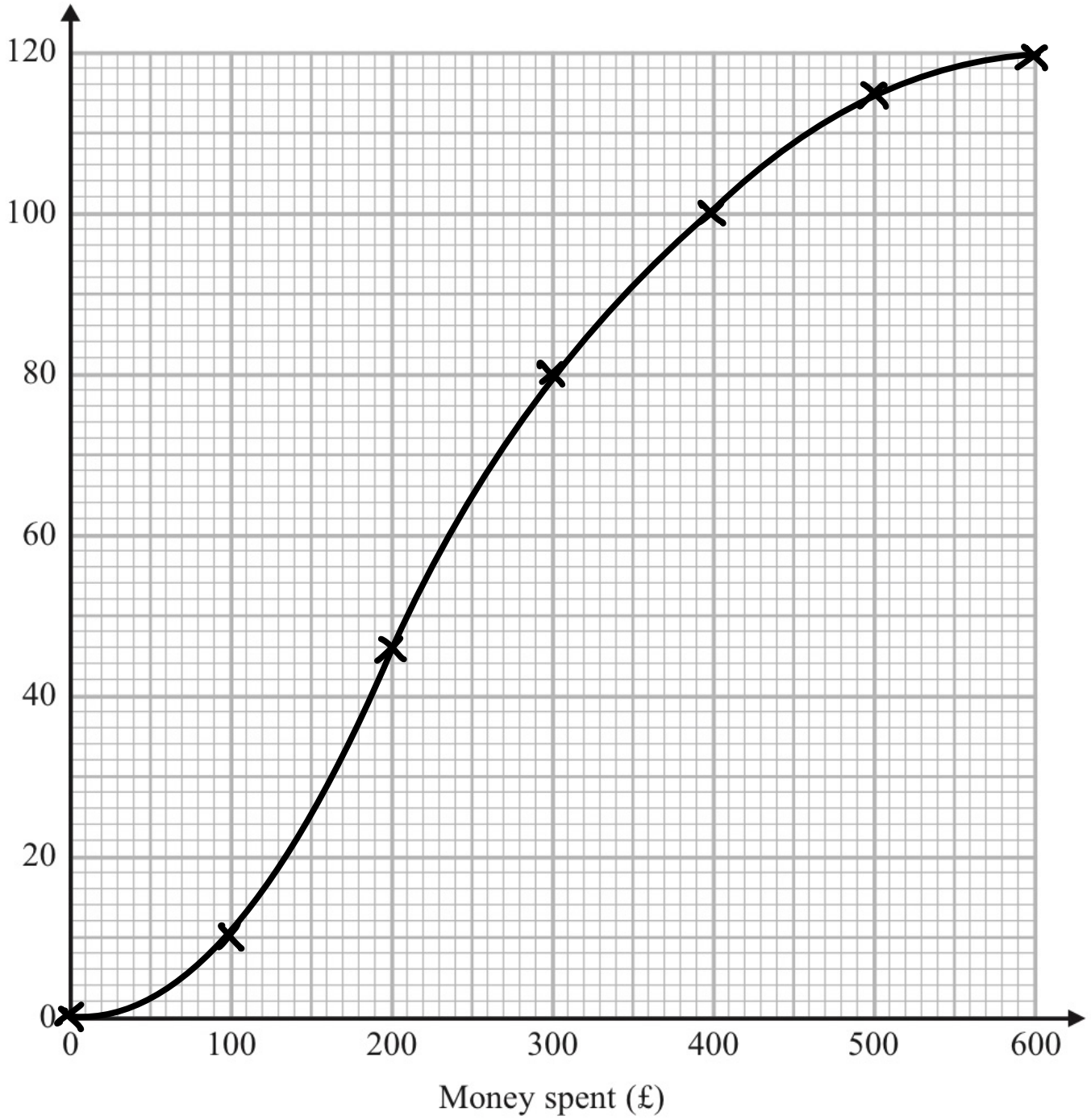
£.....

(2)

TOPMaths A* Level

41.

Cumulative frequency



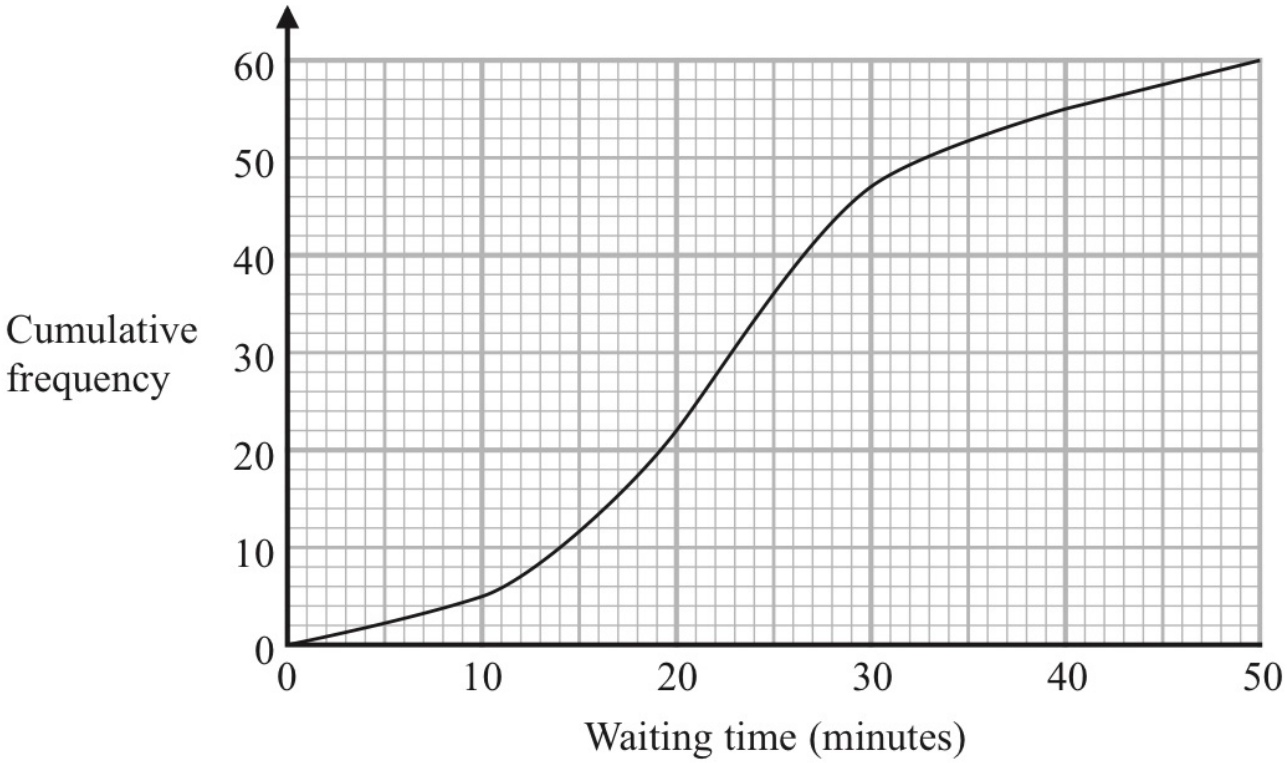
(e) Use your graph to find an estimate for the number of families that spent more than £450 on holiday.

.....

(2)

42.

The cumulative frequency graph gives information about the waiting times, in minutes, of people with appointments at Hospital A.



(a) Use the graph to find an estimate of the median waiting time at Hospital A.

..... minutes

(1)

(b) Use the graph to find an estimate of the interquartile range of the waiting times at Hospital A.

..... minutes

(2)

Hospital A

(a) median ..... 23 ..... minutes

(b) interquartile range ..... 12 ..... minutes

42.

At a different hospital, Hospital B, the median waiting time is 28 minutes and the interquartile range of the waiting times is 19 minutes.

(c) Compare the waiting times at Hospital A with the waiting times at Hospital B.

.....

.....

.....

.....

(2)

TOPMaths
A* Level

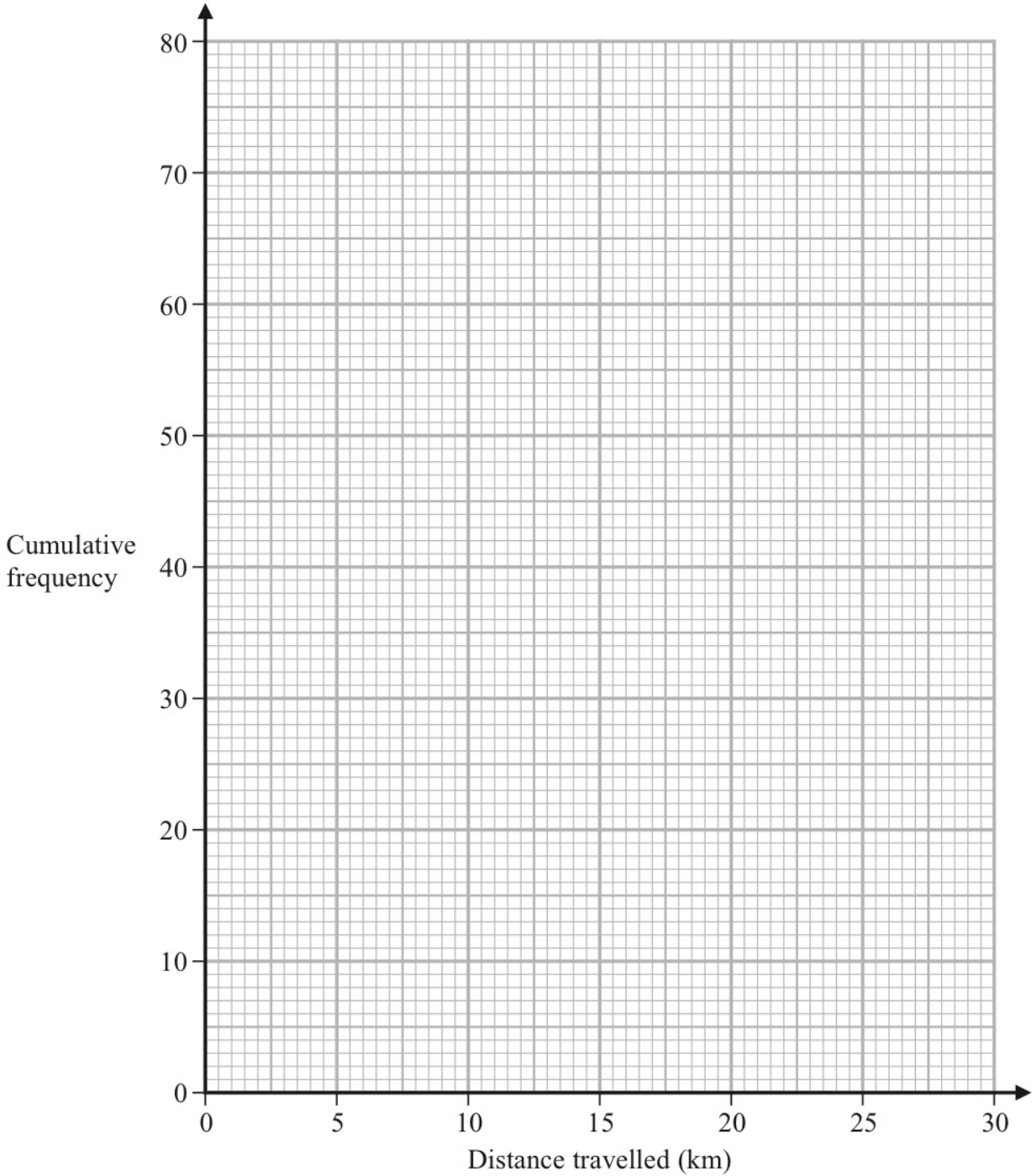
43.

The cumulative frequency table gives information about the distance, in kilometres, that each of 80 workers travel from home to work at Office *A*.

(a) On the grid below, draw a cumulative frequency graph for the information in the table.

(2)

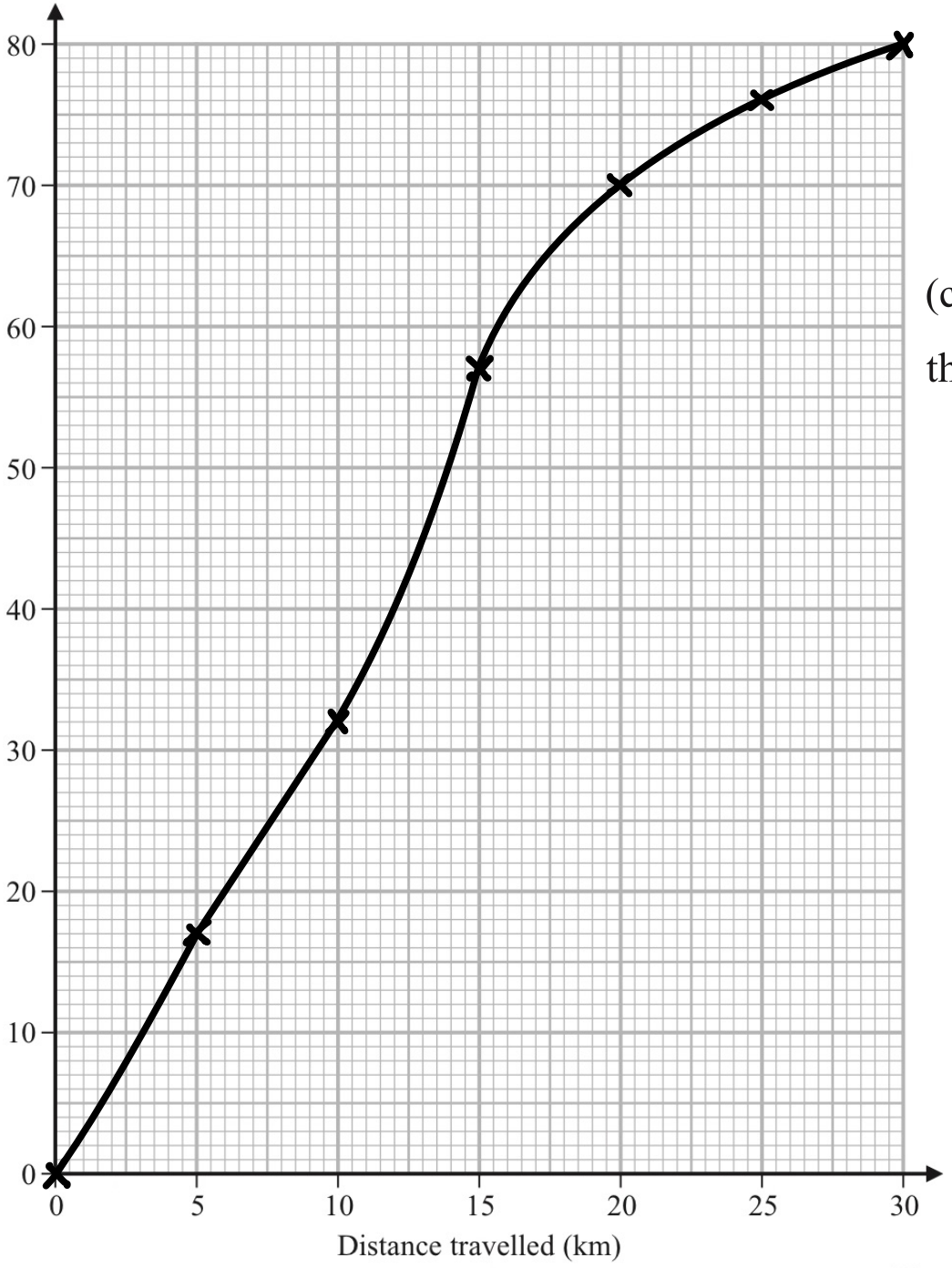
Distance travelled (d km)	Cumulative frequency
$0 < d \leqslant 5$	17
$0 < d \leqslant 10$	32
$0 < d \leqslant 15$	57
$0 < d \leqslant 20$	70
$0 < d \leqslant 25$	76
$0 < d \leqslant 30$	80



TOPMaths A* Level

43.

(b) Use your graph to find an estimate for the median distance travelled.



..... km

(1)

(c) Use your graph to find an estimate for the interquartile range of the distances travelled.

..... km

(2)

43.

Office *A*

(b) median ..... 12 ..... km

(c) interquartile range ..... 9.5 ..... km

For Office *B*, the median distance workers travel from home to work is 15 km and the interquartile range is 5 km.

(d) Use the information above to compare the distances that workers at Office *A* and workers at Office *B* travel from home to work.
Write down **two** comparisons.

1 .....

.....

.....

2 .....

.....

.....

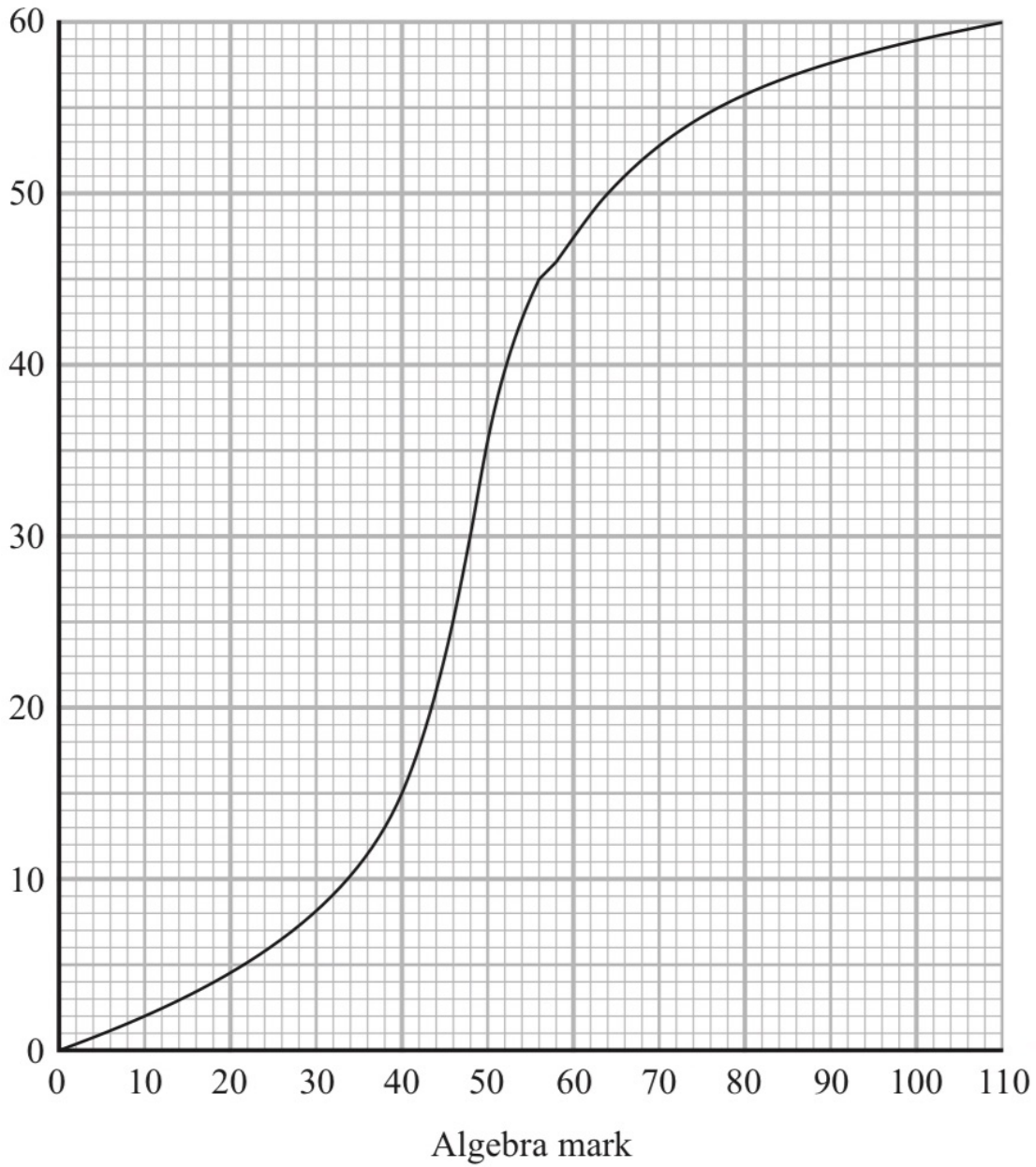
(2)

44.

A group of 60 students each sat an algebra test and a geometry test.
Each test was marked out of 110

The cumulative frequency graph gives information about the marks gained by the 60 students in the algebra test.

(a) Use the graph to find an estimate for the median mark in the algebra test.

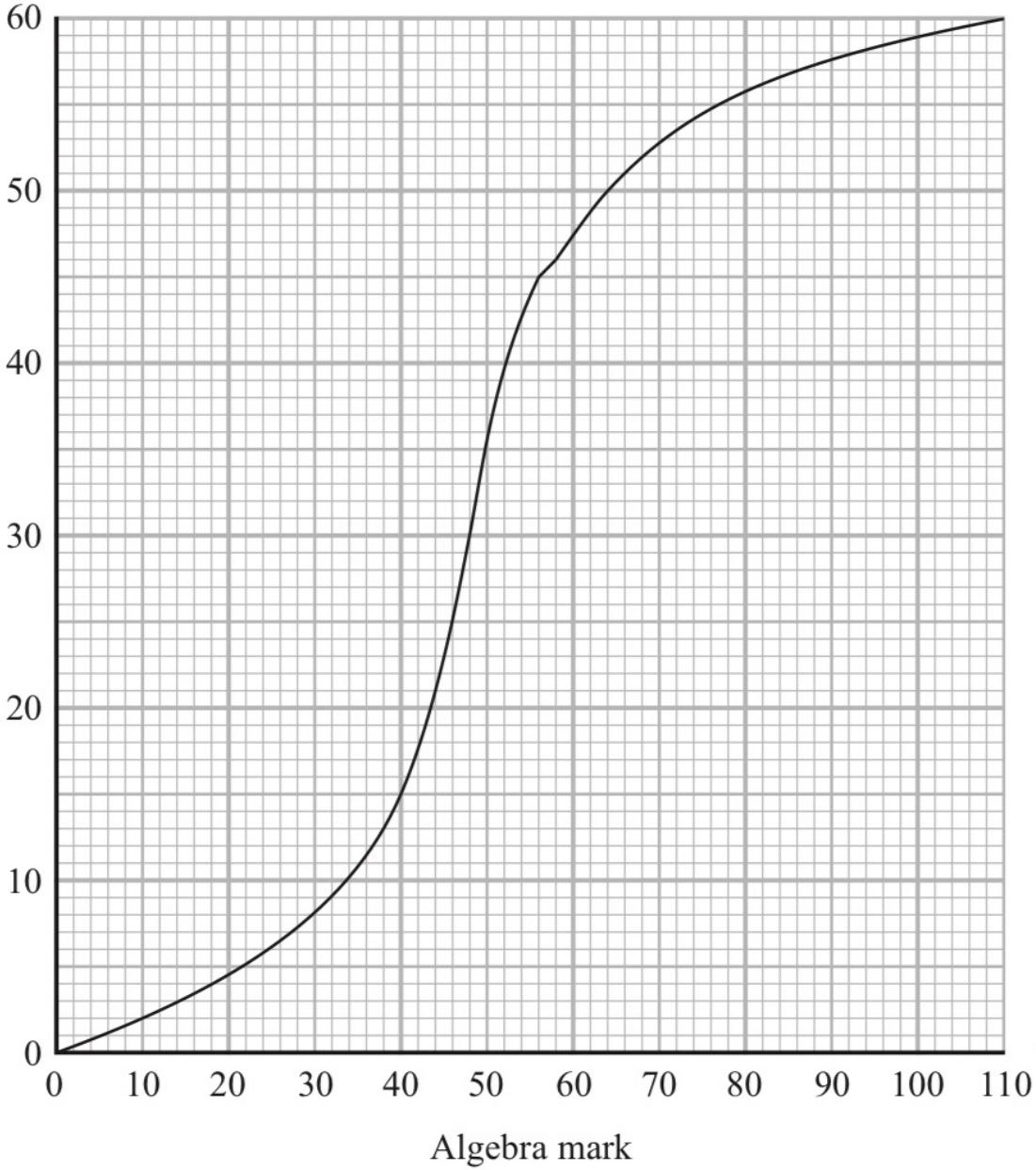


.....

(1)

44.

(b) Use the graph to find an estimate for the number of students who gained 58 marks or less in the algebra test.



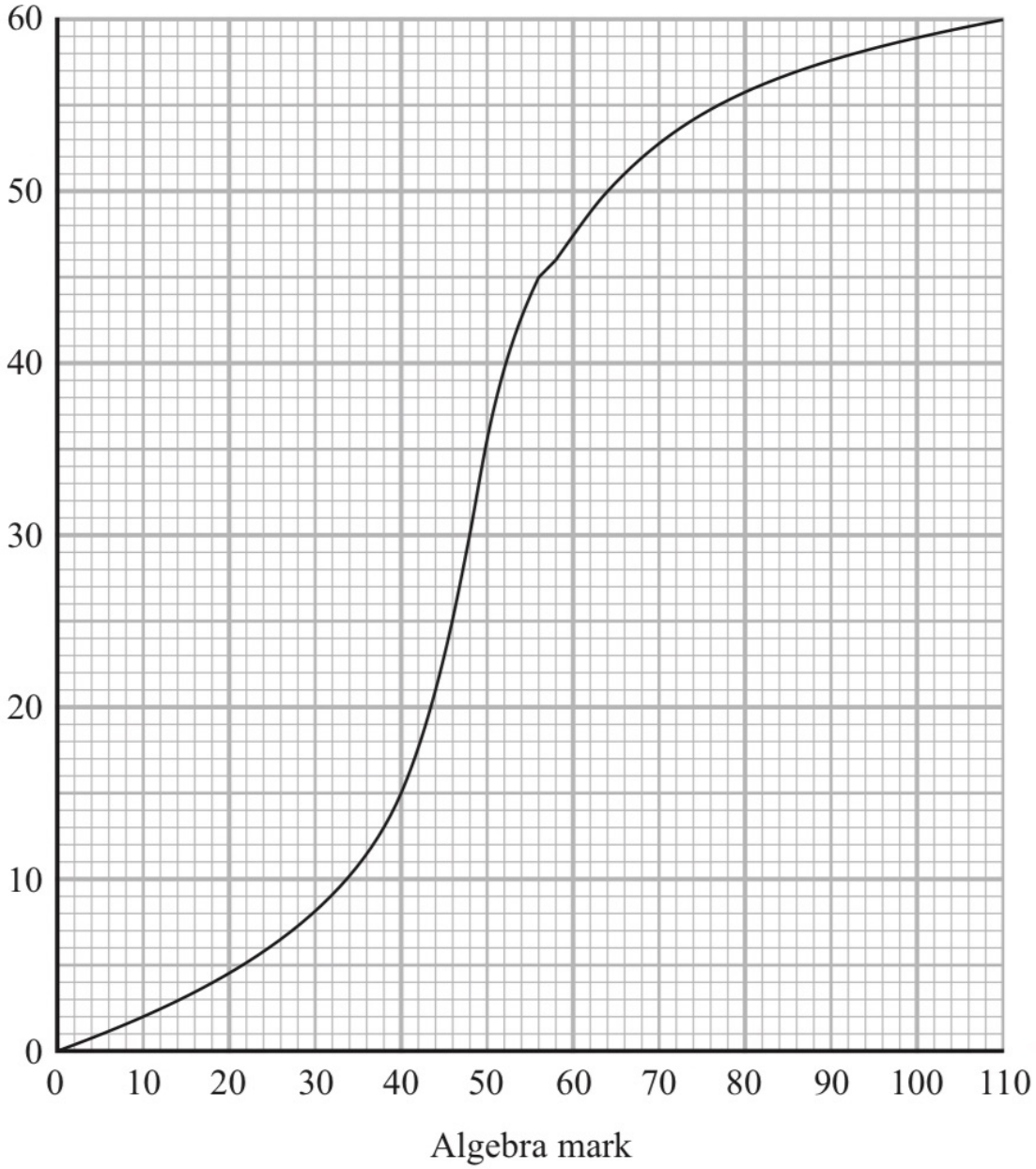
.....

(1)

(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

.....

(2)



(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

16

.....

(2)

TOPMaths
A* Level

44.

The interquartile range of the marks gained in the geometry test is 9

Luis says

"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Luis correct?
Give a reason for your answer.

.....

.....

(1)

TOPMaths A* Level

45.

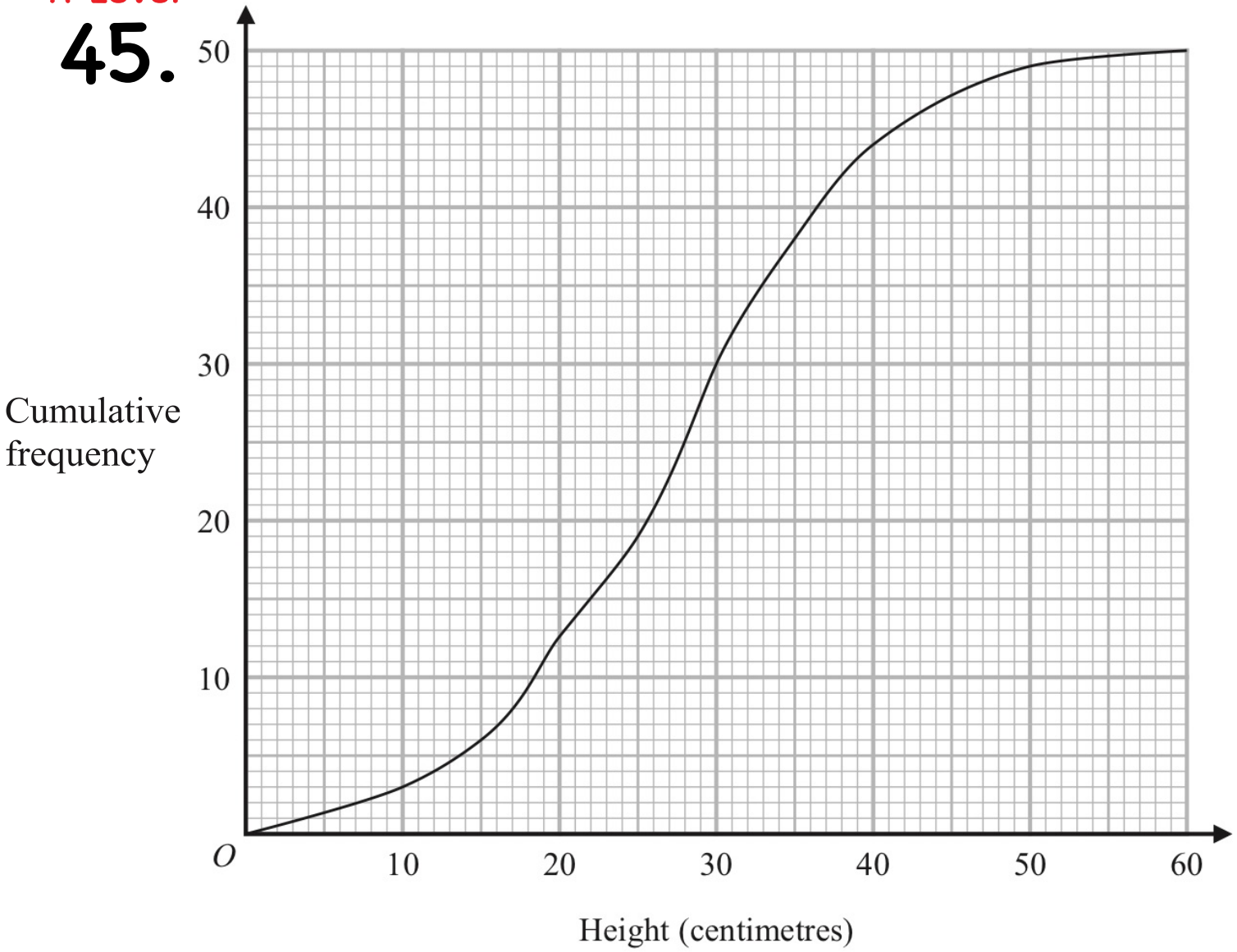
The cumulative frequency graph shows information about the heights, in centimetres, of 50 plants in a flowerbed.

(a) Use the graph to find an estimate for the median height of these plants.

..... centimetres

(1)

(b) Use the graph to find the frequency for the class interval $30 < \text{Height} \leqslant 40$



.....

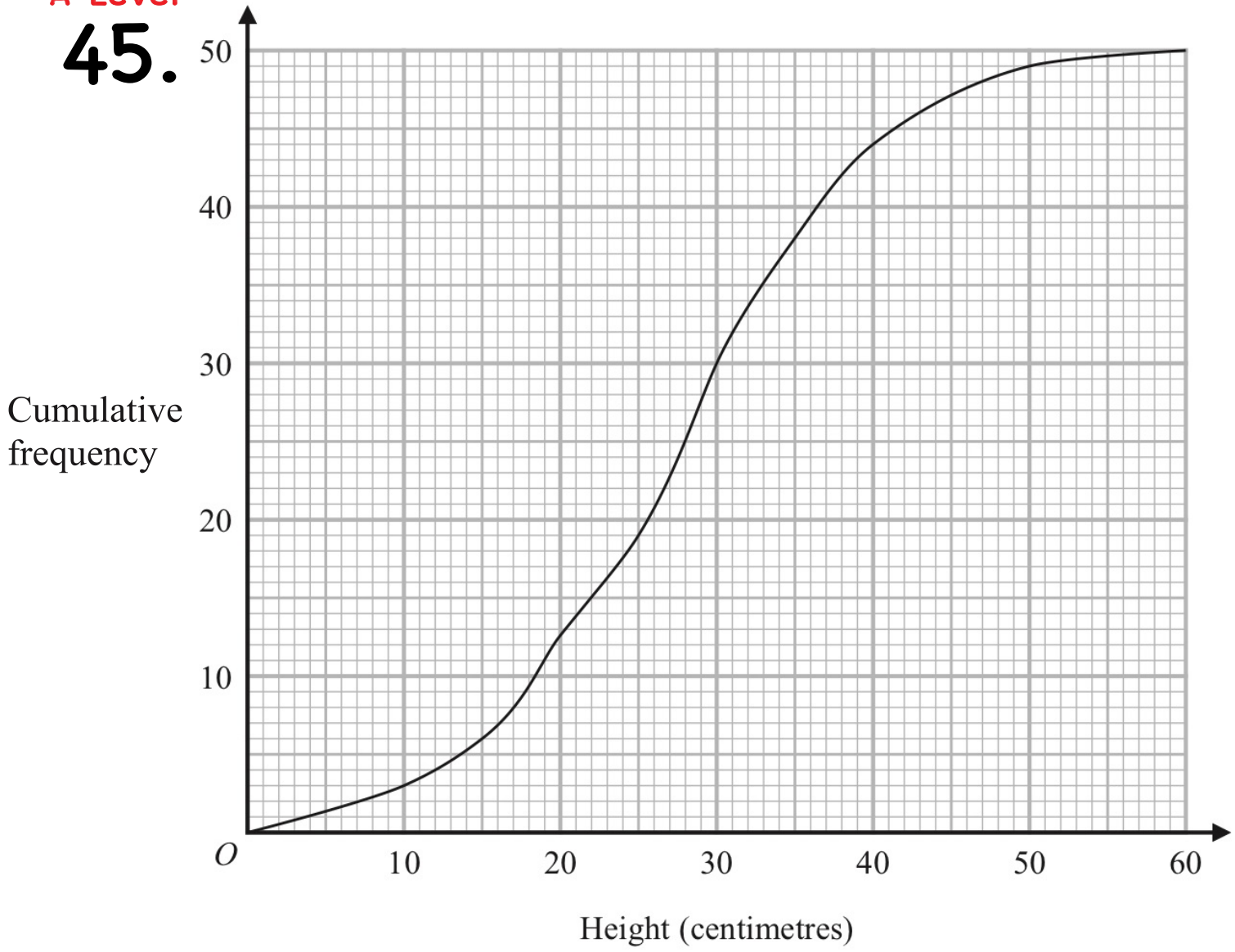
(1)

45.

(c) Use the graph to find an estimate for the number of plants with a height greater than 35 centimetres.

.....

(2)



MATHS ONLINE

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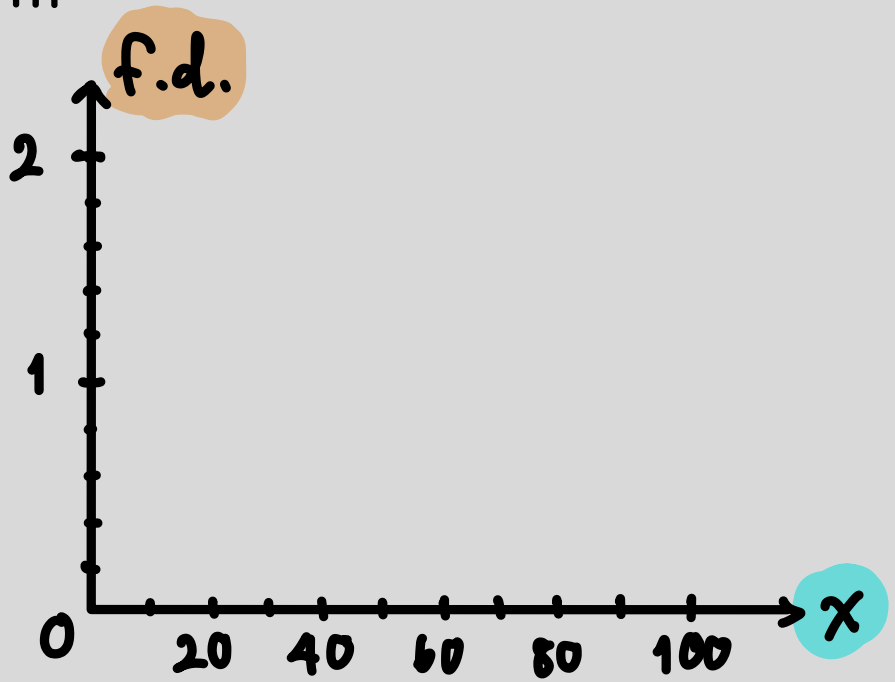
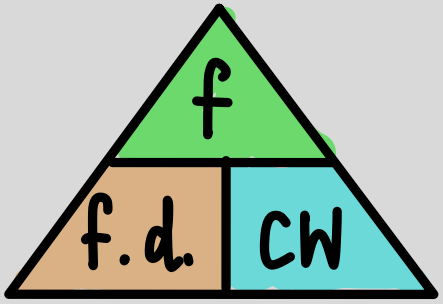
6.6 Histograms

Area of each bar represents frequency.

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

x	frequency	CW	f.d.
0 – 20	28		
20 – 40	36		
40 – 50	20		
50 – 70	30		
70 – 100	18		





f.d. = f / CW = Area

6.6 Histograms

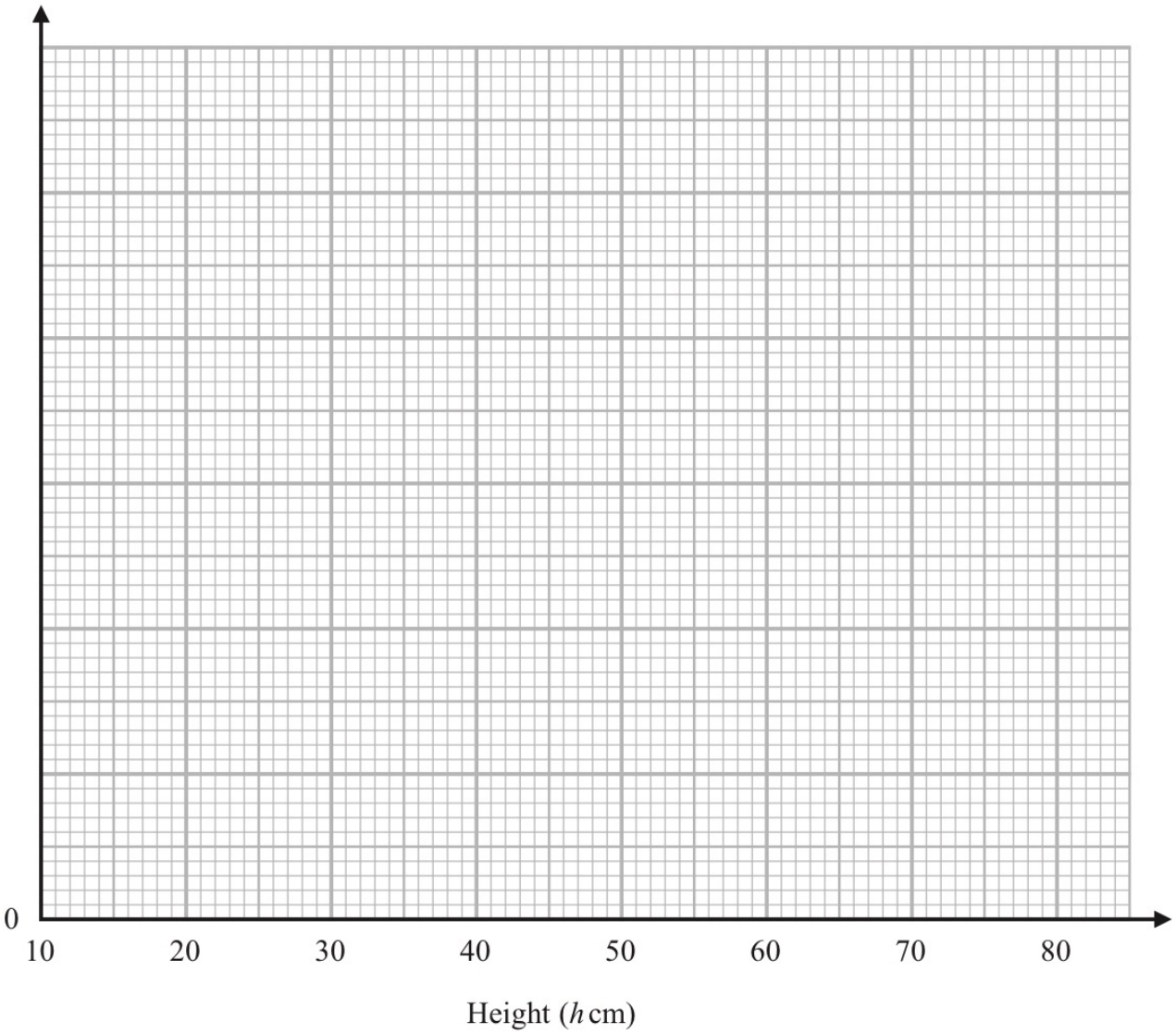
TOPMaths
A* Level

46.

The table gives information about the heights, in centimetres, of some plants.

(a) On the grid, draw a histogram for this information.

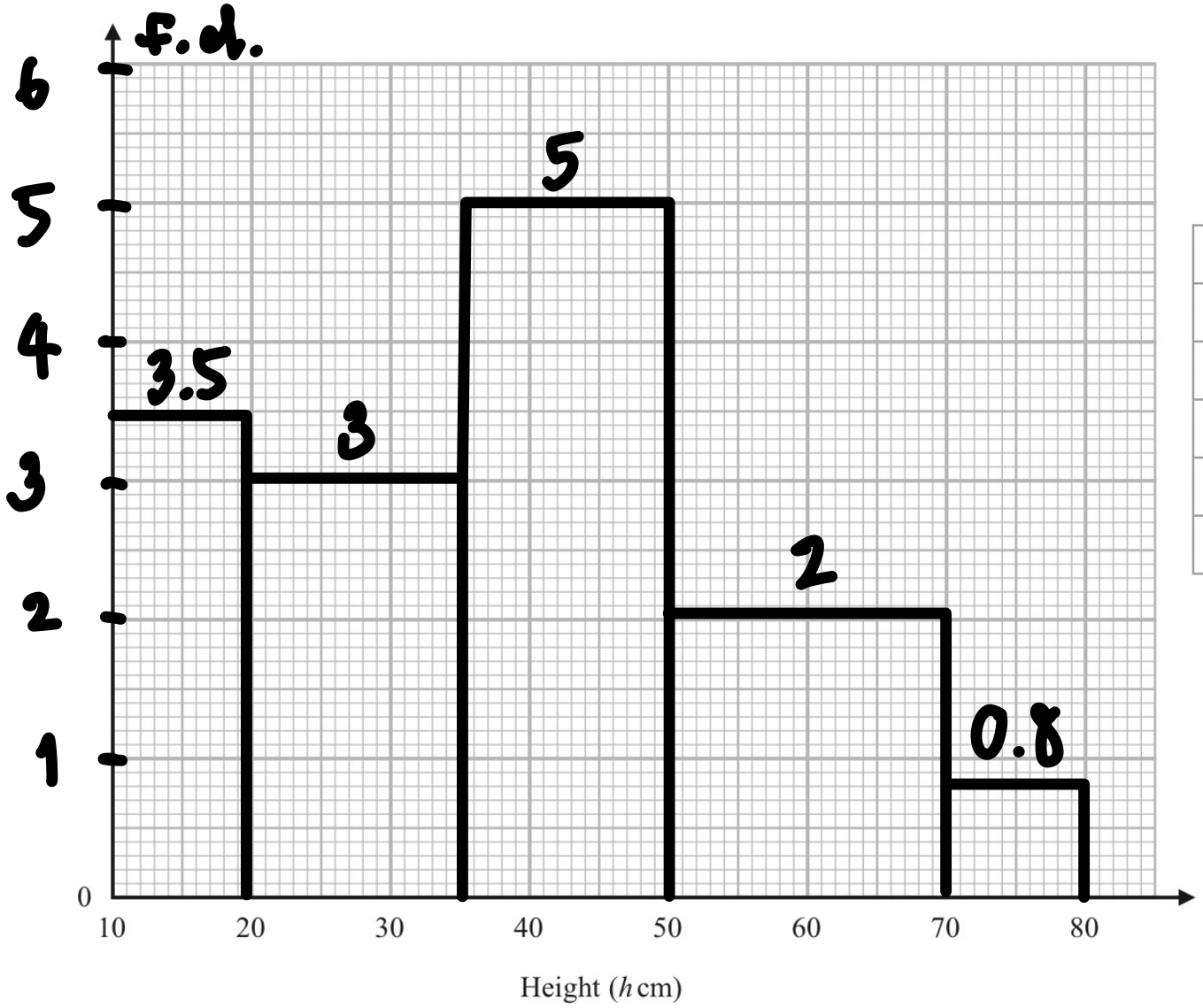
Height (h cm)	Frequency
$10 < h \leqslant 20$	35
$20 < h \leqslant 35$	45
$35 < h \leqslant 50$	75
$50 < h \leqslant 70$	40
$70 < h \leqslant 80$	8



(3)

46.

120



Height (h cm)	Frequency
$10 < h \leqslant 20$	35
$20 < h \leqslant 35$	45
$35 < h \leqslant 50$	75
$50 < h \leqslant 70$	40
$70 < h \leqslant 80$	8

(b) Work out an estimate for the number of these plants with a height greater than 40 cm.

.....

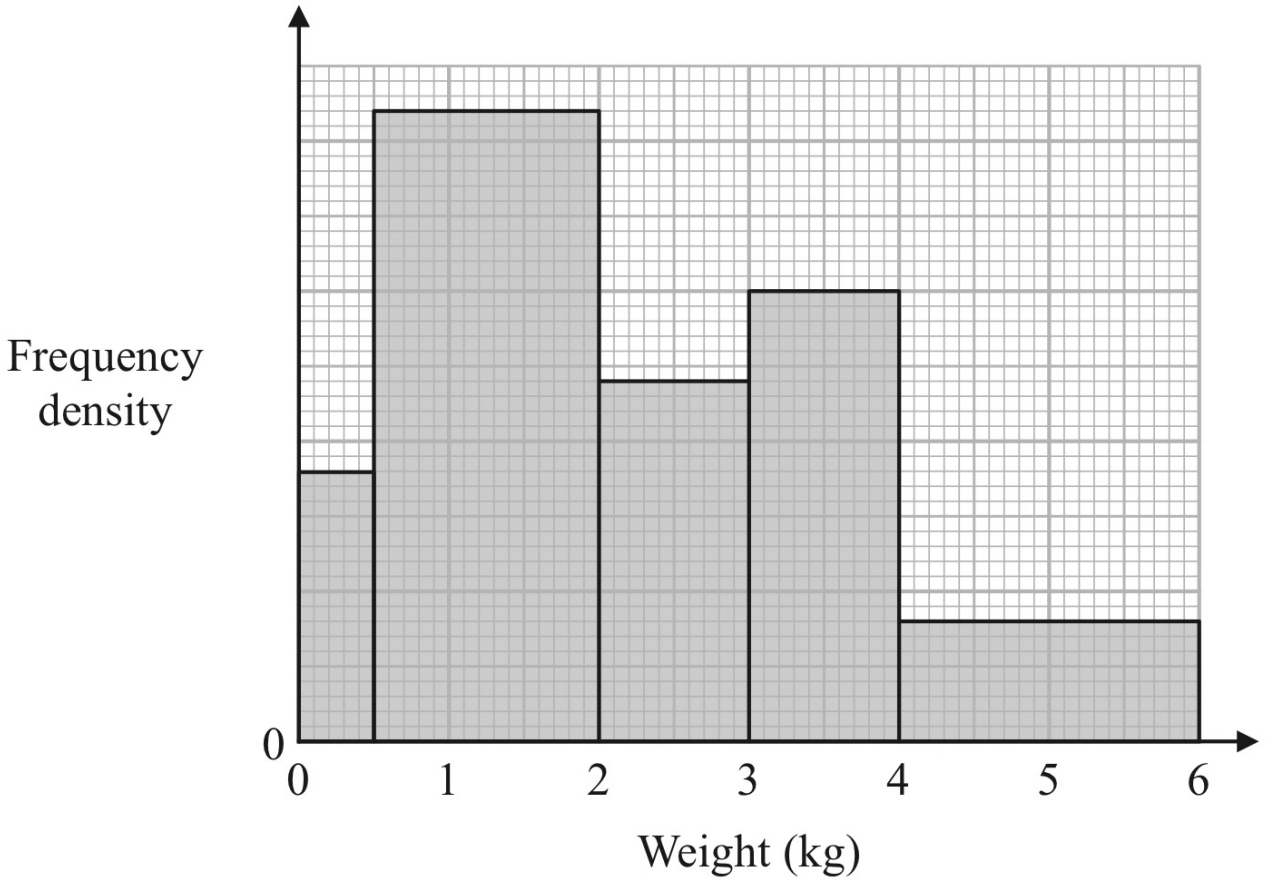
(2)

TOPMaths A* Level

47.

A postman records the weight of each parcel that he delivers.

The histogram shows information about the weights of all the parcels that the postman delivered last Monday. No parcels weighed more than 6 kg.



63 of the parcels that the postman delivered last Monday each had a weight between 0.5 kg and 2 kg.

(a) Work out the total number of parcels the postman delivered last Monday.

.....

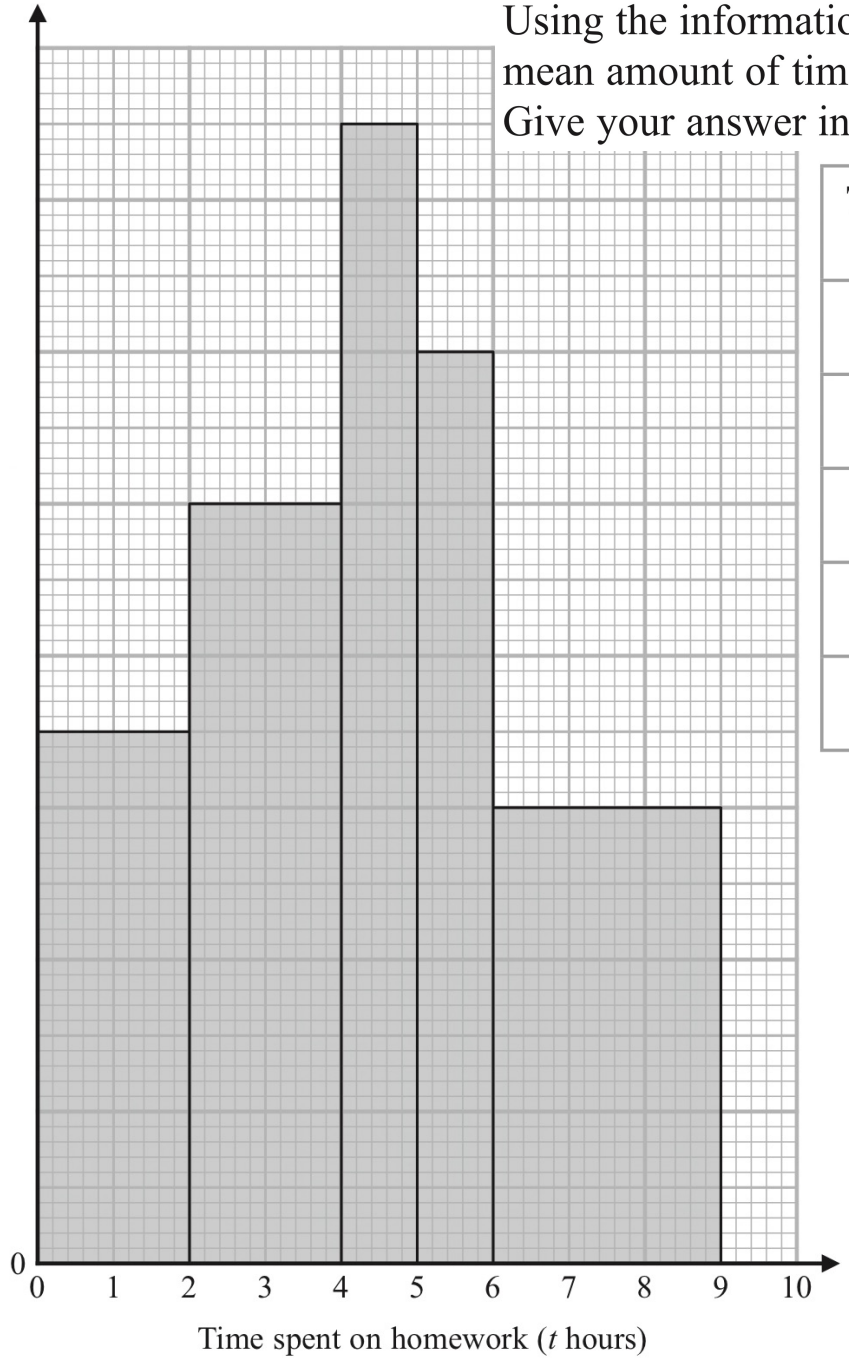
(3)

TOPMaths A* Level

48.

The histogram and the table give some information about the amounts of time, in hours, that Year 11 students at Bergdesh Academy spent, in total, on their homework last week.
No student in Year 11 spent longer than 9 hours on their homework.

Using the information in the histogram and in the table, work out an estimate for the mean amount of time the Year 11 students spent on their homework last week.
Give your answer in hours correct to 3 significant figures.



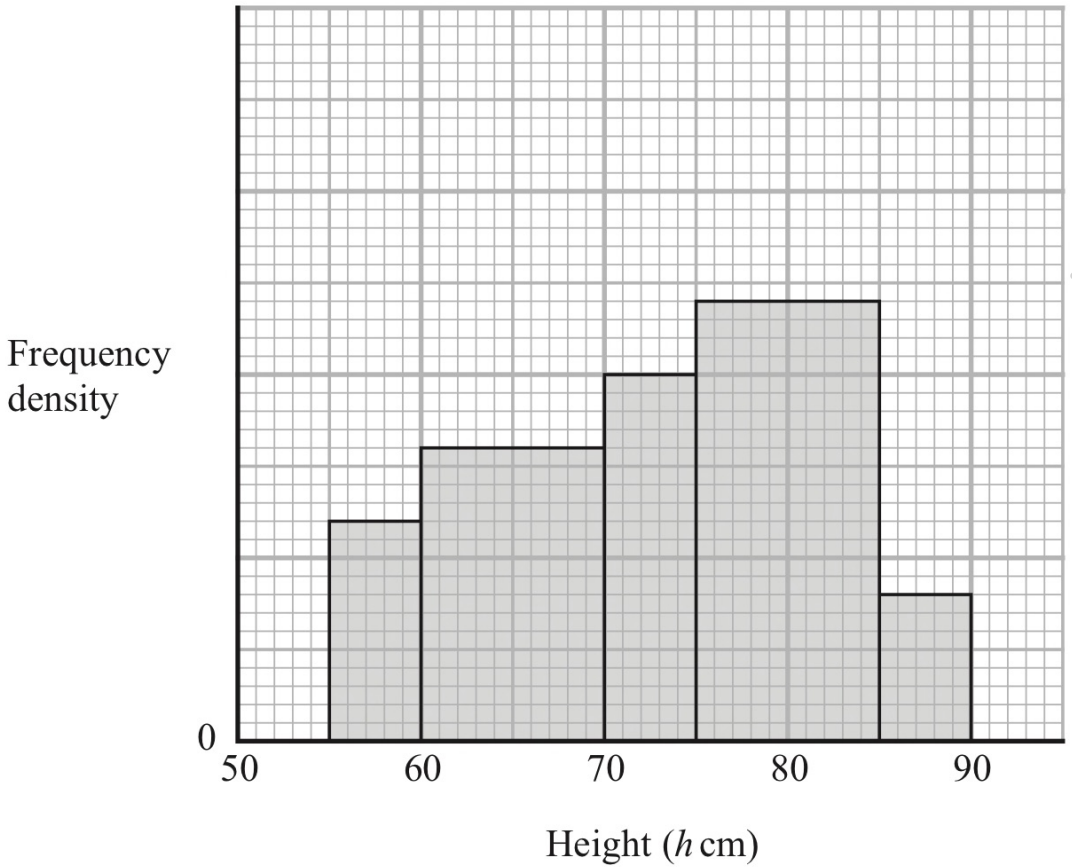
Time spent on homework (t hours)	Frequency
$0 < t \leqslant 2$	28
$2 < t \leqslant 4$	
$4 < t \leqslant 5$	
$5 < t \leqslant 6$	
$6 < t \leqslant 9$	

..... hours

(Total for Question 18 is 5 marks)

TOPMaths
A* Level
49.

The histogram gives information about the heights, h cm, of some tomato plants.



There are 12 tomato plants for which $75 < h \leqslant 85$
One of the tomato plants is selected at random.

Find an estimate for the probability that this tomato plant has a height greater than 82.5 cm

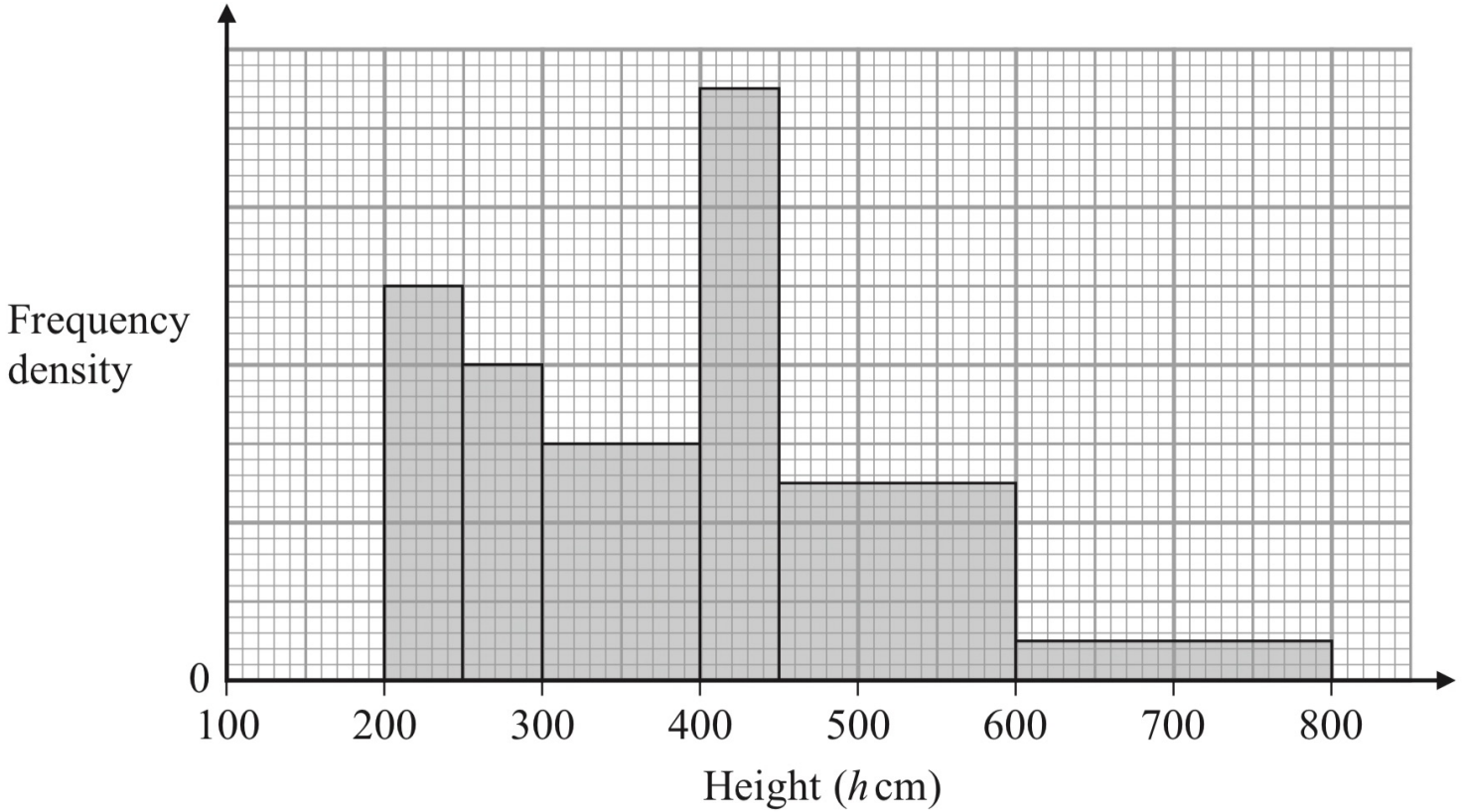
.....

(Total for Question 20 is 4 marks)

TOPMaths A* Level

50.

The histogram gives information about the height, h cm, of each tree in part of a forest.



There are no trees for which $h \leqslant 200$ and for which $h > 800$

The number of trees for which $300 < h \leqslant 400$ is 8 fewer than the number of trees for which $400 < h \leqslant 500$

Work out an estimate for the number of trees in this part of the forest that have a height greater than 500 cm.

.....

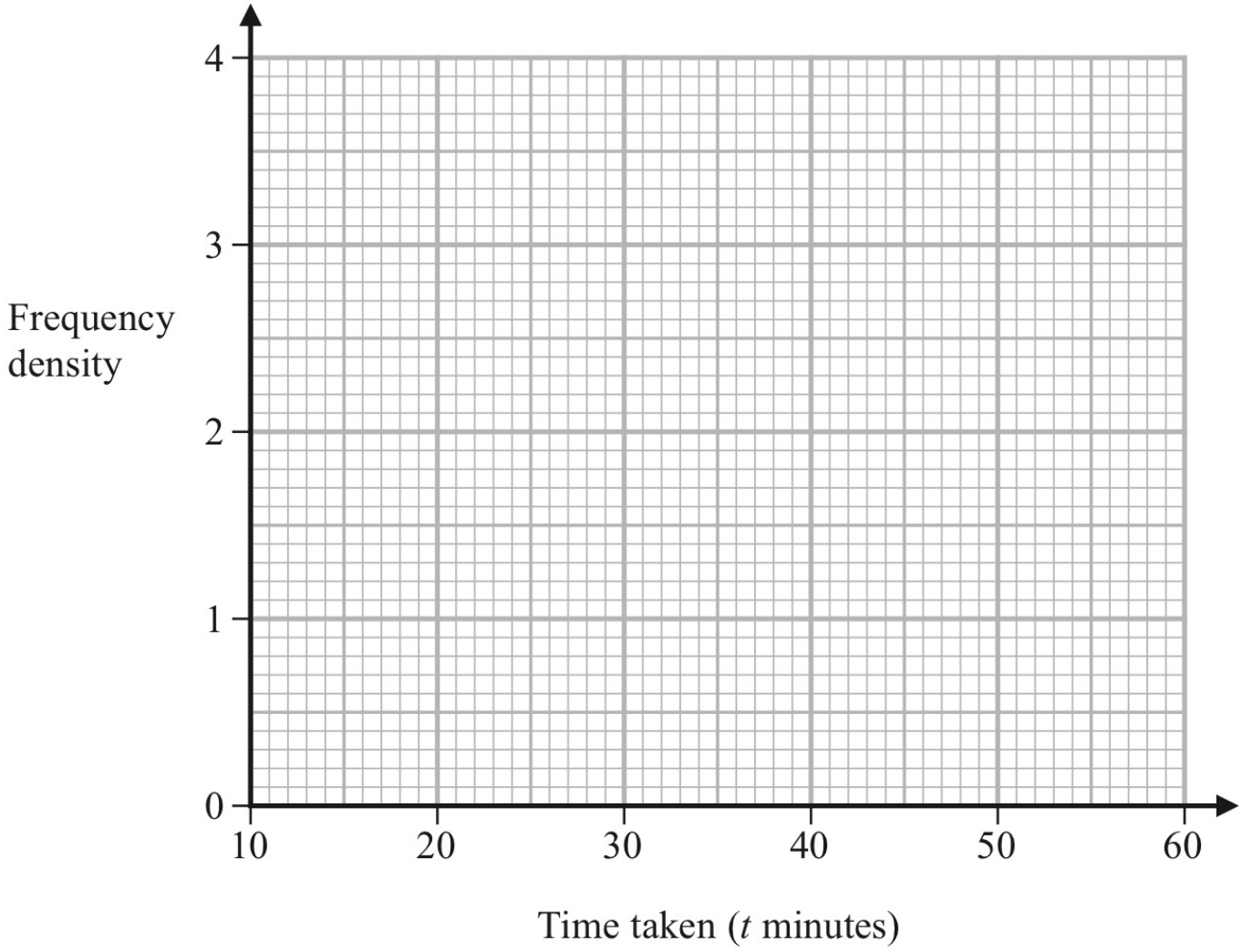
(Total for Question 19 is 3 marks)

TOPMaths
A* Level
51.

The table gives information about the time taken by each student in Year 11 to complete a homework task.

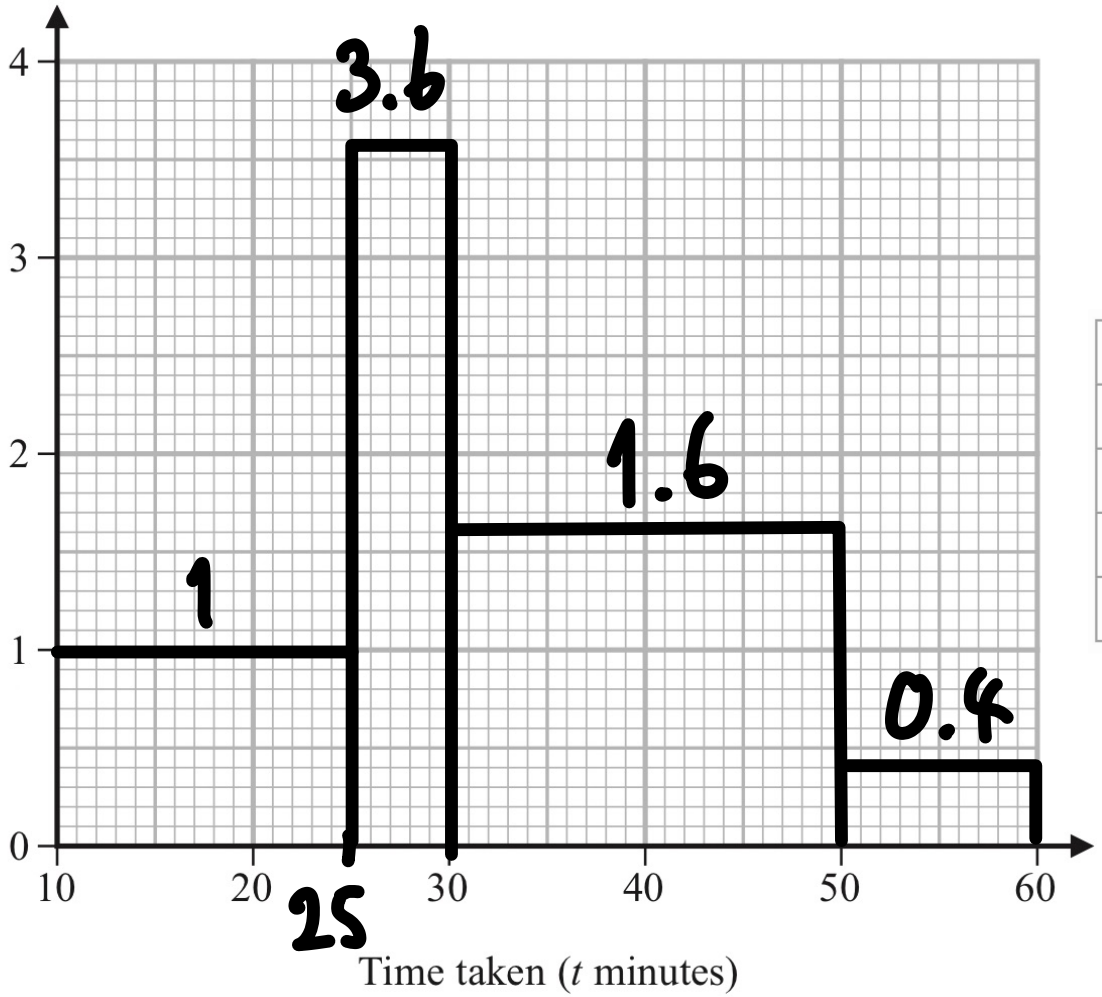
(a) On the grid, draw a histogram for this information. (3)

Time taken (t minutes)	Frequency
$10 < t \leqslant 25$	15
$25 < t \leqslant 30$	18
$30 < t \leqslant 50$	32
$50 < t \leqslant 60$	4



TOPMaths A* Level

51.



Time taken (t minutes)	Frequency
$10 < t \leqslant 25$	15
$25 < t \leqslant 30$	18
$30 < t \leqslant 50$	32
$50 < t \leqslant 60$	4

One of these students who took 50 minutes or less and more than 25 minutes to complete this homework task is chosen at random.

(b) Find an estimate for the probability that this student took 45 minutes or less to complete this homework task.

.....

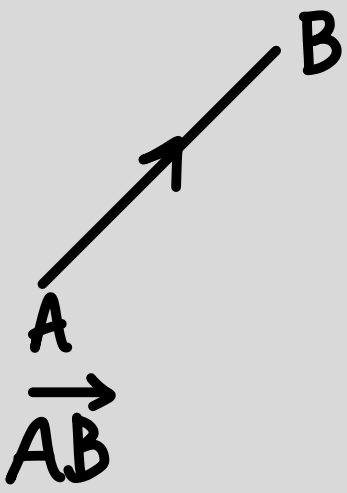
(2)

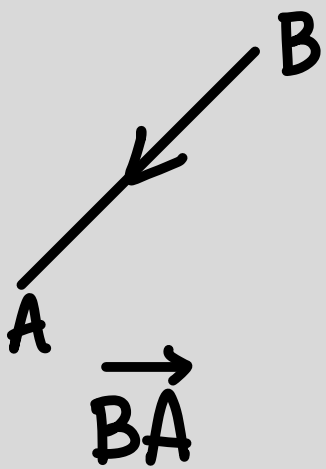
MATHS ONLINE

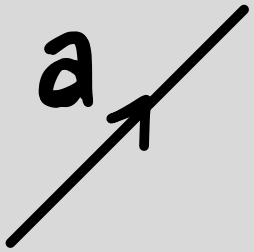
By: Kru Tar

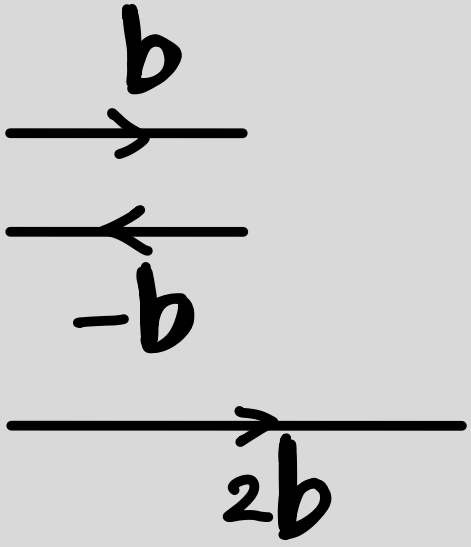
5.4 Vectors

Scalar + Direction

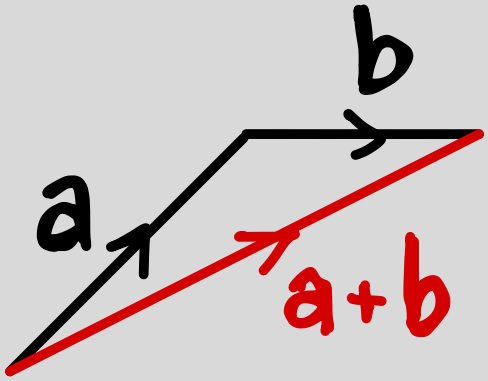




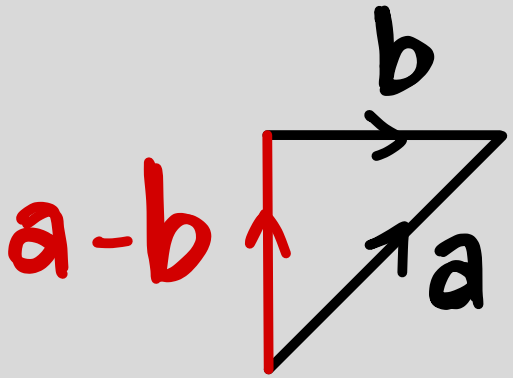




Addition



Subtract



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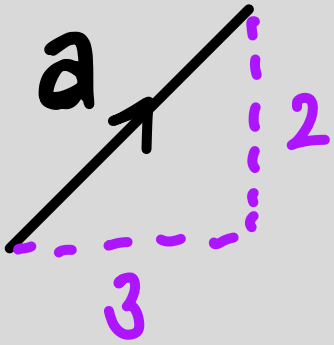
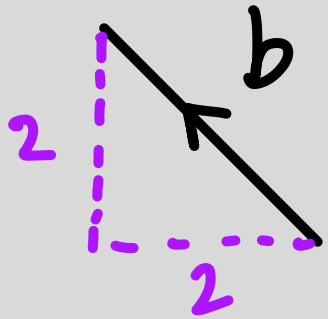
5.4 Vectors

Column vector

$$a = \begin{pmatrix} x \\ y \end{pmatrix}$$



$a =$



$b =$

a) $2a =$

b) $a + b =$

c) $a - 2b =$

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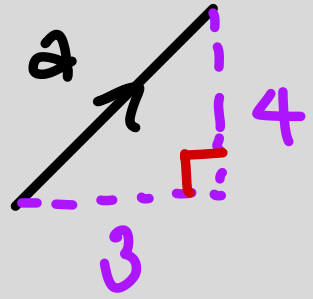
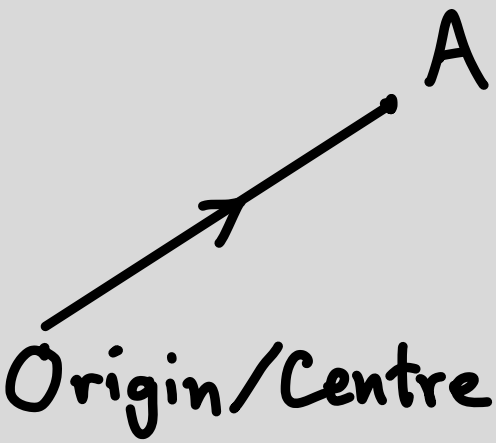
5.4 Vectors

Modulus/Magnitude

$$|\overrightarrow{AB}| \text{ or } |a| = \sqrt{x^2+y^2}$$



Position vector of a point A



Start with the origin/centre.

$$= \overrightarrow{OA}$$

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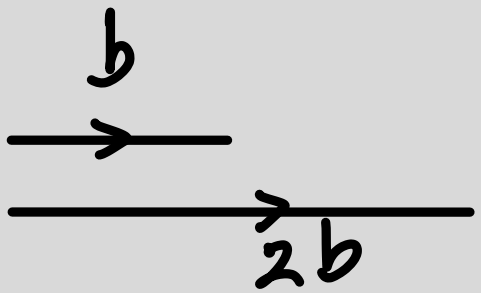
5.4 Vectors

Parallel vectors



$$\mathbf{a} = k\mathbf{b}$$

Vector **a** is a multiple of **b**.

Vector **a** and **b** are parallel.

Ex.

$$\mathbf{a} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$$

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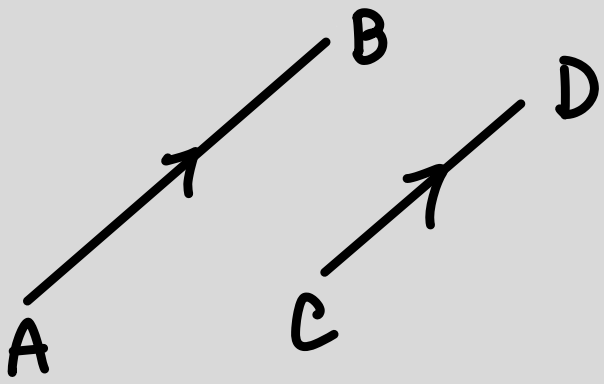
5.4 Vectors

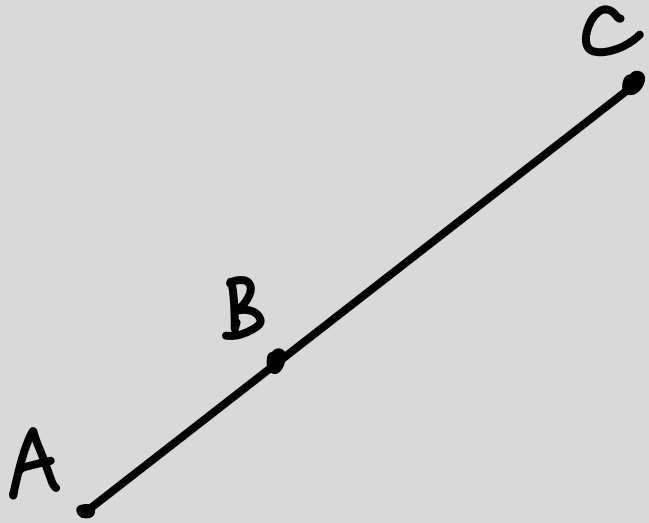
Parallel vectors



$$\overrightarrow{AB} = k\overrightarrow{CD}$$

1) AB and CD are parallel.

2) Length $AB = kCD$



$$\overrightarrow{AB} = k\overrightarrow{BC}$$

B is a common point.

A, B and C lie on the same straight line.

OR A, B and C are collinear.

5.4 Vectors

TOPMaths
A* Level

52.

$ABCD$ is a trapezium.

$\overrightarrow{DC} = 3\overrightarrow{AB}$

$\overrightarrow{DA} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$

Find the exact magnitude of $\overrightarrow{BC}$

(Total for Question 23 is 5 marks)

TOPMaths
A* Level

53.

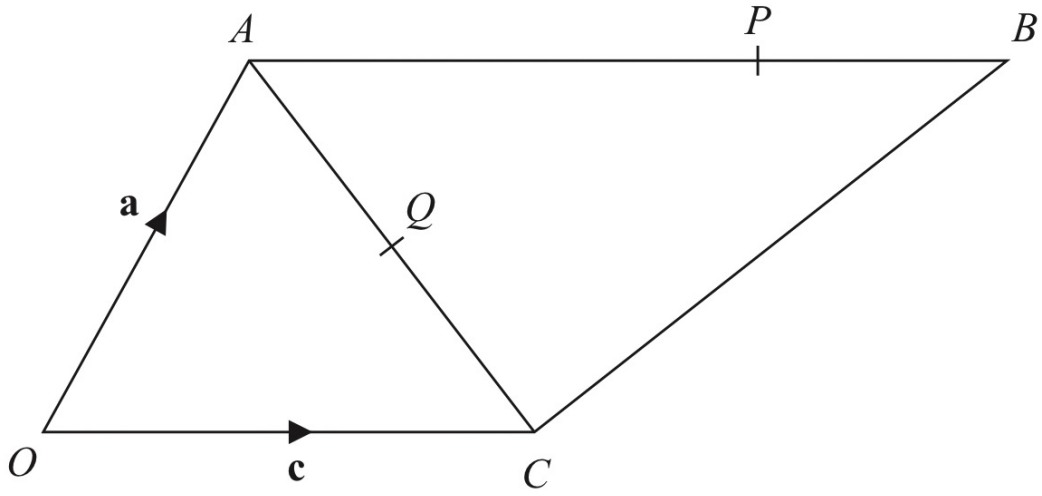


Diagram **NOT** accurately drawn

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OC} = \mathbf{c}$ $\overrightarrow{AB} = 2\mathbf{c}$

P is the point on AB such that $AP : PB = 3 : 1$
Q is the point on AC such that OQP is a straight line.

Use a vector method to find $AQ : QC$
Show your working clearly.

$AQ : QC =$ .....

(Total for Question 24 is 5 marks)